The paper was published in the proceedings of the 7th International Young Geotechnical Engineers Conference and was edited by Brendan Scott. The conference was held from April 29th to May 1st 2022 in Sydney, Australia.

Allowable bearing capacity of shallow foundations for low-density pyroclastic rocks

Capacité portante admissible des fondations peu profondes pour les roches pyroclastiques de faible densité

Carlos L. Garrido, Rubén A. Galindo & Alcibiades Serrano
Departamento de Ingeniería y Morfología del Terreno, E T S I de Caminos, Canales y Puertos, c/Profesor Aranguren nº3, Universidad Politécnica de Madrid, Spain, carlosluis.garrido.garrido@alumnos.upm.es

ABSTRACT: Volcanic rocks made from pyroclastic deposition are generally low-density and high-porosity rocks, with a geomechanical behaviour that differs from conventional rock mechanics. In this paper, a new methodology is proposed to determine the allowable bearing capacity of shallow foundations in pyroclastic rocks. The parabolic criterion is considered as the collapse failure criterion, and the study of the required parameters is also presented. By using the compression test results of a specific volcanic lithotype from the Canary Islands (welded lapilli: LP-W), all possible collapse envelopes are defined mathematically using least-squares adjustment by stress paths in a MATLAB model. Among them, two main parabolic envelopes are distinguished: the medium and minimum envelopes. With these two collapse envelopes, reliability analysis can be made, and the safety in shallow foundations can be determined. The obtention of the bearing capacity and the validation of the reliability formulation proposed is accomplished by implementing a finite-difference model in FLAC2D.

RÉSUMÉ : Les roches volcaniques issues de dépôts pyroclastiques sont généralement des roches de faible densité et de forte porosité, avec un comportement géomécanique qui diffère de la mécanique des roches conventionnelles. Dans cet article, une nouvelle méthodologie est proposée pour déterminer la capacité portante admissible des fondations peu profondes dans les roches pyroclastiques. Le critère parabolique est considéré comme le critère de rupture par effondrement, et l'étude des paramètres requis dans le critère parabolique est également présentée. En utilisant les résultats des tests de compression d'un lithotype volcanique spécifique des Îles Canaries (welded lapilli : LP-W), tout est possible les enveloppes de réduction sont définies mathématiquement à l'aide d'un ajustement des moindres carrés par des chemins de contraintes dans un modèle MATLAB. Parmi elles, on distingue deux enveloppes paraboliques principales : les enveloppes moyennes et minimales. Avec ces deux enveloppes d'effondrement, une analyse de fiabilité peut être effectuée et le facteur de sécurité dans les fondations peu profondes peut être déterminé. L'obtention de la capacité portante et la validation de la formulation de fiabilité proposée est réalisée en implémentant un modèle aux différences finies en FLAC2D.

KEYWORDS: low-density pyroclastic rocks, failure criterion, bearing capacity, shallow foundations, safety factor.

1 INTRODUCTION

Volcanic rocks made from pyroclastic deposition are known as pyroclastic rocks. The origin of pyroclastic rocks is related to explosive volcanic eruptions, in which pyroclasts are transported by air, deposited on the ground, and subjected to compaction and cementing events. Due to their formation process, pyroclastic rocks are generally fragmented rocks with low density and high porosity. This structure leads to a geomechanical behaviour that depends on their internal distribution and the welding degree between its particles. In addition to the high heterogeneity of volcanic rock masses, these phenomena make the approaches to geotechnical problems in pyroclastic rocks very singular.

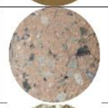
The mechanical behaviour of low-density pyroclastic rocks is between hard soils and soft rocks. At "low" enough confining pressures, these materials behave as rocks with high deformation modulus and low deformations. Nonetheless, at "high" enough confining pressures, the bonds between particles may break, and the deformation may increase considerably. This phenomenon results in a sudden decrease in its volume and a reorganization into a more compact structure, which resembles soils if its structure gets destroyed. The previous process is known as mechanical collapse and involves drastic changes in the properties of the collapsible materials, which may lead to sudden failures without significant prior warning deformations. Therefore, great interest exists in depicting a theoretical and practical framework for low-density pyroclastic rocks in the field of civil engineering in volcanic areas. Such a framework would facilitate estimating the bearing capacity on building foundations, the earth pressure exerted on walls, and the slope stability analysis on these materials.

