# Teaching numerical methods or teaching with numerical methods?

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ABSTRACT: This paper presents some of the results of a collaboration between TC103 (numerical methods) and TC306 (geoengineering education) of ISSMGE with the objective of producing high quality educational material to be shared with educators in form of figures or videos. The project stems from the idea that figures, charts and videos obtained with the use of numerical tools can enrich teaching of key concepts of geomechanics in introductory courses. Examples are visual resources obtained using DEM to illustrate micro-phenomena and the particulate nature of soils and its consequences, or using FEM/FDM to illustrate macro-phenomena such as stresses under loaded areas or bearing capacity of a foundation. Figures are submitted with metadata useful to guide the use of the material even by educators that are not experts in numerical methods. However, this raises a fundamental question: how detailed should be student's (and educator's) knowledge on numerical methods to fully understand (and use) the material? This paper discusses this issue showing how the same material, if correctly prepared, can be used with different purposes both in base and advanced courses. Pilot examples are illustrated with specific emphasis on stress under loaded areas.

*Keywords: Numerical Methods, Stress bulbs, Stress visualization, FEM*

## 1 Introduction

Figures and videos are essential to explain fundamental concepts of soil mechanics. YouTube seems to abound of educational videos and a number of websites shares figures, Power Point slides and web-app for students. However, a recent study revealed that the majority of teachers in geotechnical engineering are not satisfied with this educational material (Pantazidou & Calvello, 2023). The survey revealed that 52% of instructors were not satisfied with the materials they currently used, and 45% were not satisfied with materials found through searches on the web. Respondents expressed a significant need for materials to be used both in lectures and to engage students outside class, with case studies and laboratory-related materials being the most desired, but there is also a significant desire of photographs and videos. While no one topic stood out clearly above all others, educational material for foundation topics, particularly bearing capacity and stress distribution underneath loaded areas, will be useful to a good percentage of geotechnical engineering instructors.

Building on this, a collaborative initiative was launched between ISSMGE (International Society for Soil Mechanics and Geotechnical Engineering) technical committees TC306 on Geoenginering Education and TC103 on Numerical Methods to address these gaps, with the objective of developing peer-reviewed, high quality, and sharable educational material by means of numerical methods. Specifically, numerical methods, such as the Discrete Element Method (DEM), Finite Element Method (FEM), and Material Point Method (MPM), may be used to create short videos, animations, effective figures or hands-on spreadsheets to help visualize complex concepts and enhance student understanding. All the material will be prepared with accompanying metadata and usage guidelines to ensure clarity and usability. These resources will guide instructors, especially those unfamiliar with numerical methods, how to effectively incorporate the materials into their teaching.

This initiative brings a fundamental question: can we use this material without knowing and teaching the basics of these numerical methods? In base courses, numerical methods can be used as complementary tools to enhance students' understanding of fundamental concepts through improved visualizations, which requires only a very limited knowledge of the mathematical details of the applied numerical method. The accompanying metadata will support instructors in this process. In advanced courses, the teacher can discuss with more details the features of the models and provide assignments that require the implementation or modification of the model. In both cases, it should be avoided to present numerical methods as replacements for established theories and methods.

A preliminary survey within the members of TC103 and TC306 explored the availability of ready-to-use material that could be immediately shared, but many instructors indicated that they had prepared Excel files or scripts for their classes, but admitted that these were not ready to be shared or used by others. This confirms the private nature of teaching, and that we are still struggling to create an engaged community of teachers who publicize their educational material and take the responsibility of judging the educational material of their colleagues.

Another notable feature of this initiative is the introduction of a formal peer-review phase, novelty in this context. Additionally, the reviewed and finalized materials will be shared on the ISSMGE platform dedicated to peer-reviewed educational resources (https://www.issmge.org/education/peer-reviewed-educational-materials), marking a significant step toward elevating the quality and reliability of shared teaching materials.

The working group is still in the early stages of this effort and has chosen to first focus on small, manageable tasks such as the preparation of static figures or spreadsheets. An initial review of available resources from TC103 members indicated that most existing materials were tailored for advanced courses, with students having a relatively strong background on numerical methods or in the process of developing their numerical skills. These instructors want to keep it for this purpose, although they are available to adapt it for basic soil mechanic courses. Consequently, this project has evolved to focus on creating versatile materials suitable for both basic and advanced courses, i.e. to teach *with* numerical methods and to teach numerical methods.

