Effect of Variation of Geotechnical Properties on Stability of Retaining Walls: a Case Study in Japan

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ABSTRACT

In the present paper, the effects of variation of geotechnical properties on stability of a series of 54 retaining walls in Hodogaya Ward and Naka Ward of the Yokohama municipality area in Japan are analyzed for various earthquake conditions. The probabilistic and sensitivity analysis are carried out by Monte Carlo Simulation and *F*-test analysis respectively. It is observed that most of the retaining walls have probability of failure (P_f) between 40-60% for overturning and 10-40% for sliding modes of failure. A range of probabilistic risk factors (R_f), which simultaneously identifies the effects of P_f and also the sensitivity of geotechnical random variables on different failure modes, is proposed for different earthquake conditions. It is observed that internal angle of friction of the backfill is less sensitive to P_f compared to cohesion (c_2) of foundation soil. For PGA > 400 cm/s^2 and variation of c_2> 10%, walls having height more than 3m require a base width almost twice the height of the wall. The proposed procedure in this study can be applied in the design of retaining walls in other parts of Tokyo area as well, for similar subsurface and backfill conditions.

INTRODUCTION

Uncertainty is evident in almost every field of engineering, and geotechnical engineering is no exception. Natural soils are heterogeneous and anisotropic due to their composition and complex depositional processes. The increasing frequency of landslides and failures of earth structures and their adverse impact have led the geotechnical engineers to recognize the importance of probabilistic approaches for analysis of geotechnical structures. Conventional design of geotechnical structures is based on limit equilibrium methods (*LEM*) and on the concept of Factor of Safety (*FS*). This method holds good only when the input parameters, namely, engineering properties of soil, location of ground water table and loading conditions etc., required for design can be accurately assessed. But variation of site data from the estimated value is more common than the exception. The concept of reliability analysis is a well-established mathematical approach to account for these uncertainties of field variables.

A number of approaches are developed through years for assessing reliability of geotechnical structures in terms of reliability index (β) and probability of failure (P_f) (Harr, 1984; Kulhawy, 1992; Duncan, 2000; Hasofer and Lind, 1974; Baecher and Christian, 2003). A few approaches for evaluating sensitivity of random variables on overall failures of structures have also been reported in past research works (Frey and Patil, 2002; Saltelli et al., 1999). In geotechnical engineering, the differential analysis method (local method) for sensitivity analysis has been applied (Babu and Basha, 2008a, 2008b) to estimate the sensitivity of geotechnical random variables on failure probabilities of different structures.

Previous research works indicate that the use of partial safety factors for different geotechnical variables results economises the design compared to applying a lumped factor of safety to the structure as a whole. However, the uncertain nature of soil properties may vary these partial safety factors depending upon the amount of uncertainties, which may result in under-estimation or over-estimation of various parameters/ element dimensions. Limited approaches are available in literature to generalize this "partial factor of safety" based upon variations of different geotechnical random variables. Quite a number of different approaches are available to determine the failure probability (P_f) for variation of the random variables and assess the severity of random variables on different modes of failure. Contrarily, design approach incorporating both the effects has not yet been adequately addressed. The present study adopts a design methodology, which combines the failure probability (P_f) for variation of random variables and sensitivity of random variables (S) on different modes of failure, thereby producing more efficient and economical design.

Guharay *et al.* (2015), Guharay and Baidya (2016) formulated a new factor called the probabilistic risk factor (R_f) by combining mathematically the probability of failure (P_f) of different potential failure modes and sensitivity (S) of geotechnical random variables on each of these failure modes. The authors have applied this probabilistic risk factor based approach to a number of typical geotechnical earth structures such as cantilever retaining wall (GuhaRay *et al.*, 2014), gravity retaining wall (GuhaRay and Baidya, 2012) and sheet pile walls (GuhaRay and Baidya, 2015). The primary objective of the present paper is to apply the above methodology to failure of a series of 54 gravity retaining walls subjected to earthquake loading in Tokyo Metropolitan and Yokohama Municipality area in Tokyo, Japan. The present paper extensively studies the effect of variation of geotechnical random variables on failure and recommends widths required for safe working of the retaining walls based on different wall heights and earthquake conditions.