To face the design of shallow foundations is needed to define a safety factor to convert bearing capacity into allowable pressure. Traditionally, safety factors in soil mechanics are well known and highly contrasted with the experience acquired in this type of construction. However, using standardized safety factors in rock mechanics is not recommended because of the variability of the parameters. This fact is ever more relevant in pyroclastic rocks due to their heterogeneity, limited knowledge, risk of mechanical collapse, and the evolution of properties because of the previous process. For these reasons, low-density pyroclastics rocks require a specific treatment to obtain the allowable bearing capacity and the corresponding safety factor for shallow foundations. In this paper, the proposed methodology is developed and applied to a particular lithotype of welded lapilli (LP-W) from the Canary Islands. Some concepts, such as the geotechnical classification (Conde, 2013; Conde et al., 2015) and the collapse parabolic criterion (Serrano et al., 2016) used, are also introduced to facilitate interpretation of the proposed methodology.

2 GEOTECHNICAL CLASSIFICATION

The geotechnical classification used for volcanic pyroclasts is shown in Table 1, obtained from an extensive study performed with low-density pyroclastic rocks by Conde (2013) and Conde et al. (2015). For its development, 250 specimens were trimmed and tested in the CEDEX's Geotechnical Laboratory from 22 different blocks of lapilli, scoria, pumice, basaltic, and sialic ashes from the Canary Islands (Tenerife, El Hierro, and La Palma).

Table 1: Geotechnical classification for volcanic pyroclasts (Conde et al., 2015).

	Description	Particles size (mm)	Welding or lithification	Matrix	Lithotype	Photo (Scale: circle diameter=7±0.5 cm)	Porosity type
BASALTIC	Basaltic ashes (Ash tuff) **(BA)**	<2mm	Lithified	NO	BA-L		Matrix
			Slightly lithified		BA-SL		
			Loose		BA-Lo		
	Basaltic ashes with particles of different nature **(BA)**	Matrix:<2mm Particles: different sizes	Lithified	YES	BA-L-MP		Matrix
	Lapilli **(LP)**	2-64mm	Welded	NO	LP-W		Mixed
			Slightly welded		LP-SW		
			Loose		LP-Lo		
			Lithified	YES	LP-L-M		Matrix
	Scoria (Bombs, Blocks) (Pyroclastic breccia) **(SC)**	>64mm	Welded	NO	SC-W		Vacuolar
			Slightly welded		SC-SW		
			Loose		SC-Lo		
SALIC	Sialic ashes (Ash tuff) **(SA)**	<2mm	Lithified	NO	SA-L		Matrix
			Slightly lithified		SA-SL		
			Loose		SA-Lo		
	Salic ashes with particles of different nature **(SA)**	Matrix:<2mm Particles: different sizes	Lithified	YES	SA-L-MP		Matrix
	Pumice (Pumice-flow deposits) **(PM)**	>2mm	Welded or Lithified	NO	PM-W/L		Reticular
			Slightly welded/ lithified		PM-SW/SL		
			Loose		PM-Lo		
			Lithified	YES	PM-L-M		Matrix

The previous classification considers the following aspects for its evaluation: magma composition (basaltic or salic), particle size, welding or lithification degree, and porosity type, being this last aspect one of the most representative of pyroclastic rocks. According to Santana et al. (2008), there are four types of porosity in pyroclastic rocks: reticular (cemented particles with macropores around them), vacuolar (vitreous masses with quasi-spherical cavities inside), mixed (intermediate scenario between reticular and vacuolar), and matrix (fine grain matrix that fills the macropores). As shown afterward, the lithotype studied in this paper is classified as a welded lapilli (LP-W).

3 FAILURE CRITERION

3.1 *Mathematical formulation (parabolic criterion)*

Based on the empirical study performed on volcanic pyroclasts by Serrano et al. (2016), the parabolic criterion was suggested as a strength criterion for low-density pyroclastic materials. Before its development, authors such as Wong & Mitchel (1975), Serrano (1976), and Aversa & Evangelista (1998) proposed different yield surfaces for low-density and collapsible materials, which promoted the empirical study carried out by Serrano et al. (2016). The parabolic criterion is represented in Equation (1):

$$q_{KR}^* = M(p_{KR}^* + t^*)\left(1 - \frac{p_{KR}^* + t^*}{P_{co}^*}\right)^{\lambda} \quad (1)$$

Which in universal or canonical variables is expressed by Equation (2):

$$q_K = M p_K (1 - p_K)^{\lambda} \quad (2)$$

In failure law (2), represented in Figure 1, there are four variables: two explicit parameters (M and λ) along with two hidden parameters (t^* and P_{co}^*). M is a frictional parameter that can be determined by triaxial tests, depending on the value of the initial instantaneous friction angle (ρ_o) and a coefficient for low pressures (k). Being $k = 0$ in the parabolic criterion, the parameter M is obtained by Equation (3):

$$M = \frac{1}{1+k}\frac{6\sin\rho_o}{3-\sin\rho_o} \xrightarrow{k=0} M = \frac{6\sin\rho_o}{3-\sin\rho_o} \quad (3)$$

On the other hand, the variable λ is a shape parameter that represents a non-linear extension from the Mohr-Coulomb strength criterion, which is verified when $\lambda = 0$ as shown in Figure 1. The variable λ is obtained from experimentation, with values between $0 < \lambda \leq 1$. About the two hidden parameters, isotropic tensile strength t^* and pressure module P_{co}^* must be adjusted by isotropic traction and compression test results.