By providing ready-to-use, high-quality materials, this initiative aims to ease the burden on instructors who face time and resource constraints, while also ensuring that the materials are fully reviewed and ready for broad use. This paper explores the challenges and opportunities of developing such materials and discusses how they can be leveraged to enhance geotechnical engineering education at all levels.

## 2 Methodology

The work is organized in 4 phases carried out in synergy by the task force of members of TC306 and TC103 as show in Figure 1.

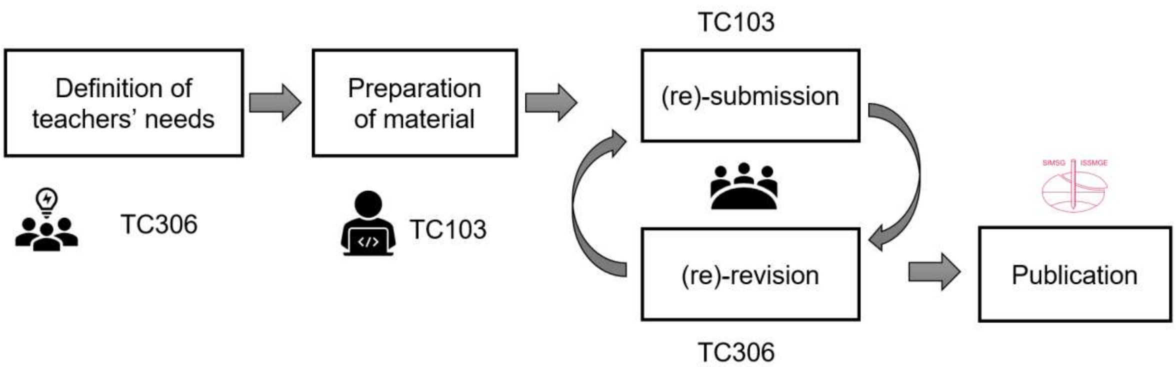


**Figure 1. Work flow of the joint TC103-306 collaboration**

The first phase of this work involved defining the teachers' needs. This was initially done within the members of TC306, which produced a list of very specific requirements. The teachers' needs should be specific, because generic requirements may generate uncertainty in the work and result in unsatisfactory material. Instead, they must be expressed in a detailed manner, with clear indications of the pedagogical objectives. Most of these needs can be addressed with numerical methods, but in this paper, only a pilot case, specifically the visualization of stresses under loaded areas, is discussed in detail. For the considered pilot case, the teacher's need is "*Stress distributions underneath loaded areas – convince my students and myself that the theory of elasticity performs well: I would like to see how the results would differ if the stress distribution was calculated with something more suitable for soil than the theory of elasticity. I want to see with my own eyes and show students that the difference is indeed small. Also, I would like to provide examples of when the differences are not negligible*."

The second phase consists in preparing the educational material. For the pilot case, the analytical solution of the Boussinesq theory is implemented in MATLAB to draw colour figures to better visualize the stress bulbs compared to black-and-white textbook figures. This is an analytical solution based on the assumption of a homogeneous elastic half-space. Equations and charts for several types of flexible surface loads based on the above assumptions are available in many geotechnical textbooks. The set of figures prepared for this pilot case focuses on the load increment generated by a strip load. A strip load is the load transmitted by a structure of finite width and infinite length on a soil surface.

The analytical solution for this case provides the following equations for vertical stress, horizontal stress and shear stress at a point in the homogeneous elastic half-space, with coordinates (x,z):

$$\Delta\sigma_z = \frac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\beta)] \tag{1}$$

$$\Delta\sigma_x = \frac{q}{\pi}[\alpha - \sin\alpha\cos(\alpha + 2\beta)] \tag{2}$$

$$\Delta\tau_{zx} = \frac{q}{\pi}[\sin\alpha\cos(\alpha + 2\beta)] \tag{3}$$

q is the applied load per unit of area and the geometric quantities and sign convention are defined in Figure 2.

