INTERNATIONAL SOCIETY FOR
SOIL MECHANICS AND
GEOTECHNICAL ENGINEERING

SIMSG ISSMGE

This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

The paper was published in the proceedings of the 7th International Young Geotechnical Engineers Conference and was edited by Brendan Scott. The conference was held from April 29th to May 1st 2022 in Sydney, Australia.

Proceedings of the 7th International Young Geotechnical Engineers Conference – Scott (Ed.)
© 2022 Australian Geomechanics Society, Sydney Australia, ISBN 978-0-9946261-5-8

An innovative energetic approach to interpret liquefaction behaviour of non-saturated sandy soils

Une approche énergétique innovante pour interpréter le comportement de liquéfaction des sols sableux non saturés.

Lucia Mele & Alessandro Flora
Department of Civil, Environmental and Architectural Engineering, University of Napoli Federico II, Italy, lucia.mele@unina.it

Stefania Lirer
Department of Engineering Sciences, University of Roma Guglielmo Marconi, Italy

ABSTRACT: The reduction of soil susceptibility to earthquake induced liquefaction is a crucial issue in most of the seismic areas around the world. Among the available techniques, induced partial saturation (IPS) is considered a promising technology well suited for densely urbanized areas, that may suffer from the use of more invasive technologies. The experimental evidence show that even a small reduction in the degree of saturation leads to a large increase of the liquefaction resistance of sandy soils. The same authors in already published papers suggested analysing the undrained cyclic behaviour of liquefiable soils in an energetic framework, proposing the specific energy to liquefaction as the key parameter ruling the liquefaction resistance of sandy soils. In this research, the experimental results carried out on non-saturated soils have been carefully analysed according to this energetic perspective: the experimental evidence prove that the specific deviatoric energy to liquefaction ($E_{s,liq}$) depends on the specific volumetric energy at liquefaction ($E_{v,liq}$), and, for a given number of cycles to liquefaction (N_{liq}=15), a correlation between these two variables has been introduced. Although the research is ongoing, the proposed energetic approach seems to be a useful tool for the prediction of the liquefaction resistance of non-saturated sandy soils that may significantly improve the model of Mele and Flora (2019).

RÉSUMÉ : La réduction de la susceptibilité des sols à la liquéfaction induite par les tremblements de terre est un problème crucial dans la plupart des zones sismiques à travers le monde. Parmi les techniques disponibles, la saturation partielle induite (IPS) est considérée comme une technologie prometteuse bien adaptée aux zones densément urbanisées, qui peuvent souffrir de l'utilisation de technologies plus invasives. Les preuves expérimentales montrent que même une petite réduction du degré de saturation conduit à une forte augmentation de la résistance à la liquéfaction des sols sableux. Les mêmes auteurs dans des articles déjà publiés ont suggéré d'analyser le comportement cyclique non drainé des sols liquéfiables dans un cadre énergétique, en proposant l'énergie spécifique à la liquéfaction comme paramètre clé régissant la résistance à la liquéfaction des sols sableux. Dans cette recherche, les résultats expérimentaux réalisés sur des sols non saturés ont été soigneusement analysés selon cette perspective énergétique : les preuves expérimentales prouvent que l'énergie déviatorique spécifique à la liquéfaction ($E_{s,liq}$) dépend de l'énergie volumétrique spécifique à la liquéfaction ($E_{v,liq}$), et, pour un nombre donné de cycles de liquéfaction (N_{liq}=15), une corrélation entre ces deux variables a été introduite. Bien que les recherches soient en cours, l'approche énergétique proposée semble être un outil utile pour la prédiction de la résistance à la liquéfaction des sols sableux non saturés qui peut améliorer considérablement le modèle de Mele et Flora (2019).

KEYWORDS: laboratory testing; liquefaction; Induced Partial Saturation (IPS); energetic approach.

1 INTRODUCTION

Liquefaction is a phenomenon marked by a temporary and rapid reduction of soil shear strength and stiffness which may occur in saturated loose sandy deposits during an earthquake because of the rapid increase of pore water pressure (Δu). The ground failures induced by soil liquefaction phenomenon (ground settlements, cracks, ground oscillations, lateral spreads) can cause extensive damages on the built environment and, as a consequence, there is a need for risk mitigation measures.

In the last decades many experimental research efforts (Yilmaz et al, 2008; Park & Kim, 2013; Sadrekarimi & Jones, 2019; Mele, 2020) have been devoted to the study of the factors ruling the liquefaction susceptibility of sandy soils (grading, relative density, stress state, fine content and plasticity index, saturation degree) and to the more effective mitigation techniques. Compared with traditional methods (soil densification (Da Fonseca et al., 2015), addition of fine (ElMohtar et al. 2014), drainage system (Flora et al., 2020)), the recently developed mitigation measures are soil grouting with nanomaterials or chemical solutions, bio-cementation and induced partial saturation (Bao et al., 2019).

Among them, induced partial saturation IPS (Eseller Bayat et al., 2012; Mele et al., 2019a; Flora et al., 2020) is considered one of the most promising and effective technologies to improve soil liquefaction resistance. It results also well suited for densely urbanized areas, that may suffer from the use of more invasive technologies.