To complement this study, Table 2 includes a compilation of outcomes obtained from Conde (2013) and Serrano et al. (2016), which shows the values of the four variables required in the parabolic criterion (1). These results provide the expected order of magnitude for each parameter considered. In addition, as observed in Table 2, almost every shape parameter λ is equal or close to one, with $\lambda = 1$ being the most generic scenario for the parabolic criterion.

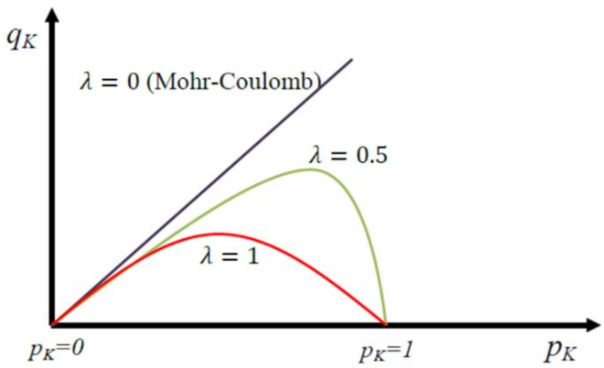


Figure 1. Parabolic criterion for volcanic pyroclasts in Cambridge dimensionless variables (p_k, q_k).

3.2 *Identification of the failure zones*

Assuming the parabolic criterion with $\lambda = 1$, the concept of an instantaneous friction angle (ρ) is defined in Equation (4) after Serrano & Olalla (1994) and using Hill-Lambe variables:

$$\sin\rho = \frac{dq}{dp} = \frac{3M(1-2p_K)}{6 + M(1-2p_K)} \quad (4)$$

Within the range of $p_k \in [0, 1]$, instantaneous friction angle ρ may be non-real. If $|\sin\rho| \leq 1$, then ρ is a real value, and the failure is plastic, forming vertical and inclined failure planes caused by the lack of confining pressure. If $|\sin\rho| > 1$, then ρ is an imaginary value, and the plastic failure does not exist. Failure occurs in this scenario by the destructuring of the specimen (mechanical collapse). Depending on the friction parameter M, two different destructuring failure mechanisms may exist along the strength criterion: compression destructuring (RCD) occurs when $M > 1.5$, and tensile destructuring (RDT) exists when $M > 3$ along with RCD. According to Table 2, the values of M are essentially between 1.5 and 3, with no value registered higher or equal to 3. Because of that, RCD is the only destructuring or mechanical collapse failure mechanism considered, as exposed in Figure 2.

Table 2: Parameters of the parabolic criterion (1) obtained by Conde (2013) and Serrano et al. (2016).

Material	Pressure module P_{co}^{*}		Tensile coefficient ζ		*Shape parameter* λ		*Frictional parameter M*	
	Mean (MPa)	*Range (MPa)*	*Mean (%)*	*Range (%)*	*Mean*	*Range*	*Mean*	*Range*
Welded lapilli (LP-W)	1.51	1.12-1.85	9.1	1.8-17.1	0.98	0.93-1	2.43	1.78-2.97
Slightly welded lapilli (LP-SW)	1.04	0.2-1.88	4.5	0-16.7	0.74	0.5-0.9	2.42	1.66-2.93
Weathered lapilli (LP-L-M)[1]	11.1	7.81-12.5	3.3	0.5-6.5	1	0.99-1	1.43	1.16-1.71
Welded pumice (PM-W)	0.29	0.2-0.39	18.4	5.1-25	0.84	0.63-1.02	2.19	1.68-2.73
Weathered pumice (PM-L-M)[1]	4.71	3.37-7.17	2.9	0.3-5.9	0.99	0.98-1	2.09	1.51-2.75
Welded scoria (SC-W)[1]	4.41	4.41	1.6	1.6	0.22	0.22	2.94	2.94
Lithified basaltic ashes (BA-L)[1]	5.26	5.26	2.3	2.3	1	1	2.62	2.62
Slightly lithified salic ashes (SA-SL)[1]	4.31	4.31	4.9	4.9	1	1	1.83	1.83
Weathered salic ashes (SA-L-MP)[1]	8.64	8.64	1	1	1	1	2.94	2.94
Red Tuff[2] (SA-L or SA-L-MP)[3]	9.27	9.27	6.1	6.1	1	1	2.99	2.99
Yellow Tuff[2] (SA-L or SA-L-MP)[3]	8	8	22.7	22.7	0.89	0.89	1.77	1.77
Pozzolana Nera[2] (SA-L or SA-L-MP)[3]	7.24	7.24	5.1	5.1	0.78	0.78	2.17	2.17
Fine-Grain Tuff[2] (SA-L or SA-L-MP)[3]	23.1	20.9-25.3	13	11.8-14.2	0.8	0.78-0.82	1.87	1.81-1.94