FORMULATION OF PROBABILISTIC RISK FACTOR

For the probabilistic analysis of the geotechnical structures, the performance function is defined as $g_i(x) = (FS)_i - 1$, where i = different failure modes. $g_i(x) < 0$ implies failure conditions. In the present study, P_f is determined by Monte Carlo Simulation by coding the limit equilibrium equations in MATLAB R2015a (The Mathworks, Inc., Naticks, MA, US) by generating 50,000 data points. A convergence study is carried out to fix the number of data points. For sensitivity analysis, the ANOVA F-test (Saltelli *et al*., 1999) is used to quantify the sensitivity of the input geotechnical parameters on the output performance function. The probabilistic risk factor (R_f) is calculated by mathematically combining probability of failure and sensitivity. If i and j represent the random variables and the number of failure modes respectively, then for each random variable, R_f may be defined as the product of the normalized failure probability (P_f') and sensitivity (S'):

$$R_f(i) = 1 + \sum_{j=1}^{n} P_f'(j) \times S'(i)$$

The original values of the random variables, when modified by these R_f values, yields corrected values of the random variables, which have variations included into them. The reader can refer the work of GuhaRay and Baidya (2012, 2014) and GuhaRay *et al.* (2014) for better understanding of the computational procedure. The computational procedure has not been discussed elaborately in this paper in order to avoid repetition.

CASE STUDY

In the present study, a series of 15 gravity retaining walls located in southern parts of Ota Ward in Tokyo Metropolitan and 39 walls in Hodogaya Ward and Naka Ward of Yokohama Municipality in Japan (Gautam and Kanda, 2009) are considered for analysis. These areas consist of little sloppy and fragile ground condition. Gautam and Kanda (2009) reported that most of the retaining walls have P_f varying from 40-60% for overturning and 10-40% for sliding. The soil properties are reported in Table 1 and the basic configuration of the retaining wall is shown in Fig. 1.

Table 1 ***Soil Properties***

	Backfill Soil	**Foundation Soil**
Type of Soil	Dry Sandy Soil	Cohesive Soil
Unit Weight (kN/m^3)	18	*
Internal Friction Angle (°)	30	*
Cohesion (kN/m^2)	-	30 – 40

*Footnote: *not reported*

During the field survey, it was reported that almost all the walls have similar arrangements in the exposed face (Gautam and Kanda, 2009). The range of heights (h) of the retaining walls was considered from the field survey. The length and thickness of the base were estimated to be 2/3rd of h and at least 20cm respectively. Length from toe to column of wall was taken to be h/8, subjected to a minimum value of 20cm. The walls are analysed against two external modes of failure viz. sliding and overturning by Mononobe-Okabe Method (Mononobe and Matsuo, 1929; Okabe, 1926). It is observed that cohesion c affects only the sliding mode of failure. P_f is negligible for overturning mode of failure when PGA is less than 300 cm/s^2. For PGA above 400 cm/s^2, P_f=1 for overturning mode of failure. Hence, it may be concluded that for PGA < 300 cm/s^2, overturning mode of failure does not contribute significantly to the total P_f and c is the governing geotechnical parameter. But φ affects both sliding and overturning mode of failure. For PGA > 400 cm/s^2, overturning contributes primarily on total P_f and φ becomes the dominating parameter.