As shown by several research (Chaney, 1978; Yoshimi et al., 1989; Ishihara et al., 2002; Yegian, 2007; Wang et al., 2016a; Mele et al., 2019a). The liquefaction resistance increases as the degree of saturation (S_r) decreases. Soil desaturation can be obtained by air injection (Eseller-Bayat et al., 2012), with chemical methods and by means of the biogas produced by bacteria. Moreover, several studies (e.g. Mahmoodi, B., & Gallant, A., 2021) have demonstrated the durability of air into pore water.

As explained by Okamura and Soga (2006), the presence of air in the voids increases the soil liquefaction resistance in two ways: during the undrained seismic loading, the soil tends to reduce its volume and the low volumetric stiffness of the gases, well lower than that of the water, reduces the pore water pressure build-up. The second mechanism is due to the matric suction of non-saturated soils, which increases the stiffness and strength of soils (Bishop and Blight, 1963) and it is relevant for low degrees of saturation.

The effectiveness of desaturation has been studied among others by Mele et al. (2019a) and Mele (2020) by performing cyclic triaxial tests on saturated and non-saturated specimens of different sandy soils: at the laboratory scale the seismic load (represented by a harmonic loading path) is quantified via the *Cyclic Stress Ratio* CSR defined as:

$$CSR = \frac{q_d}{2 \cdot \sigma'_0} \qquad (1)$$

where q_d is the cyclic deviatoric stress and σ'_0 is the confining effective stress, while N_{cyc} is the applied number of constant amplitude stress cycles.

Conventionally, the attainment of liquefaction may be identified according to stress and strain criteria. According to stress criterion, liquefaction triggering is identified when the pore water pressure ratio r_u (defined as the ratio between Δu and σ'_0) is equal to about 0.90. It should be emphasised that Δu is the excess of pore air pressure for the specimen with positive suction measurement, otherwise it is the excess of pore water pressure (Wang et al., 2016a). On the other hand, according to strain criterion, Ishihara (1993) suggested that liquefaction occurs when the double strain amplitude (ε_{DA}) is equal to 5%.

The applied CSR for which soil liquefaction is attained in a number of cycles represents the *Cyclic Resistance Ratio* CRR. The results of cyclic tests are generally reported in the plane CRR -N_{liq} where N_{liq} is the number of cycles required to attain soil liquefaction. The cyclic resistance curve CRR -N_{liq} depends on the soil state conditions: confining stress (σ'_0), relative density (D_r) and degree of saturation (S_r). These three parameters have been summarized in a synthetic one, called specific volumetric energy to liquefaction ($E_{v,liq}$), which is part of an innovative and promising energetic approach firstly proposed by Mele et al. (2019a). In this paper new considerations about this approach will be discussed to better understand the behaviour of non-saturated sandy soils in cyclic undrained conditions.

2 ENERGETIC APPROACH IN LIQUEFACTION TESTS

2.1 *Theoretical principles of energetic approach*

It is well known that soil liquefaction may occur in non-saturated conditions, too (Unno et al., 2008; Wang et al., 2016a).

To better understand the meaning of liquefaction in non-saturated soils the definition of effective stress has to be given. In this research the expression proposed by Bishop and Blight (1963) has been adopted:

$$\sigma'_{ns} = (\sigma - u_a) + \chi \cdot (u_a - u_w) \qquad (2)$$

where σ is the total stress and u_a, u_w e χ are respectively the pore air pressure, the pore water pressure and the material parameter accounting for the effect of the degree of saturation. The term (σ-u_a) is called "net stress", while (u_a-u_w) is the "matric suction" (s). In this paper, the parameter χ is assumed equal to the degree of saturation S_r ($\leq$100%).

Based on Eq. (2), it can be understood that in non-saturated soils, liquefaction occurs when both the pore air and water pressure are equal to the initial total confining pressure (Unno et al., 2008).

As the stress and strain criteria give different results in term of N_{liq} in non-saturated conditions (Mele et al., 2019a) generally, the strain criterion (ε_{DA}=5%) is preferred and used to identify the cyclic resistance curve CRR-N_{liq}.

During the undrained cyclic triaxial tests on loose non-saturated sands, volumetric strains (ε_v) increase until to reach a final value at liquefaction (when $\sigma'_{ns}\approx 0$), defined potential volumetric strain ε_v* (Okamura and Soga, 2006), as shown in Figure 1. It depends only on the initial soil state, defined for instance by the degree of saturation (S_{r0}), the void ratio (e_0) and the initial net stress (σ-u_a)$_0$. In fact, whatever the applied CSR was, ε_v* is the same for a given initial state condition. ε_v* can be easily found by simple thermodynamic considerations (isothermal condition), applying Boyle and Mariotte law reported below (Okamura and Soga, 2006):

$$\varepsilon_v * = \frac{e_0}{1+e_0} \cdot (1 - S_{r0}) \cdot \left(1 - \frac{u_{a,0}}{\sigma}\right) \qquad (3)$$

where σ is the constant confining total stress and $u_{a,0}$ is the initial air pressure.