[1] P_{co}^{*} was not reached in tests, but it was deduced from mathematical adjustment.
[2] Data from literature. Red and yellow tuffs: Tommasi & Ribacchi (1998); Pozzolana nera: Cecconi & Viggiani (1998 and 2001); Fine grain tuff: Aversa et al. (1993), Aversa & Evangelista (1998) and Evangelista et al. (1998).
[3] Assumed lithotypes in accordance with Conde et al. (2015) geotechnical classification.

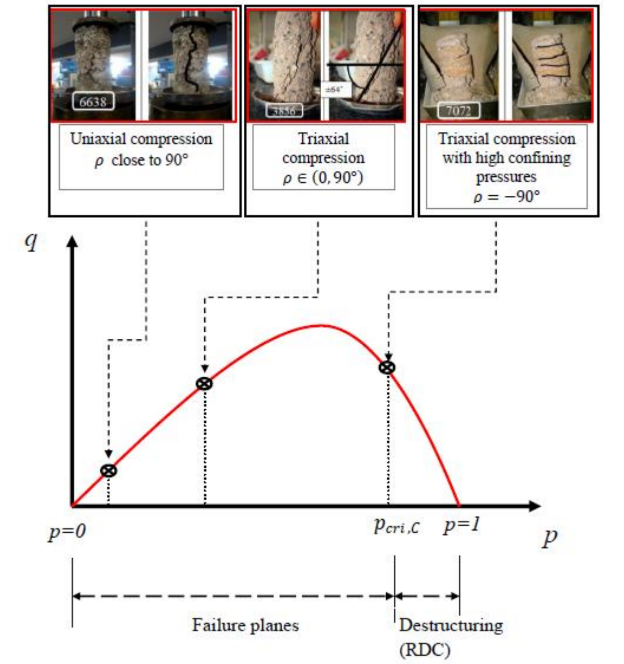


Figure 2. Failure zones of the parabolic criterion with $\lambda = 1$ obtained experimentally by Conde (2013) and Serrano et al. (2016) (p, q).

4 ALLOWABLE BEARING CAPACITY

4.1 *Methodology goal (safety factor)*

For a specific lithotype to study, this approach aims to define its safety factor (SF) as the division of two bearing capacities (q_h), as shown in Equation (5):

$$SF = \frac{q_{h,med}}{q_{h,min}} \tag{5}$$

Each of the previous bearing capacities, being ($q_{h,med}$) the medium bearing capacity and ($q_{h,min}$) the minimum bearing capacity, is related to one of the two main collapse envelopes of this methodology. To obtain these failure laws, it is necessary to carry out uniaxial, triaxial, and isotropic compressive tests, registering the studied lithotype failure points. Including every failure point in the analysis, all possible collapse envelopes are defined mathematically, using least-squares adjustment by stress paths in a MATLAB model and the parabolic criterion (1) considering $\lambda = 1$. Among them, the two principal collapse envelopes are distinguished: the medium envelope, which defines the average parameters of the parabolic criterion for the tested lithotype, and the minimum envelope, corresponding to the most unfavorable calculation scenario in which every possible envelope is above it. As a representative example, Figure 3 includes both main envelopes for the welded lapilli analyzed. Once the main envelopes are defined, they are introduced into a finite-difference model in FLAC2D to obtain the corresponding bearing capacities.