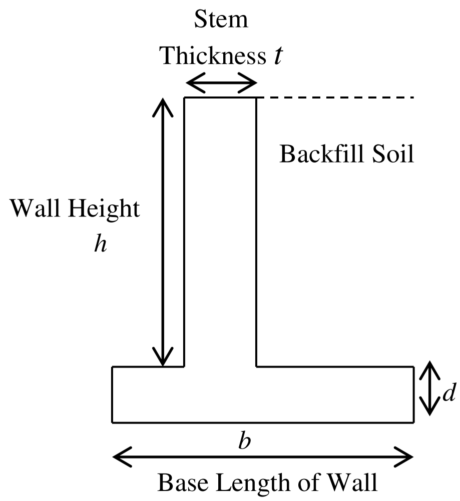

Figure. 1 Basic Configuration of Retaining Wall

Table 2 tabulates the FS and P_f for 54 retaining walls for different magnitudes of earthquake for sliding mode of failure. The results presented in Table 1 differ marginally from that reported by Gautam and Kanda (2009) because the later used First Order second Moment Method for analysis, while Monte Carlo Simulation is used in the present study. For example, for Ota ward, upto PGA 400 cm/s^2, total number of failures reported by the cited reference is 10, while that in the present study is 15.

Table 2 FS and P_f of 54 retaining walls for different PGA values (Sliding Mode of Failure)

Ret. Wall No.	h (m)	L (m)	t (m)	PGA= 100cm/s^2		PGA= 200cm/s^2		PGA= 300cm/s^2		PGA= 400cm/s^2		PGA= 500cm/s^2	
				FS	P_f	FS	P_f	FS	P_f	FS	P_f	FS	P_f
Retaining Walls of Hodogaya and Naka Ward													
Y11	2.75	1.93	0.2	2.322	0	1.605	0	1.198	0.074	0.927	0.768	0.715	0.995
Y14	2.25	1.49	0.21	2.593	0	1.788	0	1.333	0.010	1.029	0.462	0.792	0.961
Y15	2.00	1.32	0.24	2.78	0	1.913	0	1.424	0.003	1.098	0.285	0.844	0.899
Y16	1.50	0.99	0.18	3.33	0	2.289	0	1.7	0	1.308	0.042	1.000	0.549
Y17	1.25	0.83	0.18	3.769	0	2.585	0	1.917	0	1.472	0.006	1.124	0.268
Y18	1.50	0.99	0.22	3.331	0	2.286	0	1.697	0.000	1.305	0.043	0.999	0.549
Y19	2.00	1.32	0.2	2.778	0	1.914	0	1.425	0.003	1.099	0.287	0.844	0.905
Y20	2.75	1.93	0.2	2.322	0	1.605	0	1.198	0.074	0.927	0.768	0.715	0.995
YA1	1.75	1.23	0.2	3.015	0	2.075	0	1.543	0.000	1.188	0.132	0.911	0.774
Y22	3.50	2.45	0.3	2.066	0	1.429	0.00003	1.068	0.309	0.828	0.956	0.642	0.999
Y23	3.60	2.7	0.25	2.036	0	1.409	0	1.054	0.352	0.818	0.963	0.634	0.999
Y24	2.75	1.82	0.2	2.322	0	1.605	0	1.198	0.074	0.927	0.768	0.715	0.995
Y25	2.30	1.52	0.21	2.561	0	1.766	0	1.317	0.013	1.017	0.502	0.783	0.967
Y26	3.00	2.25	0.21	2.221	0	1.536	0	1.147	0.133	0.888	0.858	0.687	0.998
Y112	1.50	0.99	0.2	3.33	0	2.287	0	1.699	0.000	1.306	0.434	1.000	0.540
Y113	2.75	1.82	0.2	2.322	0	1.605	0	1.198	0.074	0.927	0.768	0.715	0.995
Y29	1.60	1.06	0.22	3.193	0	2.194	0	1.629	0.000	1.254	0.070	0.960	0.656
YA2	3.25	2.44	0.2	2.134	0	1.477	0	1.104	0.212	0.856	0.919	0.662	0.999
Y33	2.50	1.65	0.2	2.444	0	1.687	0	1.259	0.031	0.973	0.631	0.750	0.986
Y34	2.50	1.65	0.22	2.445	0	1.688	0	1.259	0.029	0.973	0.628	0.750	0.986
YA3	2.25	1.49	0.21	2.593	0	1.788	0	1.333	0.009	1.029	0.462	0.792	0.961
YA4	2.25	1.49	0.21	2.593	0	1.788	0	1.333	0.009	1.029	0.462	0.792	0.961
YA5	2.35	1.55	0.21	2.53	0	1.745	0	1.301	0.017	1.005	0.531	0.774	0.973
Y50	2.20	1.45	0.18	2.625	0	1.811	0	1.35	0.009	1.042	0.429	0.802	0.952

