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Effect of fiber and cement additives on the small-strain stiffness behavior of Toyoura sand

Effet des additifs pour fibres et ciments sur le comportement de rigidité aux petites déformations du sable de Toyoura

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ABSTRACT: In this study, monotonic triaxial drained compression tests are performed on medium dense specimens of Toyoura sand-cement-fiber mixtures with different percentages of fiber and cement (e.g., 0%–3%) additives. The experimental results indicate that behavior of the mixtures is significantly affected by the concentration of fiber and cement additives. Based on a comprehensive set of test results, modifications to the series of equations were developed that can be used to evaluate the shear modulus and mobilized stress curves at small-strain levels. The experimental results and model comparison show that the elastic threshold strain (γ_e), reference strain (γ_r), increases with fiber and cement additives. In addition, the range of curvature parameter, from 0.88 to 1.0, provides a good comparison with the results of small-strain measurements. Overall, the comparison of the results and model shows that the small-strain measurements obtained using local strain transducers fall within the range of model upper and lower bound curves. The results of the unreinforced, fiber, and cemented sand shows a close agreement with the model mean curve, but fiber-reinforced cemented sand shows a good comparison with model upper bound.

RÉSUMÉ : Dans cette étude, des essais de compression monotone triaxiale drainée sont effectués sur des échantillons de densité moyenne de mélanges sable-ciment-fibre de Toyoura avec différents pourcentages d'additifs de fibres et de ciment (par exemple, 0 % - 3 %). Les résultats expérimentaux indiquent que le comportement des mélanges est significativement affecté par la concentration des additifs de fibres et de ciment. Sur la base d'un ensemble complet de résultats d'essais, des modifications de la série d'équations ont été développées qui peuvent être utilisées pour évaluer le module de cisaillement et les courbes de contrainte mobilisée à des niveaux de faible déformation. Les résultats expérimentaux et la comparaison des modèles montrent que la déformation élastique seuil (γ_e), déformation de référence (γ_r), augmente avec les additifs fibre et ciment. De plus, la plage de paramètres de courbure, de 0.88 – 1.0, fournit une bonne comparaison avec les résultats des mesures de petites déformations. Dans l'ensemble, la comparaison des résultats et du modèle montre que les mesures de petites déformations obtenues à l'aide de transducteurs de déformation locaux se situent dans la plage des courbes limites supérieure et inférieure du modèle. Les résultats du sable non renforcé, fibreux et cimenté montrent un accord étroit avec la courbe moyenne du modèle, mais le sable cimenté renforcé de fibres montre une bonne comparaison avec la limite supérieure du modèle.

KEYWORDS: small-strain stiffness; ground improvement; ground remediation; local strain; triaxial test

1 INTRODUCTION

The Great East Japan earthquake of 2011 generated a huge quantity of disaster waste and tsunami deposits, which required proper treatment and disposal. To effectively use these waste soils in sustainable geotechnical infrastructures, it is essential to understand the mechanical behavior in their native (pure) or mechanically stabilized form (amended with cement and fiber). Laboratory and field testing has shown that the stress-strain behavior of sands can be highly nonlinear, even at stresses well below the peak strength of the material. One of the first comprehensive studies where the parameters that control nonlinear soil behavior were identified was the study by Hardin and Drnevich (1972a and b). The empirical equations proposed by Hardin and Drnevich (1972b) account for the effects of plasticity index, overconsolidation ratio and confining pressure mainly through adjusting reference strain. The effect of soil type, number of loading cycles, loading frequency and saturation, amongst other aspects have also been taken into consideration (e.g. Darendeli, 2011; Hurtado and Newson, 2016). Iwasaki et al., (1978) studied the impact of confining pressure, but the study was limited to observations on clean narrow graded sands tested at low pressures.