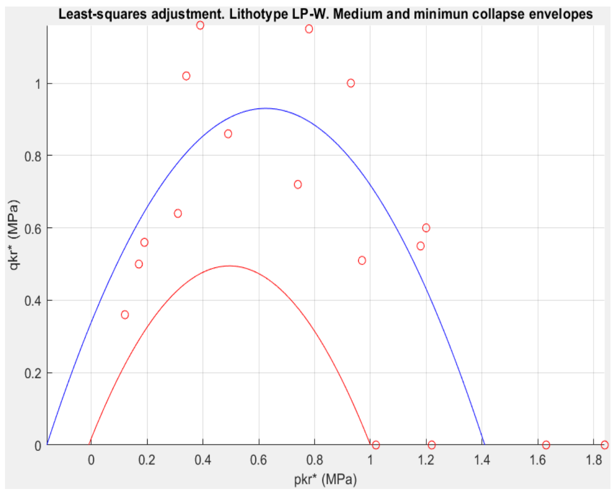


Figure 3. Medium (blue) and minimum (red) main collapse envelopes (LP-W data from Conde, 2013) (p_{KR}^{*}, q_{KR}^{*}).

4.2 *Initial data (LP-W)*

Table 3 illustrates the failure points taken from Conde (2013) and Serrano et al. (2016). Four samples of welded lapilli (WA, WB, WC, and WD), and a total of nineteen specimens tested, have been selected, with a variable height between 10 and 13 cm and a diameter between 5 and 7 cm. They were first saturated and then maintained in a freezer for 75 hours before cutting them with a diamond cutter. The samples were dry before testing due to 48 hours of heating inside an oven. In addition, the triaxial and isotropic tests required a strong rubber of oilcloth type, over two or three rubber normal impervious membranes (Serrano et al., 2016).

Table 3: Initial data of welded lapilli (LP-W) from the Canary Islands obtained by Conde (2013).

Sample	Test	$p^*_{KR}\ (MPa)$	$q^*_{KR}\ (MPa)$	Group
WA-1	Uniaxial	0.19	0.56	G1
WA-2	Triaxial	1.20	0.60	G2
WA-3	Isotropic	1.63	0.00	G3
WB-1	Uniaxial	0.39	1.16	G1
WB-2	Uniaxial	0.34	1.02	G1
WB-3	Triaxial	0.78	1.15	G2
WB-4	Triaxial	1.18	0.55	G2
WB-5	Triaxial	0.93	1.00	G2
WB-6	Isotropic	1.84	0.00	G3
WB-7	Isotropic	1.63	0.00	G3
WB-8	Isotropic	1.02	0.00	G3
WC-1	Uniaxial	0.17	0.50	G1
WC-2	Triaxial	0.97	0.51	G2
WC-3	Isotropic	1.22	0.00	G3
WD-1	Uniaxial	0.12	0.36	G1
WD-2	Triaxial	0.74	0.72	G2
WD-3	Triaxial	0.49	0.86	G1
WD-4	Triaxial	0.31	0.64	G1
WD-5	Isotropic	1.02	0.00	G3

4.3 *Groups of data*

Before proceeding with the reliability analysis, it is necessary to classify the failure points into groups. Grouping the data is one of the most controversial steps in this approach and requires precise treatment, as not every combination of failure points is consistent with the parabolic criterion and could lead to miscalculations. Therefore, the proposed methodology considers one failure point for each group when generating the collapse envelopes. As shown afterward, it is necessary to differentiate a minimum of three groups to ensure the correct definition of the parabolic criterion, one for each unknown variable.

As represented in Figure 2, the uniaxial, triaxial, and isotropic compressive tests have specific locations along the parabolic criterion. Because of that, the failure points of these tests will also have that same situation, and they will be easier to classify as well. Accordingly, the following four general groups of data are distinguished: low confining tests (uniaxial and triaxial tests on the growing side of the parabolic criterion), intermediate confining tests (triaxial tests near to the maximum value of variable q along the collapse envelope), high confining tests (triaxial tests on the decreasing side of the parabolic criterion) and very high confining tests (isotropic tests).

With the previous general groups and the data of welded lapilli from Table 3, Figure 4 shows the three groups selected in this scenario: G1 for low confining tests, G2 for intermediate and high confining tests, and G3 for very high confining tests. The blue lines observed in Figure 4 correspond to the stress paths for each failure point, with a 3:1 slope in a p^*_{KR} - q^*_{KR} diagram. Considering this concept, if the triaxial tests had been carried out with the same confining pressure, the failure points would have been represented on the same stress path, and the groups of data would have been easier to define.

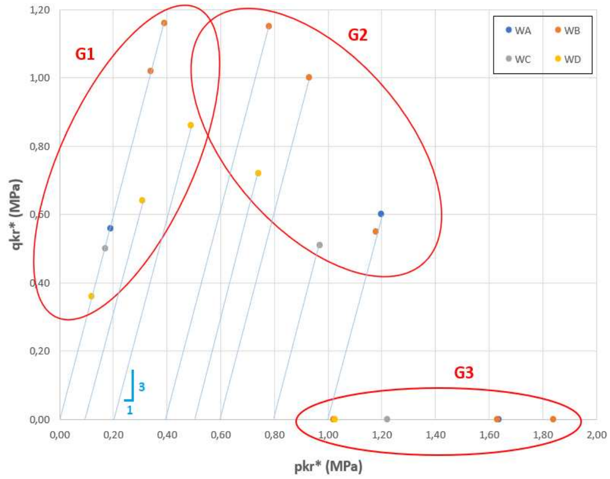


Figure 4. Groups of data (LP-W data from Conde, 2013).