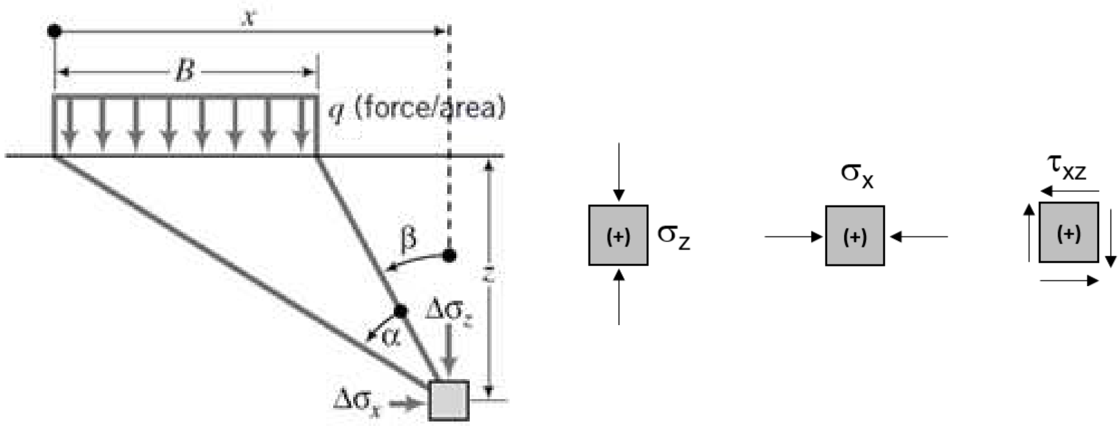


**Figure 2. Definitions for the calculation of stress increments due to a strip load at a point in the homogeneous elastic half-space with coordinates (x, z), a and b are positive counterclockwise**

These equations are implemented in Matlab to prepare three contour figures of normalized stress increments ($\Delta\sigma_z/q$, $\Delta\sigma_x/q$, $\Delta\tau_{zx}/q$) with normalized coordinates (X/B, Z/B) with origin in the centre of the loaded area. Results are also exported in a normalized format to Excel, which instructors can use to produce additional figures for their classes or provide assignments to students.

The same problem is then solved with the Finite Element Method (FEM). To produce high-quality figures, the software must be flexible enough to adapt the rendering of the results, and many commercial software packages lack this flexibility, thus choosing the right tool is important and could be time-consuming. For this study, GeoStudio 2024 is used. The 2D plane strain problem is simulated considering 6 different models where geometry, material constitutive model and material parameters are varied as follows (Table 1):

- Model 1 serves as a benchmark for analytical comparison, featuring a 100 kPa load on a 1 m-wide strip with a linear elastic material and no gravity.
- Models 2 and 3 introduce gravity and a wider 2 m strip load. Both use a linear elastic material, but with different values of Young Modulus (E). Model 2 represents a medium dense sand, while Model 3 simulates soft clay.
- Model 4 incorporates elasto-plastic behaviour using the Mohr-Coulomb failure criterion.

- Models 5 and 6 implement the Modified Cam Clay model with the load applied at the surface or at the bottom of a 0.5m-deep excavation.

The output visualization is adjusted within the software to obtain a figure that can be easily compared with the one produced in MATLAB or data are extracted to produce graphs depending on the purpose of the model. Finally, the figures are attached to a PowerPoint slide, and text or shapes are added to improve the clarity of the presentation.

A word file with metadata is prepared to improve the usability of the material. For the time being, this word file contains the following information:

- A brief title and a descriptive figure title,
- Soil mechanics issue/concept,
- Course chapter/topic,
- Material type,
- Brief articulation of figures' main idea,
- Name and contact of the author,
- Additional material (including detailed explanation of figures and suggestions for instructors),
- License of use.

However, the structure of this document is still under discussion and will be improved in the future.

The third phase is the review phase, in which members of TC306 submitted their recommendations to improve the educational material. Finally, the educational material is revised and resubmitted until a final version is obtained. Note that this process is commonly used for research papers, but it is rarely applied for educational material that are produced in the context of a research project.

The last phase is the publication for the educational material on the ISSMGE website (https://www.issmge.org/education/peer-reviewed-educational-materials).