The initial stiffness of a composite material was affected by the different characteristics of the steel and polyamide fibers (e.g., stiffness, roughness, rigidity, size etc). Previous research with mixtures of steel fibers and sand (Michalowski and Zhao, 1996) indicated that even larger fiber concentrations (e.g., 1.25% by volume) had no adverse effect on the initial stiffness. In addition, steel fiber had a reinforcement effect only slightly higher than less stiff polyamide fiber of the same geometry. It was further concluded that, this difference might be attributed to a larger interfacial friction angle of steel fibers compared to polyamide fibers. In addition, it was reported that the strain levels or mobilization resistance for steel fiber (e.g. stiff) is greater than that of polyamide fibers (e.g. flexible) due to their greater stiffness. The literature review on cemented sand shows that natural or artificial cementation increases the small-strain stiffness behavior (G_0) of sands (Sharma and Fahey 2004) Sharma and Fahey (2004) reported that small-strain stiffness (G_0) to be for cemented sands practically independent of the mean stress and dependent on cementation until it was reached a threshold stress corresponding to the onset of major structure degradation. Yun and Santamarina (2005) indicated for artificially cemented soils an increase of G_0 with increasing stress after yielding and the values of G_0 remained higher than for the reconstituted soils. Cementation appears to control only

G_0 of clays below isotropic or vertical yield stress and the pressure dependency appears to prevail at higher stresses. The latter findings lead to the conclusion that the stiffness of the cemented soils is strongly increased by cementation and independent of confining pressure. Mair (1993) and Xu et. al. (2014) proposed that the stiffness of a soil is constant below a strain level of 0.001 % (e.g. 10-5) and reduces significantly with an increase in strain level (i.e., above 0.001 %). In recent decades, researchers have attempted to validate the approximate relationship between stiffness and strain level by employing different instruments, as shown in Fig. 1.

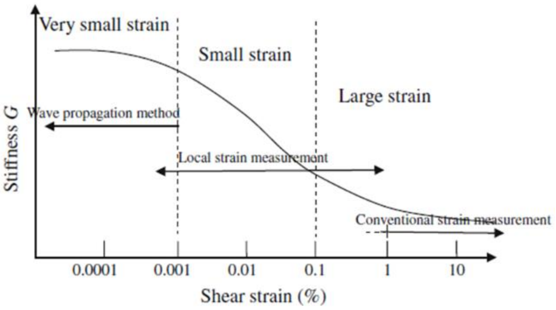


Figure 1. Typical modulus degradation measurement of soil stiffness in laboratory test (after Mair (1993)).

Fahey and Carter (1993) proposed a hyperbolic model to characterise modulus reduction as a function of shear strength mobilization, as seen in Equation (1). This function requires maximum shear stress (τ_{max}), and an estimate of the small-strain shear modulus (G_0) to be the value of shear modulus at shear strain of 10-6 and is assumed to be constant below this value, as well as empirical parameters f and g. Fahey and Carter, (1993) showed some success fitting this three-parameter model to the data of a wide range of uncemented soils.

$$\frac{G}{G_0} = 1 - f\left(\frac{\tau}{\tau_{max}}\right)^g \qquad (1)$$

Darendeli, (2001) proposed a modified hyperbolic model (Equation 2)based on testing of intact sand-gravel sample;

$$\frac{G}{G_0} = \left[\frac{1}{(1+\gamma/\gamma_r)^a}\right] \qquad (2)$$

where a is called the curvature parameter, and γ_r is the reference strain value at which G/G0 = 0.50. This model uses only two parameters, and the reference strain provides an efficient normalization of the shear strain. To better understand the non-linear elastic behavior of sands, and produce a generalized functional relationship, Oztoprak and Bolton, (2013) conducted a metastudy of the secant shear modulus degradation curves of 454 tests of uncemented sands from the literature. This curve-fitting process led to new interpretations and definitions that enable better predictions of the shear modulus degradation of sands with strain, based on soil classification data.

In order to enhance the current database on small-strain stiffness behavior and stiffness degradation of amended soils, there is a need to further investigate the effect of fiber and cement additives on the small-strain stiffness (G_0) of sands. In addition, the aim of the work is to develop a rational method and propose few modifications to the series of equations that can be used to evaluate the shear modulus and mobilized stress curves at small-strain levels. Furthermore, the experimental results of amended soils are compared with the Oztoprak and Bolton, (2013) upper bound and lower bound stiffness degradation models. The modified version of hyperbolic equation for amended soils (e.g. fiber only, cement only and fiber reinforced cemented sands) leads to a wide range of values for elastic threshold strain ($\boldsymbol{\gamma_e}$), the reference strain ($\boldsymbol{\gamma_r}$), the range of curvature parameter (a).