4.4 *Reliability analysis*

Once the groups of failure points have been established, the reliability analysis consists of selecting one failure point per group and adjusting them to the parabolic criterion by least-squares methodology. The envelopes used for the adjustment verify Equation (1) with $\lambda = 1$, obtaining the other three variables as a result (M, t^* and P^*_{co}). This procedure is repeated with all potential combinations of points between groups until all possible collapse envelopes are defined.

The least-squares approach seeks to minimize the square distance between the points considered and the adjustment envelope with the minimum accumulated error. Depending on the definition of the previous distance, different least-squares methodologies exist. In this study, three alternatives were evaluated: vertical distance, perpendicular distance to the adjustment envelope (Euclidean distance), and distance according to the stress path of the points to be adjusted (see stress paths of the failure points in Figure 4). Of all of them, the stress paths approach is the one that provided the best fit and is the one considered for this methodology. Least-squares adjustment by stress paths is expressed by Equation (6) as follows:

$$E_{LSA} = \sum_{j=1}^{n_p} d_j^2 = \sum_{j=1}^{n_p} \left(q^*_{KR,ej} - q^*_{KR,j}\right)^2 + \left(p^*_{KR,ej} - p^*_{KR,j}\right)^2 \qquad (6)$$

Being E_{LSA} the accumulated error and d the distance between the failure point (p^*_{KR}, q^*_{KR}) and the point from the envelope evaluated $(p^*_{KR,e}, q^*_{KR,e})$ that intersects the corresponding stress path. Equation (6) has been programmed numerically in a MATLAB model until reaching the minimum value of E_{LSA} for each combination of failure points between groups. For the welded lapilli data used from Table 3, Figure 5 shows all the 252 adjustment envelopes obtained from this least-squares approach. With this number of envelopes (n = 252), reliability analysis can be performed, being the failure probability associated with the allowable bearing capacity the inverse value of the previous one ($f_{PR} \leq 1/n$ = 0.00397 = 0.397%).

4.5 *Main collapse envelopes*

After finishing the reliability analysis, the next step requires the definition of the main collapse envelopes (medium and minimum), which are necessary to obtain the corresponding bearing capacities and the safety factor according to Equation (5). The medium and minimum envelopes verify the parabolic criterion (1) with $\lambda = 1$. The final parameters of both main envelopes are represented in Table 4.

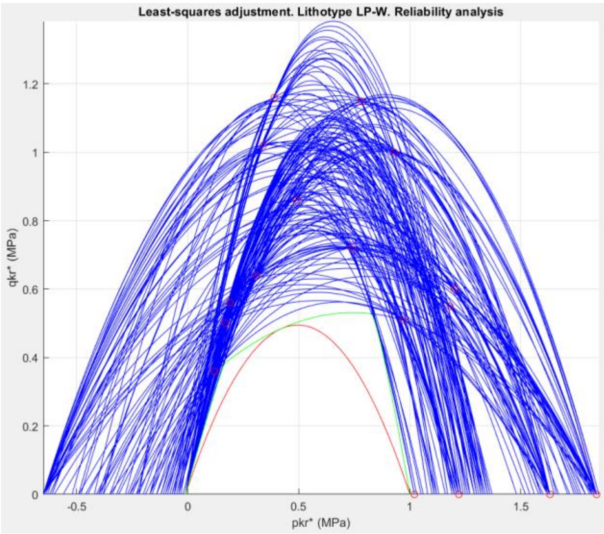


Figure 5. Reliability analysis (blue; $n = 252$) with the minimum collapse envelope (red) (LP-W from Conde, 2013) (p^*_{KR}, q^*_{KR}).

The minimum envelope is obtained directly from the reliability analysis. This curve is the one that leaves all the envelopes obtained from the least-squares adjustment above it ($n = 252$), as previously represented in Figure 5. Variables t^* and P^*_{co} of the minimum envelope correspond to the maximum value of t^* and the minimum value of P^*_{co} registered in the reliability analysis. With them, the resulting variable M is the one that makes the minimum envelope tangent to all adjustment envelopes from underneath.