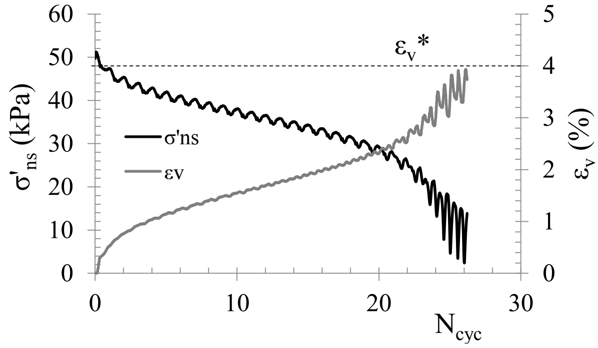


Figure 1. Effective stress (σ'_{ns}) and volumetric strain (ε_v) versus number of cycles (tests performed by Mele et al., 2019a).

Mele et al. (2019a), performing different cyclic triaxial tests on non-saturated sandy specimens, identified the analytical law that links the stress changes and the corresponding values of volumetric strains ε_v. In Figure 2, the results have been plotted in the non-dimensional plane $\varepsilon_v/\varepsilon_v$* - $\sigma'_{ns}/\sigma'_{ns,0}$ with the fitting curve.

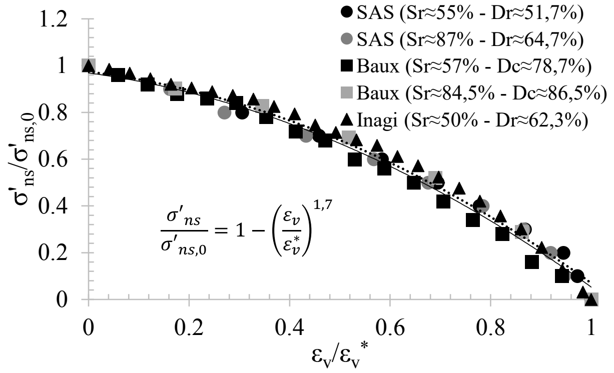


Figure 2. Dimensionless stress ($\sigma'_{ns}/\sigma'_{ns,0}$) versus dimensionless volumetric strain ($\varepsilon_v/\varepsilon_v$*) (modified after Mele et al., 2019a).

Starting from thermodynamic considerations, where a non-saturated soil can be considered as a thermodynamically open system and under the further hypothesis of isothermal process, constant mass and air as an ideal gas, the specific energy to reach liquefaction ($E_{tot,liq}$) can be introduced, defined as the sum of the specific volumetric energy at liquefaction ($E_{v,liq}$) and the specific deviatoric energy at liquefaction ($E_{s,liq}$).

$$E_{tot,liq} = E_{v,liq} + E_{s,liq} \qquad (4)$$

$E_{v,liq}$ is the energy that the soil, under cyclic loading in undrained conditions, spends to change its volume (it is null in undrained cyclic tests on saturated specimens). $E_{v,liq}$ can be defined as the sum of three components: the volumetric specific energy to liquefaction of soil skeleton ($E_{v,sk,liq}$), the volumetric specific energy to liquefaction of water ($E_{w,liq}$), the volumetric specific energy to liquefaction of air ($E_{air,liq}$). They represent the work done by the volumetric deformation of the soil skeleton and the work caused by the flow of mass of water and air into the system of pores, respectively.

The volumetric specific energy of soil skeleton ($E_{v,sk,liq}$) can be found using the following equation:

$$E_{v,sk,liq} = \int_0^{\varepsilon_{v,(Nliq)}} [(\sigma - u_a) + sS_{r0}] \cdot d\varepsilon_v \quad (5)$$

where (σ-u_a) is the net stress, s is the suction; S_r the degree of saturation, while $d\varepsilon_v$ is the increment of volumetric strain during undrained cyclic loading. $E_{v,sk,liq}$ depends just on confining stress, void ratio and degree of saturation ($E_{v,sk,liq} = f(\sigma'_0, e_0, S_{r0})$). This integral can be evaluated using the average curve $\varepsilon_v - \sigma'_{ns}$ of the soil that may be found from Figure 1, known $\sigma'_{ns,0}$ and the potential volumetric strain (eq. 3).

Energetic contributions of water and air are given by the following equation, respectively:

$$E_{w,liq} = -\int_{Sr0}^{Sr,liq} \frac{e(S_r)}{1 + e(S_r)} s(S_r) \cdot dS_r \quad (6)$$

$$E_{air,liq} = \frac{e_0}{1 + e_0}(1 - S_{r,0}) u_{a,liq} \left(\ln \frac{V_{air,0}}{V_{air,liq}} \right) \quad (7)$$

Where $u_{a,liq}$ is the air pore pressure and $V_{air,0}$ and $V_{air,liq}$ are the initial and at liquefaction volume of air, respectively.

$E_{v,liq}$ is a state parameter which depends on confining stress, void ratio and degree of saturation, while it does not depend on the applied CSR. It can be considered as a synthetic parameter which summarizes the three state parameters that identify a cyclic resistance curve (e, σ' and S_r) of a non-saturated soil.