**Table 1. Features of the numerical models implemented**

| Feature | Model 1 (Analytical Comparison) | Model 2 (Elastic - Medium Dense Sand) | Model 3 (Elastic - Soft Clay) | Model 4 (Mohr-Coulomb - Medium Dense Sand) | Model 5 (Modified Cam Clay) | Model 6 (Modified Cam Clay - Excavation) |
|---|---|---|---|---|---|---|
| Load Entity | 100 kPa | 100 kPa | 100 kPa | 100 kPa | 100 kPa | 100 kPa |
| Load Width | 1 m | 2 m | 2 m | 2 m | 2 m | 2 m (applied at excavation bottom) |
| Material Model | Linear Elastic | Linear Elastic | Linear Elastic | Elastic-Perfectly Plastic (Mohr-Coulomb) | Modified Cam Clay | Modified Cam Clay |
| Material parameters | E=30000 kPa, $\nu$=0.3 | E=30000 kPa $\nu$=0.3 | E=1000 kPa $\nu$=0.3 | E=30000 kPa $\nu$=0.3 $\varphi$=35° | Table 2 | Table 2 |
| Soil Unit Weight | 0 | 20 kN/m³ | 20 kN/m³ | 20 kN/m³ | 20 kN/m³ | 20 kN/m³ |
| Notes | No gravity, used for comparison with analytical solution | Simulates medium dense sand | Simulates soft clay | Shear failure mechanism included in the model | This model has convergence issues | Load applied at excavation bottom |

**Table 2. Material parameters and initial state parameters of Modified Cam Clay model. *the slope of the critical state line in the p′-q plane can be calculated as $M = 6\frac{sin(\varphi)}{3-sin(\varphi)} = 1.11$**

| Parameter | | Value |
|---|---|---|
| Initial void ratio [-] | $e_0$ | 1.0 |
| Overconsolidatio Ratio [-] | OCR | 2.0 |
| Poisson ratio [-] | $\nu$ | 0.3 |
| Slope of critical state line [-] | $\lambda$ | 0.32 |
| Slope of loading reloading line [-] | $\kappa$ | 0.07 |
| Unit weigth [kN/m$^3$] | $\gamma$ | 20 |
| Friction angle [°]* | $\varphi$ | 28 |

## 3 Results of the pilot case

The process presented in the previous section generated the following material, which can be downloaded (https://researchdata.cab.unipd.it/id/eprint/1549) as supplementary material of this paper:

- Contour figures of normalized stress increments ($\Delta\sigma_z/q$, $\Delta\sigma_x/q, \Delta\tau_{zx}/q$) with normalized coordinates (X/B, Z/B) with origin in the centre of the loaded area calculated analytically. These are in .fig and .jpg format. Files in .fig can only be opened in MatLab by teachers familiar with this program and offer the opportunity to modify or interact with the figures.
- Excel sheets with normalized stress increments ($\Delta\sigma_z/q$, $\Delta\sigma_x/q, \Delta\tau_{zx}/q$) with normalized coordinates (X/B, Z/B). Teachers can use it to create additional figures for their classes or provide assignments to students.
- Contour figures of vertical stresses and displacement obtained with FEM using different material parameters and constitutive models.
- PowerPoint slides with the analytical and numerical solutions. These slides can be opened in PowerPoint and modified by the teacher or used as is in their classes.
- A Word file with metadata and instructions for users.
- GeoStudio models. Teachers familiar with the program can inspect and modify them or use to give assignments to students.

This material offers multiple teaching possibilities in both base and advanced courses. In basic courses, teachers can show colour figures of stress under loaded areas instead of classic black-and-white pressure bulbs (Figure 3). Distribution of horizontal, shear, mean and deviatoric stresses are also provided, which are rarely shown in textbooks, to offer a more comprehensive understanding of the problem. The Excel file with normalized stress increments can be used to ask students to plot stress variation with distance in vertical or horizontal direction and explore the effect of the size of the loaded area and load magnitude. Students may be asked to implement a similar spreadsheet or a code as an exercise, most undergraduate students certainly have the knowledge to do so.