The medium envelope requires obtaining the average values of parameters M, t^*, and P^*_{co} from the analyzed lithotype. Several possibilities exist to calculate the medium envelope. Regardless of the chosen method, these parameters must be representative of the studied material. In this study, three alternatives were considered:

- *Medium envelope of samples*: parameters M, t^*, and P^*_{co} are adjusted for each sample (WA, WB, WC, and WD). With them, average parameters of the medium envelope are obtained.
- *Medium envelope of groups*: average values of p^*_{KR} and q^*_{KR} are obtained for each group (G1, G2, and G3). With them, average parameters M, t^*, and P^*_{co} of the medium envelope are adjusted.
- *Medium envelope of lithotype*: considering every failure point of the studied lithotype (p^*_{KR}, q^*_{KR}), average parameters M, t^*, and P^*_{co} of the medium envelope are adjusted.

Of all of them, the *Medium envelope of groups* is the one chosen and represented in Figure 3. However, the three methods were calculated and contrasted with each other, obtaining very similar outcomes.

Table 4: Adjusted parameters (M, t^*, and P^*_{co}) and bearing capacity of the medium ($q_{h,med}$) and minimum ($q_{h,min}$) envelopes.

Main envelope	M	$t^*(MPa)$	P^*_{co} (MPa)	$\boldsymbol{q_h}$ $\boldsymbol{(MPa)}$
Medium	2.53	0.10	1.39	**1.17**
Minimum	1.96	0.01	1.00	**0.85**

4.6 *Bearing capacity*

The parabolic criterion is a singular failure law that needs to be implemented in FLAC2D to operate with it. Once implemented, a model that simulates an assumed shallow foundation on the studied lithotype was built. That model was evaluated for each of the two main envelopes, calculating the bearing capacity in both of them. Table 4 shows the outcomes of this analysis. With them, and according to Equation (5), the resulting safety factor for the studied welded lapilli is $SF = 1.17/0.85 = 1.376$, with a failure probability of $f_{PR} = 0.397\%$ and an allowable bearing capacity of $q_{h,min} = 0.85\ MPa$.

5 CONCLUSIONS

The proposed methodology allows obtaining the allowable bearing capacity of shallow foundations for low-density pyroclastic rocks, which are unique materials with limited knowledge and variable properties. In addition, its safety factor and failure probability can also be determined, adding further value to this approach.

Welded lapilli (LP-W) data from Conde (2013) and Serrano et al. (2016) was used to evaluate the proposed methodology. The outcomes obtained show a safety factor of $SF = 1.376$, which is not very high for a low-density rock. This result is justified by the low dispersity of the data used in the analysis. The higher the heterogeneity of the data, the higher the difference between the medium and the minimum collapse envelopes, and the higher the safety factor results. Additionally, failure probability has a value of $f_{PR} = 0.397\%$, which is not very high. This result depends on the number of failure points available and the groups of data differentiated. Because of that, to reach a minimum value of f_{PR}, sufficient tests would be necessary to evaluate all ranges of confining pressures, combining uniaxial, triaxial, and isotropic compressive tests as previously shown. Lastly, the allowable bearing capacity obtained is $q_{h,min} = 0.85\ MPa$, which is an expected value for a shallow foundation on a soft rock.

6 ACKNOWLEDGEMENTS

I would like to thank the Universidad Politécnica de Madrid for giving me the opportunity to join this project and the Fundación José Entrecanales Ibarra for supporting my doctoral thesis. I would also like to thank Professor Serrano for sharing his knowledge with the Departamento de Ingeniería y Morfología del Terreno and Professor Galindo for his support and collaboration in all my research.

7 NOTATION

The notation used in this document, which is summarized in Table 5, is defined as follows:

- The superscript asterisk (*) indicates that the parameters or variables have physical dimensions.
- Basic parameters:
  - Isotropic tensile strength: t^*
  - Isotropic compressive strength: P^*_c
  - Pressure module: $P^*_{co} = P^*_c + t^*$
  - Isotropic tensile coefficient: $\zeta = t^*/P^*_{co}$
- Variables:
  - Major main stress: σ^*_{1R}
  - Minor main stress: σ^*_{3R}
  - Hill-Lambe variables: $p^*_R = (\sigma^*_{1R} + \sigma^*_{3R})/2$
    $q^*_R = (\sigma^*_{1R} - \sigma^*_{3R})/2$
  - Cambridge variables: $p^*_{KR} = (\sigma^*_{1R} + 2\sigma^*_{3R})/3$
    $q^*_{KR} = \sigma^*_{1R} - \sigma^*_{3R}$
- For variables, the subscript "R" represents real values. If the isotropic tensile strength (t^*) is added to the real value, then the variable does not have the subscript "R" and corresponds to the so-called physical values.
- If the values are divided by the pressure module (P^*_{co}), dimensionless parameters or variables are obtained and represented without an asterisk. Dimensionless physical variables are called universal or canonical variables.