The second contribution in the total specific energy to liquefaction (eq. 4) is given by $E_{s,liq}$. It is due only to soil skeleton and it is defined as the sum of the areas of all the cycles in the ε_s-q plane (D_{cyc}) up to liquefaction, where ε_s is the deviatoric strain and q is the cyclic deviatoric stress. $E_{s,liq}$ can be formally written with the following equation:

$$E_{s,liq} = \sum_{N_{cyc}=1}^{N_{cyc}=N_{liq}} \iint_{D_{cyc}} dq \cdot d\varepsilon_s \quad (8)$$

Unlike $E_{v,liq}$, $E_{s,liq}$ is strongly dependent on the applied CSR, in addition to soil properties and soil state. While the volumetric part can identify the position of the cyclic resistance curve of a soil, the deviatoric part is the energetic variable that, for a given value of $E_{v,liq}$ defines the cyclic resistance CRR and thus N_{liq} (Mele and Flora, 2019).

Total specific energy to liquefaction results to be the parameter ruling the liquefaction behaviour of sandy soils. In particular, $E_{v,liq}$ is a state parameter which can identify the position of a cyclic resistance curve in non-saturated conditions. As reported by Mele and Flora (2019), it can be linked to the difference between the CRR of non-saturated soil (CRR_{ns}) and the CRR of a saturated soil (CRR_s) evaluated at 15 cycles, according to the following equation:

$$\Delta CRR_{15} = -105.7 \cdot \left(\frac{E_{v,liq}}{p_a} \right)^2 + 10.2 \cdot \frac{E_{v,liq}}{p_a} \quad (9)$$

where p_a is the atmospheric pressure, which is introduced to make the relationship non-dimensional. Obviously, when $E_{v,liq}$ increases ΔCRR_{15} increases, as well. In other words, $E_{v,liq}$ can indicate the increment of resistance of a non-saturated soil compared with a saturated one at the same conditions.

On the other hand, $E_{s,liq}$ defines the CRR. A strong correlation between $E_{s,liq}$ and $E_{v,liq}$ has been found by Mele and Flora (2019), whose equation is reported below:

$$E_{s,liq} = 0.297 \cdot p_a \cdot e^{-16.7 \cdot CRR^{ctx} \cdot \left(1 - 5 \cdot \frac{E_{v,liq}}{p_a}\right)^{10}} \quad (10)$$

where all the coefficients have been calibrated by Mele and Flora (2019) to obtain the best fitting of experimental data.

In the attempt to find a relationship between the specific energy spent to liquefaction and N_{liq}, Mele and Flora (2019) proposed the following correlation:

$$\frac{CRR^{ctx}}{\left(1 + \frac{E_{tot,liq}}{p_a}\right)^6} = -0.039 \cdot \ln(N_{liq}) + 0.285 \quad (11)$$

where $E_{tot,liq}$ is given by:

$$E_{tot,liq} = E_{v,liq} + 0.297 \cdot p_a \cdot e^{-16.7 \cdot CRR^{css} \cdot \left(1 - 5 \cdot \frac{E_{v,liq}}{p_a}\right)^{10}} \quad (12)$$

2.2 *Practical application of energetic approach*

The results reported in the previous paragraph highlight as the proposed energetic model may be successfully used to predict the liquefaction resistance of non-saturated sandy soils. It should be specified that soil grading is not taken into account in the two approaches. It could be a relevant effect in the prediction of liquefaction resistance of non-saturated sandy soils. However, the experimental results are not sufficient to introduce explicitly such a dependency. With this limitation, the two approaches may be used as suggests:

1) The *first approach* uses only the volumetric component of specific energy ($E_{v,liq}$). It consists in upwards translation of saturated curve by means of eq. (9); in other words, known the liquefaction resistance of saturated soils (CRR_s), that of non-saturated soil (CRR_{ns}) is given by: $CRR_{ns} = CRR_s + \Delta CRR$.
2) The *second approach* takes into account the specific deviatoric energy, too. Fixed N_{liq}, liquefaction resistance may be estimated via eq. (11), where $E_{tot,liq}$ should be substituted by its expression reported in eq. (12).

The *first approach* needs only the cyclic resistance curve of saturated soils, however, being based on a simple translation of the cyclic resistance curve (ΔCRR does not depend on N_{liq}) it is not able to catch the possible change of curvature of the non-saturated cyclic resistance curve.

On the other hand, the *second approach* needs a few calculation steps more than the first one, being based on the calculation of the total specific energy and not only of its volumetric component. Moreover, the solution of eq. (11) (CRR) may be not immediate, requiring some efforts. However, it has the advantage of not requiring the knowledge of the saturated liquefaction resistance curve to predict the behaviour of the non-saturated soil. The result is not a translation of the CRR-N_{liq} curve, and any shape may be obtained, depending on the combination of specific volumetric and deviatoric energies to liquefaction (Mele and Flora, 2019).

In order to verify and, if necessary, improve the energetic model proposed by Mele et al. (2019a) and Mele and Flora (2019), further non-saturated tests have been analysed according to an energetic perspective. The results have been presented in the following section.