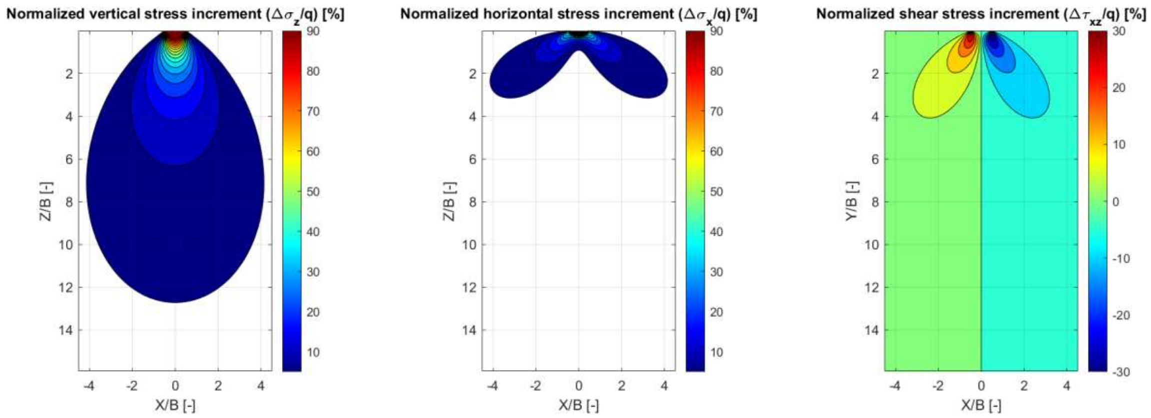


**Figure 3. Contour of the analytical solution to calculate normalized stress increments ($\Delta\sigma_z/q$, $\Delta\sigma_x/q$, $\Delta\tau_{zx}/q$) with normalized coordinates (X/B, Z/B) (Model 1 in Table 1). Note that corrdinate X is translated with respect to coordinate x used in Equations 1-3 and Figure 2**

In order to present the comparison between the analytical solution and the finite element solution, students should be familiar with FEM. In many bachelor courses, students become familiar with FEM before or during the same period of the soil mechanics course, thus the teacher can exploit this knowledge to further discuss the similarities and differences between the two approaches. The author of this paper suggests to avoid the presentation of FEM figures to students that do not have any basic knowledge of the method or to do it with care in order to avoid that this method is perceived as superior, more realistic and always preferable to analytic solutions. Indeed, the quality of FEM results depends on the ability of the engineer to define correctly the model, i.e., geometry, boundary conditions, material parameters, calculation parameters. As an example, for the considered case, if the model boundaries are too close to the loaded area, the results are not in agreement with the analytical solution (Figure 4). It is good to encourage students to compare results of FEM with analytical solutions, discuss differences and similarities, advantages and limitations.