2 MATERIALS AND METHODS

2.1 *Materials*

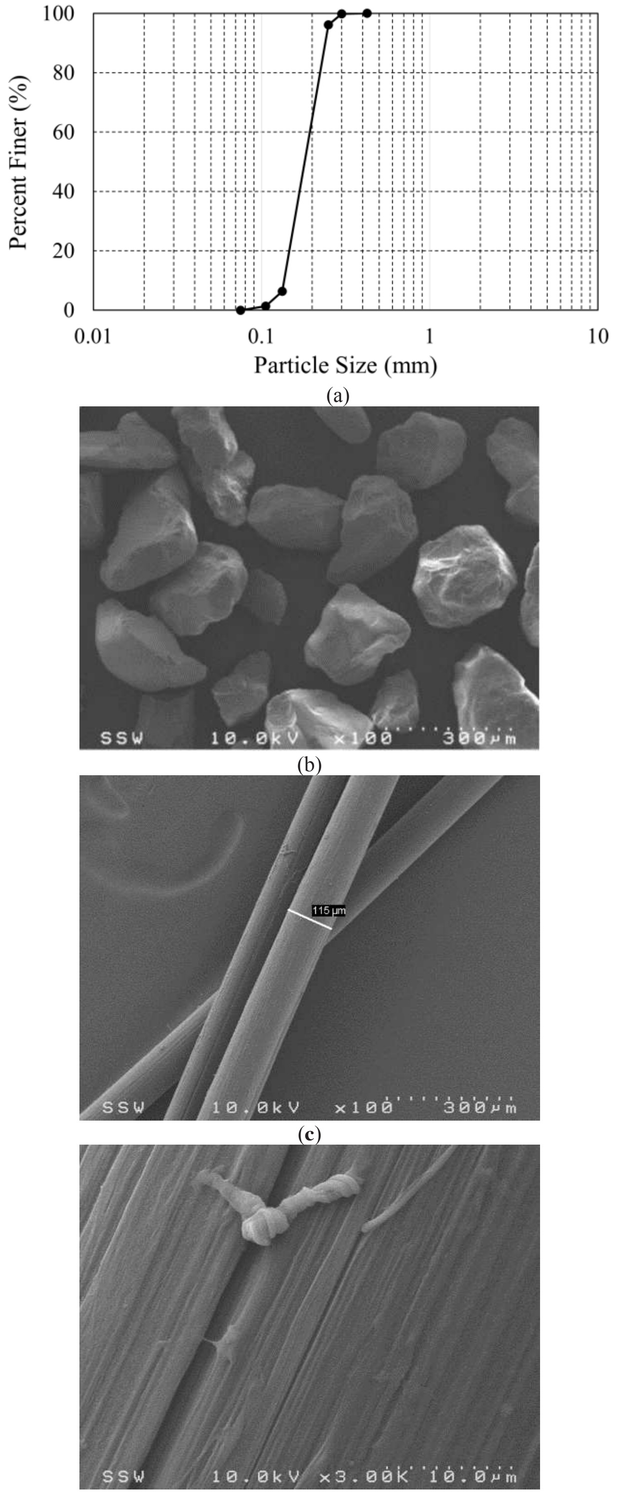
The three different types of materials [e.g. Toyoura sand, polyvinyl alcohol (PVA) fibers, and ordinary Portland cement (OPC)] were employed in this study. Toyoura sand has been previously used as a benchmark material in a number of investigations and is composed of 75% quartz, 22% feldspar, and 3% magnetite. It can be found primarily in the coastal regions of the Pacific Ocean in Japan. The soil has a uniformity coefficient (C_u) of 1.24, a minimum void ratio (e_{min}) of 0.62, a maximum void ratio (e_{max}) of 0.95 and a specific gravity of 2.65. A typical grain size distribution of Toyoura sand is presented in Figure 2a. Toyoura sand is described as having angular to sub-angular particles, is fine grained and poorly graded, which is confirmed by the low coefficient of uniformity and coefficient of curvature, according to the classification of SP by the Unified Soil Classification System (USCS) (Schmidt, 2015).