3 ENERGETIC INTERPRETATION OF LIQUEFACTION TESTS

A dataset composed by 33 non-saturated cyclic triaxial tests performed by Mele (2020) have been processed from an energetic point of view. The triaxial apparatus presents a double with the Linkage Double Cell System to measure volume change in undrained condition (Wang et al., 2016b). The specimens (d=50mm and h=100mm) where prepared with moist tamping technique (Dr ranging between 30 and 72%) and consolidated at 50 kPa. For sake of brevity, the testing programme is not reported in the paper being all the tests information available in Mele (2020).

Non-saturated tests have been performed on five sands, whose grain size distribution curves and physical properties have been reported in Figure 3 and Table 1, respectively. Figure 4a reports the experimental results in the $E_{s,liq}$ – CRR^{ctx} plane, confirming that the value of CRR attained in each test, for each soil and initial state, is uniquely related to $E_{s,liq}$. Since state conditions of non-saturated soils during cycling tests are well represented by $E_{v,liq}$, a much more general interpretation can be obtained by plotting the experimental data in the normalized plane in Figure 4b, in which a unique, non-linear relationship links $E_{s,liq}$ to the term $(CRR^{ctx} \cdot (1\text{-}5 \cdot E_{v,liq}/p_a)^{10})$, confirming the effectiveness of the equation proposed by Mele and Flora (2019) (eq. 10).

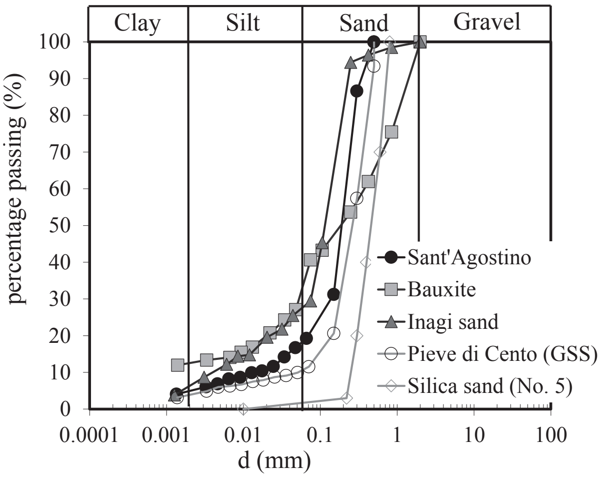
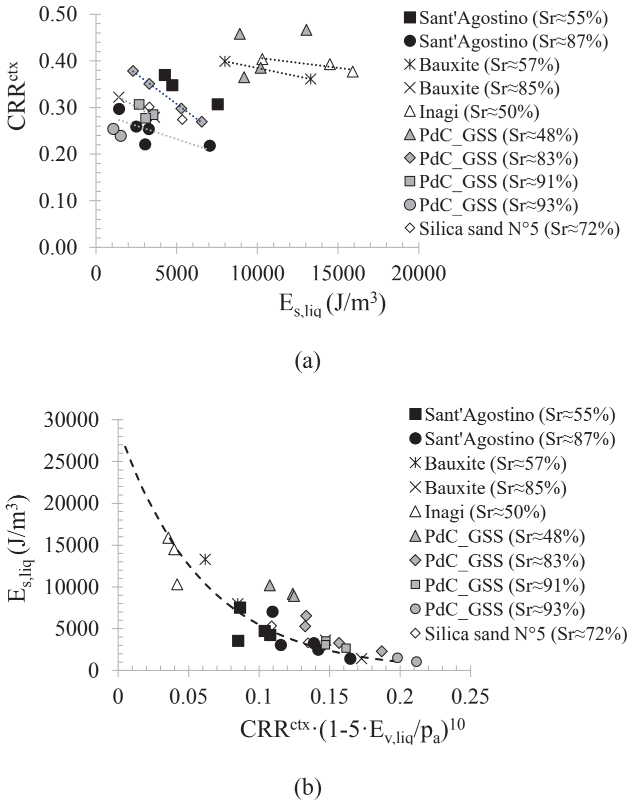


Figure 3. Grain size distribution curves of tested materials.

Table 1. Physical properties of tested soils.

Sand	G_s	U_c	e_{max}-e_{min}	D_{50} (mm)	FC (%)
Sant'Agostino	2.67	16.7	1.01-0.37	0.200	20.0
Bauxite	2.64	400	-	0.200	40.6
Inagi	2.66	30	1.64-0.91	0.115	29.5
Pieve di Cento	2.65	5	0.88-0.44	0.300	12.0
Silica sand N.5	2.64	1.9	1.11-0.69	0.471	0.0



(a)

(b)

Figure 4. CRR^{ctx} vs $E_{s,liq}$ (a); $CRR^{ctx} \cdot (1\text{-}5 \cdot E_{v,liq})^{10}$ vs $E_{s,liq}$ (b).

A strong dependence of $E_{s,liq}$ on N_{liq} is clearly shown in Figure 5a. For each value of specific volumetric energy to liquefaction a linear relationship between $E_{s,liq}$ and N_{liq} may be found (Fig. 5a). It should be noted that for saturated soils ($E_{v,liq} = 0$), $E_{s,liq}$ is approximately constant with N_{liq}, confirming the results of Mele et al. (2019b). On the contrary, when $E_{v,liq}$ increases - and then the liquefaction resistance of soil increases – two aspects may be observed:

- $E_{s,liq}$ increases, congruently with the results shown in Figure 4b;
- the slope of the relationship N_{liq} – $E_{s,liq}$ increases. In other words, for higher $E_{v,liq}$, the dependence of $E_{s,liq}$ on N_{liq} becomes relevant.