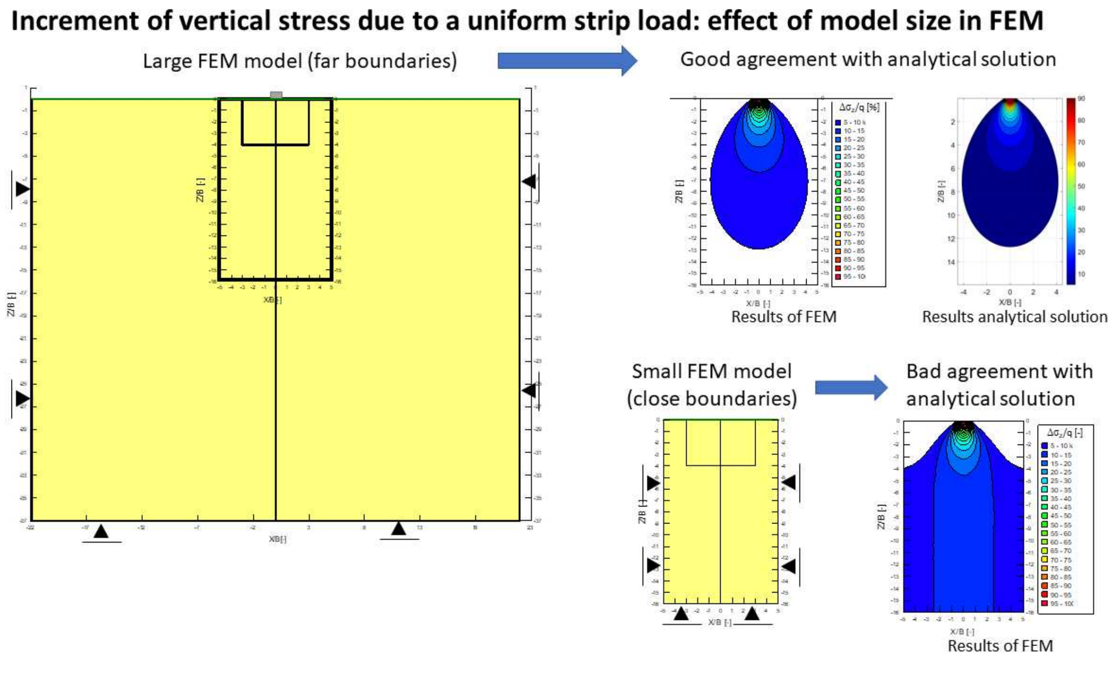


**Figure 4. Effect of distance of model boundaries on FEM results (Model 1 in Table 1). High quality figure can be downloaded from https://researchdata.cab.unipd.it/id/eprint/1549**

While the analytical solution is developed under the hypothesis of homogenous, isotropic, linear elastic half-space, FEM can be used to explore many different conditions such as stratified soil, the effect of soil constitutive model, material parameters, type of load (e.g. non uniform, inclined, applied on rigid or semi-rigid foundation). Teachers of advanced courses can certainly assign this task to their students. This pilot case explores the effect of Young modulus and constitutive model, in particular the use of an elastic-perfectly plastic (EP) model with Mohr-Coulomb failure criterion and the Modified Cam Clay model (MCC).

The results that illustrate the effect of Young modulus are presented in terms of vertical stress increment and vertical displacement with depth below the centreline and surface displacement (Figure 5). Students should grasp that even if the stress increment is identical, the displacements are proportional to soil compressibility and this relationship is linear because of the assumption of linear elasticity. These results may seem obvious and trivial to students; thus, the teacher should encourage a discussion in the room before showing the results, with special attention to practical implications.

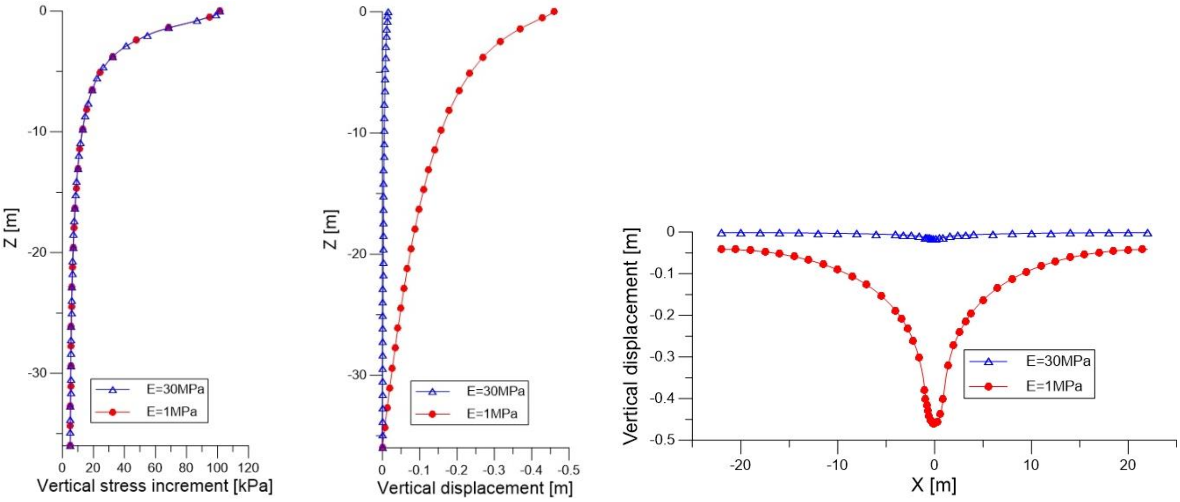


**Figure 5. Effect of Young modulus on (a) vertical stress increment at the centerline, (b) vertical displacement at the centerline, (c) vertical displacement at the surface (Model 2 and Model 3 in Table 1)**