Y51	1.75	1.16	0.2	3.015	0	2.075	0	1.543	0.000	1.188	0.132	0.911	0.774
Y52	2.00	1.32	0.2	2.778	0	1.914	0	1.425	0.003	1.099	0.287	0.844	0.905
Y58	1.80	1.19	0.2	2.963	0	2.039	0	1.517	0.001	1.168	0.160	0.896	0.804
Y61	2.75	1.82	0.22	2.323	0	1.605	0	1.198	0.071	0.927	0.769	0.715	0.994
Y62	2.25	1.49	0.22	2.594	0	1.788	0	1.332	0.010	1.029	0.464	0.792	0.961
Y63	2.00	1.40	0.2	3.015	0	2.075	0	1.543	0.000	1.188	0.132	0.911	0.774
Y64	3.50	2.45	0.2	2.06	0	1.427	0	1.067	0.309	0.828	0.956	0.641	0.999
Y65	3.50	2.45	0.2	2.06	0	1.427	0	1.067	0.309	0.828	0.956	0.641	0.999
Y66	1.00	0.66	0.2	4.42	0	3.02	0	2.235	0	1.713	0.000	1.305	0.067
Y67	1.50	0.99	0.2	3.33	0	2.287	0	1.699	0.000	1.306	0.434	1.000	0.547
Y68	2.00	1.32	0.2	3.015	0	2.075	0	1.543	0.000	1.188	0.132	0.911	0.774
Y70	2.50	1.65	0.3	2.45	0	1.688	0	1.258	0.032	0.972	0.629	0.75	0.987
Y73	4.25	3.19	0.2	1.89	0	1.311	0.00037	0.982	0.618	0.763	0.994	0.593	1.000
Y86	2.85	1.88	0.28	2.284	0	1.577	0	1.177	0.092	0.91	0.808	0.703	0.997
Y87	2.75	1.82	0.2	2.322	0	1.605	0	1.198	0.074	0.927	0.768	0.715	0.995
Retaining Walls of Ota Ward													
C11	1.25	0.83	0.18	3.769	0	2.585	0	1.917	0	1.472	0.006	1.124	0.268
C12	2.10	1.39	0.22	2.7	0	1.86	0	1.385	0.005	1.069	0.365	0.822	1.000
C13	3.50	2.45	0.22	2.061	0	1.427	0	1.067	0.312	0.828	0.312	0.641	0.999
C15	1.65	1.09	0.2	3.13	0	2.152	0	1.6	0.000	1.231	0.088	0.943	0.694
C16	1.75	1.16	0.2	3.015	0	2.075	0	1.543	0.000	1.188	0.132	0.911	0.774
C17	1.75	1.16	0.2	3.015	0	2.075	0	1.543	0.000	1.188	0.132	0.911	0.774
C18	4.00	2.8	0.22	1.941	0	1.345	0.0001	1.007	0.522	0.782	0.987	0.607	1.000
C21	2.50	1.65	0.22	2.445	0	1.688	0	1.259	0.029	0.973	0.628	0.75	0.986
C23	2.25	1.49	0.2	2.593	0	1.788	0	1.333	0.010	1.029	0.461	0.792	0.959
C26	3.75	2.89	0.25	1.999	0	1.384	0.0007	1.035	0.421	0.803	0.976	0.623	0.999
C27	4.15	3.11	0.3	1.915	0	1.326	0.0001	0.993	0.5767	0.771	0.992	0.599	1.000
C30	3.75	2.63	0.22	1.997	0	1.383	0.0007	1.035	0.4175	0.803	0.973	0.623	0.999
C36	3.25	2.28	0.2	2.134	0	1.477	0	1.104	0.2119	0.856	0.919	0.662	0.999
C37	3.00	1.98	0.3	2.227	0	1.537	0	1.147	0.1303	0.888	0.863	0.687	0.998
C40	2.50	1.65	0.2	2.444	0	1.687	0	1.259	0.0312	0.973	0.631	0.75	0.986