The second observation is clearer by plotting for N_{liq}=15, $E_{v,liq}$ versus $E_{s,liq}$ in Figure 5b. The relationship is linear and the relative equation is reported in the same figure.

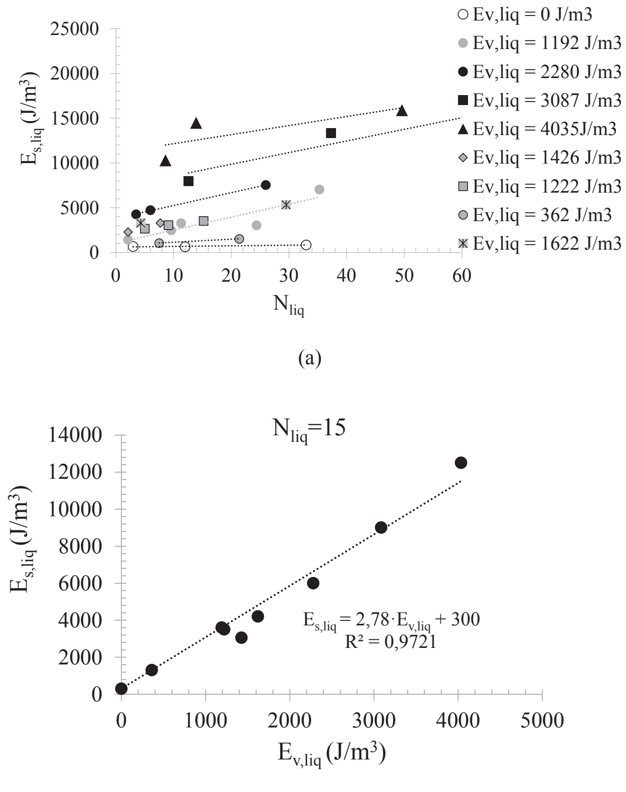
These findings are extremely important from a practical point of view because allow to simplify the second approach of the energetic model to predict liquefaction resistance of non-saturated soils (§ 2.2). In fact $E_{tot,liq}$ may be computed by means of the following simple expression:

$$E_{tot,liq} = 3.78 \cdot E_{v,liq} + 300 \qquad (13)$$

Substituting eq. (13) in eq. (11):

$$\frac{CRR^{ctx}}{\left(1 + \frac{3.78 \cdot E_{v,liq} + 300}{p_a}\right)^6} = -0.039 \cdot \ln\left(N_{liq}\right) + 0.285 \quad (14)$$

Eq. (14) makes simpler the use of the second approach, evaluating CRR_{ns} (for a fixed N_{liq}), just knowing $E_{v,liq}$, evaluated by means of first approach.



(a)

(b)

Figure 5. $E_{s,liq}$ with N_{liq} for fixed values of $E_{v,liq}$ (a) and $E_{s,liq}$ with $E_{v,liq}$ for N_{liq} = 15 (b).

The experimental results have also been plotted in Figure 6 in the plane N_{liq} - $CRR^{ctx}/(1+E_{tot,liq}/p_a)^6$. Once again, the experimental data confirm the correlation proposed in previous research work, and then eq. (11).

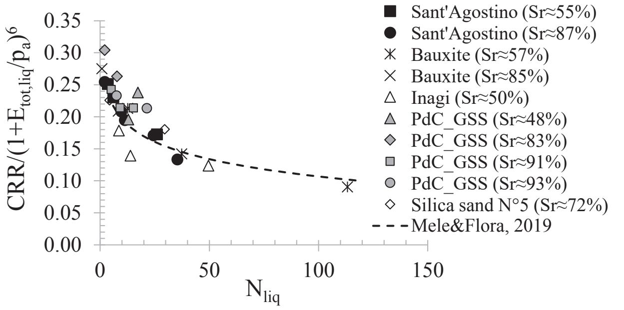


Figure 6. Normalized cyclic resistance curve for cyclic triaxial tests.

4 PREDICTION OF LIQUEFACTION RESISTANCE OF NON-SATURATED SOILS

In order to validate the proposed energetic approaches to predict liquefaction resistance of non-saturated sandy soils, some experimental data published by Tsukamoto et al., (2014) have been used. The tests have been performed on Inagi sand, whose grain size distribution curve and physical properties have been already reported in Figure 3 and Table 1.

All tests - on saturated and non-saturated soils - have been consolidated isotropically at 98 kPa and then subjected to cyclic loading. Tsukamoto et al. (2014) reported the results of cyclic triaxial tests (Inagi sand with D_r=80%) in the plane S_r – $CRR_{,20}$, where $CRR_{,20}$ is the cyclic resistance ratio evaluated at 20 cycles (Fig. 7). They showed as liquefaction resistance increases when S_r decreases, highlighting the effectiveness of desaturation as mitigation technique against liquefaction.