Elastic perfectly plastic models with Mohr-Coulomb failure criterion are often adopted to simulate the behaviour of coarse grained soils and they may be perceived as an enhancement of linear elasticity. However, in calculating the settlements of shallow foundations, since the load applied for settlement calculation is typically much smaller than the ultimate bearing capacity of the foundation (we often use safety factors between 2 and 3), the soil behaviour is still elastic and the displacements obtained with the two models are identical. This fundamental understanding of soil behaviour and the difference between serviceability limit state (SLS) and ultimate limit state (ULS) is an important aspect that student should learn. To convince students, the teacher can show the vertical displacements calculated with FEM for linear elastic (Model 2 of Table 1) and elastic-perfectly plastic model (Model 4 of Table 1). At this point, the teacher should ask if this constitutive model is realistic and what are the limitations of the assumptions. The discussion in class is certainly useful both in basic and advanced coursed, although the level of complexity should be adapted to the background of the students.

Soil behaviour, especially in soft clays, is not linear elastic, thus a model including non-linear compressibility should be used. A possibility is the use of Modified Cam Clay (MCC) (Model 5 in Table 1), which is often included in many soil mechanic courses. The replacement of the MCC model in the previous FEM model leads to non-convergence of the results because at low confining stress the stiffness is too small. The teacher can discuss with students what could be the possible solutions, for example: consider an excavation, consider the presence of a dry crust of stiffer soil at the surface. In this pilot case, the excavation is implemented (Model 6 in Table 1). Now the model converges, and the results can be compared with the ones obtained with the Linear Elastic model and Young modulus 1000 kPa (Figure 6). Students should realize that, even if the distribution of vertical stresses is similar in the two cases, due to non-linear soil compressibility, the maximum displacement is higher with the MCC model, but the extent of displacement is much more limited, i.e. with the MCC model the vertical displacements are concentrated closer to the foundation (Figure 7). A discussion on the practical implications of these observations is encouraged.

In basic courses these results can be instrumental to improve the fundamental understanding of soil behaviour under loaded areas and the consequences for practical applications. In advanced courses, these slides may serve as examples, and then students may be encouraged to implement their own FEM codes or to use a commercial software to analyse the same problem and then compare the results with the analytical solution. Several variations can be assigned in terms of geometries, boundary conditions and constitutive models.

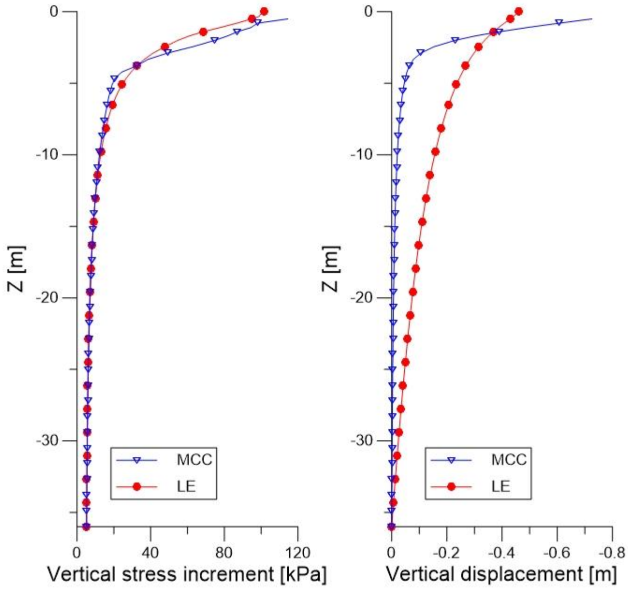


**Figure 6. Comparison between linear elasticity (Model 3 in Table 1) and Modified Cam Clay (Model 6 in Table 1): Vertical stress increment and vertical displacement with depth at the centerline**