It can be observed from Table 2, that the failure has initiated from PGA = 300 cm/s^2. P_f is within the range of 10^{-3} (which is considered to be safe according to US Army corps of Engineers, 2001) for PGA upto 200 cm/s^2. For PGA = 300 cm/s^2, 16 walls out of 54 is safe against sliding i.e. have $P_f < 0.001$, while only 52 out of 54 has $FS > 1$. In other words, from deterministic analysis, 52 out of 54 walls is safe, while the variation of c is considered, 16 out of 54 is safe (for $P_f < 0.001$). For PGA = 400 cm/s^2, 1 out of 54 walls is safe against sliding i.e. have $P_f < 0.001$, while 28 out of 54 has $FS > 1$. For PGA = 500 cm/s^2, no wall is safe against sliding i.e. all 54 walls have $P_f > 0.001$, while 6 out of 54 has $FS > 1$. Tables 3a and b show the number of failure cases for the two different wards for *COV* of c = 25%.

Table 3a Failure Distribution in Hodogaya and Naka Ward (for *COV* of *c* = 25%)

PGA (cm/s^2)	100	200	300	400	500
No. of walls with $FS < 1$	0	0	1	16	34
No. of walls with $P_f > 0.001$	0	0	27	38	39

Table 3b Failure Distribution in Ota Ward (for *COV* of *c* = 25%)

PGA (cm/s^2)	100	200	300	400	500
No. of walls with $FS < 1$	0	0	1	9	14
No. of walls with $P_f > 0.001$	0	0	11	15	15

Hence, variation in cohesion plays significant role in safety of the structure when PGA exceeds 200 cm/s^2. Hence, the breadth (b) of the retaining walls needs to be increased, keeping in mind the variation of cohesion value at that place, in order to maintain safety against PGA > 200cm/s^2. So instead of using b = 2/3rd*h for all cases, it is recommended to increase the heel length L_h, keeping "a" and "L_t" constant (b_{min} = 2/3rd *h).

Tables 4a and b shows the R_f values for c and φ respectively for different intensities of earthquake. From Table 4a, it is seen that the R_f value for c is 1 for PGA < 200 cm/s^2 for all heights of retaining walls, while it increases with increase in PGA values. On the other hand, from Table 4b, it is seen that R_f value for φ is 1 for PGA < 300 cm/s^2 for all heights of retaining walls. These R_f values are incorporated in design and design recommendations for breadth of the retaining walls for variations of c and different intensities of earthquake are presented in Table 5. These values may be directly implemented in design, for known height of the retaining wall and earthquake intensity. From Table 5, it is evident that the retaining walls are safe for PGA < 200 cm/s^2. But if the earthquake intensity exceeds 200 cm/s^2, the retaining wall has to be redesigned to ensure safety. Thus, apart from indicating the P_f of each retaining wall, the present risk factor based design approach helps to identify the breadth required for different retaining walls based on their heights, variation of geotechnical random variables and different earthquake intensities.