Both first and second energetic approaches have been used to estimated $CRR_{,20}$ of Inagi sand (D_r=80%) in non-saturated conditions (70 ≤S_r (%)≤ 95). Firstly, the specific volumetric component of energy at liquefaction has been evaluated via eqs. (5-6-7), where the volumetric potential strain has been evaluated according to eq. (3). $E_{v,liq}$ computed for 70, 80, 90 and 95% is enough to apply first approach (§ 2.2), which consist in translating the saturated curve to a quantity, function of $E_{v,liq}$ (eq. 9). In this case, $CRR_{,20}$ has been estimated summing $\Delta CRR_{,20}$ ($\approx\Delta CRR_{,15}$) to the liquefaction resistance of saturated sand (0.12).

In Table 2 potential volumetric strain, $E_{v,liq}$ and $CRR_{,20}$ evaluated according to first approach is summarized.

In second approach $CRR_{,20}$ of non-saturated soils may be estimated directly from eq. (14), obviously fixing N_{liq}=20. These results are summarized in Table 2 for each degree of saturation.

Table 2. Volumetric energetic components of total specific energy at liquefaction and $CRR_{,20}$ evaluated according to two approaches.

			1st Approach	2nd Approach
S_r (%)	ε_v^* (%)	$E_{v,liq}$ (J/m^3)	$CRR_{,20}$	$CRR_{,20}$
70	8.12	4914	0.360	0.560
80	5.41	3300	0.350	0.370
90	2.70	1559	0.264	0.242
95	1.35	655	0.188	0.200

In Figure 7 $CRR_{,20}$ evaluated by means of first and second approach have been plotted with S_r together with the experimental curve of Tsukamoto et al. (2014).

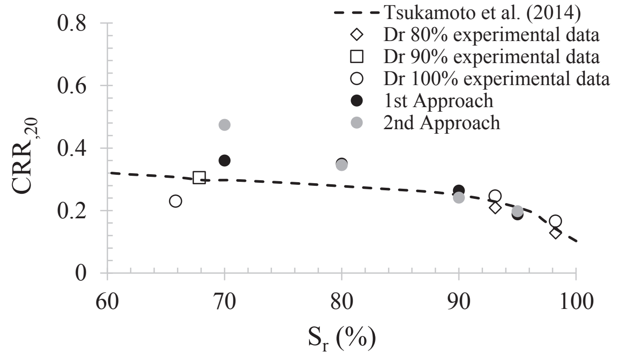


Figure 7. Experimental relationship between S_r and $CRR_{,20}$ and theoretical dots simulated according to energetic approaches (Inagi sand, D_r=80%).

The results show that the two approaches tend to provide similar results for high degrees of saturation ($S_r \geq 80\%$), while when S_r is lower than 80%, the second approach returns much higher $CRR_{,20}$ and thus, resulting not conservative. On the contrary, the first approach seems to fit well the interpolating curve of experimental results shown by Tsukamoto et al. (2014).

4.1 *Recommendations for practitioners*

The energetic interpretation of liquefaction tests has allowed to introduce two different approaches to predict liquefaction resistance of non-saturated soils. They result as a useful and promising design tool, which allows to establish the degree of saturation to apply in situ to have a fixed value of CRR.

However, it should be emphasized that shown results have been obtained performing laboratory tests and then, the model should be validated at large scale, too. In the meantime, the proposed model can be used scaling the CRR for a value, which is generally taken equal to 0.70.

Regarding the comparison between the two approaches, the results shown in this paper highlight that:

- for low degrees of saturation ($S_r < 80\%$), the use of the first approach is recommended;
- for high degrees of saturation ($S_r \geq 80\%$), the two approaches return similar results. However, if the cyclic resistance curve is needed the second approach is strongly recommended, on the contrary, the simpler one can be used. Obviously, in the latter case, the liquefaction resistance of saturated (non-treated) soils should be known.

5 CONCLUSIONS

Liquefaction triaxial tests on non-saturated sandy soils have been interpreted from an energetic point of view. The results confirm the correlation proposed in previous research works, but allow to further simplify the energetic approaches to predict liquefaction resistance of non-saturated sands. The energetic approaches have been used to simulate the liquefaction resistance of non-saturated sand, which has been tested by other authors. The results show that the model satisfactorily predict liquefaction resistance of non-saturated soil. However, a comparison between the two possible approaches is presented. They return similar results with high degrees of saturation, on the contrary, the second approach is less conservative at low degrees of saturation.

Further laboratory tests should be performed to investigate the possible effect of grain size in the proposed model and finally, the model should be validated at large scale by performing centrifuge tests or in field trial.

6 ACKNOWLEDGEMENTS

Non-saturated tests have been performed at University of Tokyo under the supervision of Prof. Junichi Koseki. The authors deeply thank him and his research team for their suggestions and discussions on tests results.

7 REFERENCES

Bao, X., Jin, Z., Cui, H., Chen, X., & Xie, X. 2019. Soil liquefaction mitigation in geotechnical engineering: An overview of recently developed methods. *Soil Dynamics and Earthquake Engineering*, *120*, 273-291.

Bishop A. W. and Blight G. E. 1963. Some aspects of effective stress in saturated and partly saturated soils. *Gèotechnique*, 13(3): 177-197.