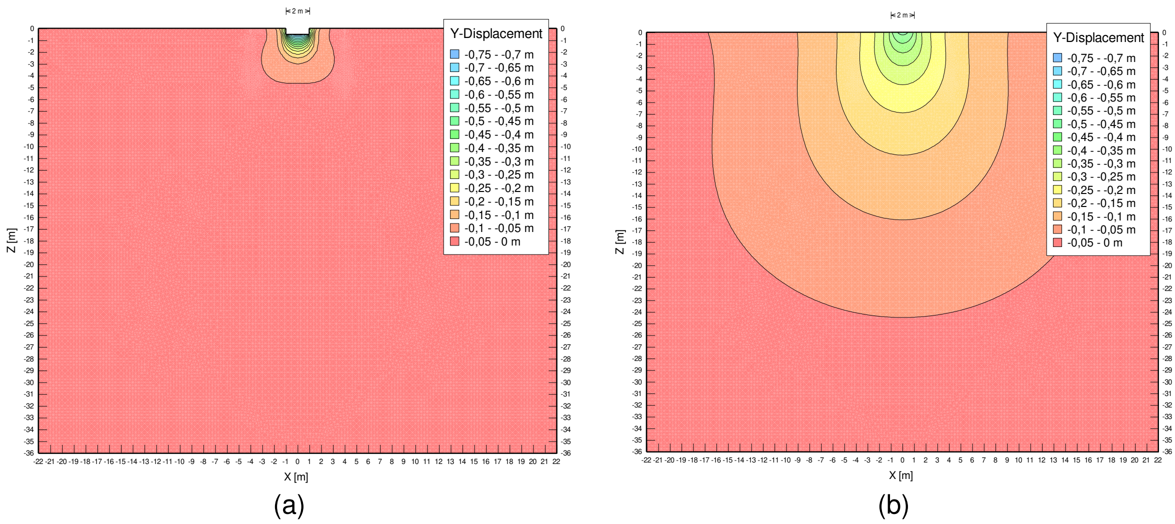


**Figure 7. Vertical displacement contour for soft clay modelled with (a) Modified Cam Clay (Model 6 in Table 1), and (b) linear elasticity (Model 3 in Table 1)**

## 4 Concluding remarks

This paper presents a collaborative initiative between TC306 (Geoengineering Education) and TC103 (Numerical Methods) of ISSMGE to develop high-quality, peer-reviewed educational materials using numerical methods. The pilot case discussed in this paper demonstrates how numerical methods can be used to create effective figures, spreadsheets, and presentations to illustrate fundamental concepts in soil mechanics, such as stress distributions under loaded areas.

While numerical methods can be powerful tools to visualize and better understand complex phenomena, it is crucial for teachers to avoid giving students the impression that numerical models are inherently superior to other solutions. Every model has its limitations, and as models become more advanced, the complexity of input parameters and modelling assumptions increases. Modelers should exercise caution and critically evaluate the appropriateness and limitations of the chosen model for the specific problem at hand.

It is important to note that while numerical methods can provide valuable insights and visualizations, they should be used judiciously and in conjunction with analytical solutions and theoretical foundations. The goal should be to leverage numerical methods as complementary tools to enhance students' understanding of fundamental concepts, rather than presenting them as replacements for established theories and methods.

Preparing high-quality educational materials requires a significant effort, involving coding, model setup, post-processing, and documentation. As an example, for the author of this paper, implementing the MATLAB code required approximately 1.5 hours, implementing and calculating all the FEM models took less than 2 hours, and preparing the output visualization, figures, charts and slides required more than 10 hours, writing the documentation required about 5 hours. This amount of time for a single topic is rarely available to a teacher. However, undertaking this task as a community can help mitigate the solitude often experienced in teaching and improve students' learning. By fostering collaboration and peer-review processes within the geotechnical engineering education community, the burden of material development can be shared, and the quality of the resulting resources can be enhanced.

## References

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# Author’s bio

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Francesca Ceccato is associate professor at the University of Padua (Italy), where she currently teaches “Geological and Geotechnical Engineering” and “Numerical Modelling in Geomechanics” in the master programs of Water and Geological Risk Engineering and Civil Engineering. She has also been involved in the international teaching program Engineering for Africa within the cooperation between the University of Padua (Italy) and the Ecole National the Traveaux Publiqe (Camerun). She is the secretary of TC103-Numerical methods of ISSMGE, and member of TC-306-Geoengineering education. Her main research topics include numerical modelling in geomechanics, with particular interest in the Material Point Method (MPM), assessment of river embankment vulnerability, soil characterization and the preservation of historical heritage, in particular the study of ancient foundations of the city of Venice. She is member of the core developer team of the open-source code Anura3D, based on MPM, and participates intensively in the activities of the Anura3D MPM research community.