Table 4a Risk Factors (R_f) for *c* with variation of *c*, *h* and PGA

Ht. of Wall *h* (m)	PGA<100 cm/s^2		PGA=100 – 200 cm/s^2		PGA=200 – 300 cm/s^2		PGA=300 – 400 cm/s^2		PGA=400 – 500 cm/s^2	
	COV of *c*									
	<10 %	10-25%	<10%	10-25%	<10%	10-25%	<10%	10-25%	<10 %	10-25%
<1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.25	1.75	1.4	1.85
1.5-2.0	1.0	1.0	1.0	1.0	1.0	1.3	1.25	1.75	1.4	1.85
2.0-2.5	1.0	1.0	1.0	1.0	1.0	1.3	1.25	1.75	1.4	1.85
2.5-3.0	1.0	1.0	1.0	1.0	1.0	1.3	1.25	1.75	1.4	1.85
3.0-3.5	1.0	1.0	1.0	1.0	1.1	1.3	1.25	1.75	1.4	1.85
3.5-4.0	1.0	1.0	1.0	1.0	1.1	1.3	1.25	1.75	1.4	1.85
4.0-4.5	1.0	1.0	1.0	1.0	1.1	1.3	1.25	1.75	1.4	1.85
4.5-5.0	1.0	1.0	1.0	1.0	1.1	1.3	1.25	1.75	1.4	1.85

Table 4b Risk Factors (R_f) for φ with variation of *c*, *h* and PGA

Ht. of Wall *h* (m)	PGA<100 cm/s^2		PGA=100 – 200 cm/s^2		PGA=200 – 300 cm/s^2		PGA=300 – 400 cm/s^2		PGA=400 – 500 cm/s^2	
	COV of φ									
	<10%	10-25%	<10%	10-25%	<10%	10-25%	<10%	10-25%	<10%	10-25%
<1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0
1.5-2.0	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0
2.0-2.5	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0
2.5-3.0	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0
3.0-3.5	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0
3.5-4.0	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0
4.0-4.5	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0
4.5-5.0	1.0	1.0	1.0	1.0	1.0	1.0	1.95	2.0	2.0	2.0

Table 5 Breadth of wall (*b*) for variation of *c*, *h* and PGA

Ht. of Wall *h* (m)	Original Breadth (m)	PGA<100 cm/s²		PGA=100 – 200 cm/s²		PGA=200 – 300 cm/s²		PGA=300 – 400 cm/s²		PGA=400 – 500 cm/s²	
		COV* of *c									
		<10 %	10-25%	<10 %	10-25%	<10%	10-25%	<10%	10-25%	<10%	10-25%
<1.5	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.2	1.45	1.6	3.15
1.5-2.0	1.3	1.3	1.3	1.3	1.3	1.3	1.4	1.65	2.5	3.25	6.5
2.0-2.5	1.65	1.65	1.65	1.65	1.65	1.65	2.15	2.5	3.5	6.15	-
2.5-3.0	2.0	2.0	2.0	2.0	2.0	2.0	2.85	3.75	7.0	-	-
3.0-3.5	2.3	2.3	2.3	2.3	2.3	2.75	3.85	5.25	-	-	-
3.5-4.0	2.65	2.65	2.65	2.65	2.65	3.5	4.75	7.25	-	-	-
4.0-4.5	3.0	3.0	3.0	3.0	3.0	4.3	5.75	-	-	-	-
4.5-5.0	3.3	3.3	3.3	3.3	3.3	5.15	6.85	-	-	-	-

CONCLUSIONS

In the present study, the probabilistic risk factor based approach is applied to a series of retaining walls and design recommendations are provided for their safe functioning corresponding to different intensities of earthquake. It is observed that the cohesion of the foundation soil is the dominating factor in failure of the retaining walls for peak ground acceleration (PGA) less than 300 cm/s^2, while the internal angle of friction plays a significant role in stability when PGA exceeds 300 cm/s^2. Seismic reliability analysis also indicates that for variation of cohesion upto 25%, the walls are for PGA upto 200 cm/s^2. For PGA > 400 cm/s^2 and variation of c_2> 10%, walls having height more than 3m require a base width almost twice the height of the wall. The proposed procedure in this study can be applied in the design of retaining walls in other parts of Tokyo area as well, for similar subsurface and backfill conditions.

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