Chaney R. 1978. Saturation effects on the cyclic strength of sands. *Proc., ASCE Special Conf. on Earthquake Engineering and Soil Dynamics*, New York, 342-358.

Da Fonseca, A. V., Soares, M., & Fourie, A. B. (2015). Cyclic DSS tests for the evaluation of stress densification effects in liquefaction assessment. *Soil Dynamics and Earthquake Engineering*, *75*, 98-111.

El Mohtar C.S., Bobet A., Drnevich V.P., Johnston C.T., Santagata M.C. Pore pressure generation in sand with bentonite: from small strains to liquefaction. Gèotechnique, 2014, 64 (2): 108-107.

Eseller-Bayat E., Yegian MK., Alshawabkeh A., Gokyer 2012. Prevention of liquefaction during earthquakes through induced partial saturation in sands. *Geotechnical engineering: new horizons*. IOS Press, Amsterdam, 188-194.

Flora, A., Bilotta, E., Chiaradonna, A., Lirer, S., Mele, L., & Pingue, L. 2020. A field trial to test the efficiency of induced partial saturation and horizontal drains to mitigate the susceptibility of soils to liquefaction. *Bulletin of Earthquake Engineering*, 1-30.

Ishihara K., Tsukamoto Y., Nakazawa H., Kamada K., and Huang Y. 2002. Resistance of partly saturated sand to liquefaction with reference to longitudinal and shear wave velocities. *Soil Foundation.,* 42(6): 93-105.

Ishihara, K. 1993. Liquefaction and flow failure during earthquakes. *Geotechnique*, *43*(3), 351-451.

Mahmoodi, B., & Gallant, A. (2021). Assessing persistence of entrapped gas for induced partial saturation. *Journal of Geotechnical and Geoenvironmental Engineering*, *147*(3), 04020184.

Mele L. 2020. Experimental and theoretical investigation on cyclic liquefaction and on the effects of some mitigation techniques. *PhD Thesis,* Università degli Studi di Napoli Federico II, Napoli, Italy.

Mele L., Tan Tian J., Lirer S., Flora A., Koseki J. 2019a. Liquefaction resistance of unsaturated sands: experimental evidence and theoretical interpretation. *Géotechnique*. DOI: 10.1680/jgeot.18.p.042

Mele, L., & Flora, A. 2019. On the prediction of liquefaction resistance of unsaturated sands. *Soil Dynamics and Earthquake Engineering*, *125*, 105689.

Mele, L., Lirer, S., Flora, A. 2019b. The specific deviatoric energy to liquefaction in saturated cyclic triaxial tests. *7th International Conference on Earthquake Geotechnical Engineering, 7ICEGE, Rome (Italy)*, 17-20 June 2019. ISBN: 978-0-367-14328-2 (Hbk), eISBN: 978-0-429-03127-4 (Book).

Okamura M. and Soga Y. 2006. Effects of pore fluid compressibility on liquefaction resistance of partially saturated sand. *Soils and Foundations* 46: 695-700.

Park, S. S., & Kim, Y. S., 2013. Liquefaction resistance of sands containing plastic fines with different plasticity. *Journal of geotechnical and geoenvironmental engineering*, *139*(5), 825-830.

Sadrekarimi A. & Jones, 2019. Effect of overburden pressure on cyclic resistance of Fraser River sand. *7th International Conference on Earthquake Geotechnical Engineering, 7ICEGE, Rome (Italy)*, 17-20 June 2019. ISBN: 978-0-367-14328-2 (Hbk), eISBN: 978-0-429-03127-4 (Book).

Tsukamoto, Y., Kawabe, S., Matsumoto, J., & Hagiwara, S. 2014. Cyclic resistance of two unsaturated silty sands against soil liquefaction. *Soils and Foundations*, *54*(6), 1094-1103.

Unno T., Kazama M., Uzuoka R., Sento N. 2008. Liquefaction of unsaturated sand considering the pore air pressure and volume compressibility of the soil particle skeleton. *Soils and Foundation* 48(1): 87-99.

Wang H., Koseki J., Sato T., Chiaro G., and Tan Tian J. 2016a. Effect of saturation on liquefaction resistance of iron ore fines and two sandy soils. *Soils and Foundations* 56(4): 732–744.

Wang H., Sato T., Koseki J., Chiaro G., Tan Tian J. (2016b) A system to measure volume change of unsaturated soils in undrained cyclic triaxial tests. Geotechnical Testing Journal.**39**(**4**): 532–542.

Yegian M. K., Eseller-Bayat E., Alshawabkeh A. and Ali S. 2007. Induced-Partial Saturation for liquefaction mitigation: experimental investigation. *Journal of geotechnical and geoenviromental engineering ASCE* 133(4): 372-380.

Yilmaz, Y. Ü. K. S. E. L., Mollamahmutoglu, M. U. R. A. T., Ozaydin, V., & Kayabali, K. 2008. Experimental investigation of the effect of grading characteristics on the liquefaction resistance of various graded sands. *Engineering geology*, *100*(3-4), 91-100.

Yoshimi Y., Yanaka K., and Tokimatsu K. 1989. Liquefaction resistance of partially saturated sand. *Soils Found.*, 29(2): 157-162.