- The subscript "K" is added to the Cambridge variables to differentiate them from Hill-Lambe variables.

Table 5: Notation of the variables used.

Variables	Main stress		Cambridge		Hill-Lambe	
	With dimensions	*Dimensionless*	*With dimensions*	*Dimensionless*	*With dimensions*	*Dimensionless*
Real	σ_{1R}^* σ_{3R}^*	$\sigma_{1R}=\frac{\sigma_{1R}^*}{P_{co}^*}$ $\sigma_{3R}=\frac{\sigma_{3R}^*}{P_{co}^*}$	q_{KR}^* p_{KR}^*	$q_{KR}=\frac{q_{KR}^*}{P_{co}^*}$ $p_{KR}=\frac{p_{KR}^*}{P_{co}^*}$	q_R^* p_R^*	$q_R=\frac{q_R^*}{P_{co}^*}$ $p_R=\frac{p_R^*}{P_{co}^*}$
Physical	$\sigma_1^*=\sigma_{1R}^*+t^*$ $\sigma_3^*=\sigma_{3R}^*+t^*$	$\sigma_1=\frac{\sigma_1^*}{P_{co}^*}=\frac{\sigma_{1R}^*+t^*}{P_{co}^*}=\sigma_{1R}+\zeta$ $\sigma_3=\frac{\sigma_3^*}{P_{co}^*}=\frac{\sigma_{3R}^*+t^*}{P_{co}^*}=\sigma_{3R}+\zeta$	$q_K^*=q_{KR}^*$ $p_K^*=p_{KR}^*+t^*$	$q_K=q_{KR}$ $p_K=p_{KR}+\zeta$	$q^*=q_R^*$ $p^*=p_R^*+t^*$	$q=q_R$ $p=p_R+\zeta$

8 REFERENCES

Aversa S, Evangelista A, Lerouei S, Picarelli L (1993): Some aspects of the mechanical behaviour of “structured” soils and soft rocks. *Proceedings of the International Symposium on Geotechnical Engineering of Hard Soils and Rocks, Athens, Greece, pp 359-366.*

Aversa S, Evangelista A (1998): The mechanical behaviour of a pyroclastic rock: failure strength and “destruction” effects. *Rock Mech Rock Eng 31:25-42*.

Cecconi M, Viggiani G (1998): Physical and structural properties of a pyroclastic soft rock. *Proceedings of the 2nd International Symposium on Hard Soils-Soft Rocks, Naples, pp 85-91.*

Cecconi M, Viggiani G (2001): Structural Features and Mechanical Behaviour of a Pyroclastic Weak Rock. *Int J Numer Anal Meth Geomech 25:1525-1557.*

Conde M (2013): Caracterización geotécnica de materiales volcánicos de baja densidad (PhD Thesis). *Universidad Complutense de Madrid, Spain.*

Conde M, Serrano A & Perucho A (2015): New geotechnical classification proposed for low density pyroclastic rocks. *4th Congress on Volcanic Rocks and Soils. Ischia, Italy, Sep. 2015.*

Evangelista A, Aversa S, Pescatore T & Pinto F (1998): Soft rocks in southern Italy and the role of volcanic tuffs in the urbanization of Naples. *Proceedings of the 2nd International Symposium on Hard Soils-Soft Rocks, Naples, Italy, pp 1243-1267*.

Santana M, Santiago C, Perucho A, & Serrano A (2008): Relación entre características químico-mineralógicas y propiedades geotécnicas de piroclastos canarios. *Proceedings of the VII Congreso Geológico, Las Palmas de Gran Canaria, Spain, pp 1567-5172*.

Serrano A (1976): Aglomerados volcánicos en las Islas Canarias. *Proceedings of Memoria del Simposio Nacional de Rocas Blandas, Madrid, Spain, pp 47- 53*.

Serrano A, Olalla C (1994): Ultimate bearing capacity of rock masses. *Int J Rock Mech Min Sci 31:93*.

Serrano A, Perucho A, Conde M (2016): Yield criterion for low-density volcanic pyroclasts. *Int J Rock Mech Min 86:194-203.*

Tommasi P, Ribacchi R (1998): Mechanical behaviour of the Orvieto tuff. *Proceedings of the 2nd International Symposium on Hard Soils-Soft Rocks, Naples, Italy, pp 901-909.*

Wong P K, Mitchell R J (1975): Yielding and Plastic Flow of Sensitive Clay. *Geotechnique 25:763-782*.