Figure 2b shows a scanning electron microscopic (SEM) image of Toyoura sand and provides an indication of the size, shape and texture of the particles (Schmidt, 2015). The Polyvinyl alcohol (PVA) fibers used in this study have a specific gravity of 1.3. Nominal dimensions of the individual fibers are 12 mm long, with a diameter of 0.11 mm (Safdar et al., 2020).

The fibers have a Young's modulus of 28 GPa and a tensile strength of 1200 MPa (Kuraray Cooperation Limited, Japan). Ordinary Portland Cement Type-I (OPC-I) shipped from Ube-Mitsubishi Cement Corporation in Japan was used as a cementing agent and added as a percent by mass to each specimen. OPC-I has a specific gravity of 3.15 and a composition consisting of approximately 63% tricalcium silicate, 12% di-calcium silicate, 5% tri-calcium aluminate, 11% tetra-calcium alumino-ferrite (ASTM C150/C150M-12). These cementitious and fibrous additives have been previously used to model the monotonic and cyclic properties of amended Toyoura sand (Nakamichi and Sato, 2013; Schmidt, 2015; Safdar, 2018, Safdar et al, 2020).

2.2 *Sample Preparation, Testing Apparatus, Testing Procedure, and Testing Program*

The under-compaction moist tamping technique was employed for sample preparation (Ladd, 1978; Diambra, 2010). Cylindrical specimens were formed in five layers with a height of 100 mm and a diameter of 50 mm (ASTM Standard D7181). Most of the samples were prepared to a target dry density value of ρ_d = 1.49 g/cm^3. This density was selected to replicate a field condition (i.e. medium dense state) for the compacted soil and for comparison with previously published studies ((Nakamichi and Sato, 2013; Schmidt, 2015). Unreinforced, fiber only, cement only, and fiber reinforced cemented Toyoura sand samples were prepared and mixed at 10 percent of water content by dry mass of soil. Figure 2 shows a local strain transducer mounted on a typical sample. All Cemented samples were cured for 3 days.



(a)

(b)

(c)

(d)

Figure 2. (a) Grain size distribution curve for Toyoura sand (Safdar 2018) (b) Toyoura sand 100× optical zoom (c) PVA fiber 100× optical zoom (d) PVA fiber 3000x optical zoom (after Schmidt (2015)).

Past research on cemented sands has focused almost exclusively on longer curing durations (e.g., 7-28 days) and higher cement contents (e.g., 0-16%). Overall, sand-cement-fiber composites have been observed to be more effective when specimens are cured for longer durations. These findings are likely to be due to a better contact between the sand-cement-fiber matrix bonding, cement hydration, and improved interaction due to a longer curing period. Limited studies are reported to determine the stiffness and strength of sand-cement-fiber composites for shorter curing duration (e.g., 3 days) and lower cement content (e.g. 0-3% by dry mass of soil). Hence, further laboratory investigations on lower cement content (e.g., 0-3%) and short curing duration is essential in relation to field applications.

Table 1 summarizes the testing program used to evaluate the effect of fiber and cement content on the small-strain shear behavior. A unique test ID is used for the representation of a test [i.e. LSM-C0F0M0 represents local strain measurement (LSM) for cement (C) = 0%, fiber (F) = 0% and silt (M) = 0%]. A GDS triaxial apparatus was employed to conduct consolidated drained (CD) compression triaxial tests to investigate the behavior of unreinforced, fiber only, cement only, and fiber reinforced cemented Toyoura sand specimens.

Hall effect local strain transducers were mounted in the middle third of the sample (see Fig. 3), which is less restrained than the end zones. It is highly desirable that axial deformations are measured locally, if small deformations moduli are to be found. The range, resolution and accuracy of Hall effect transducer is ± 0.3 mm, <0.1 μm, 0.2% respectively (GDS Instruments). Triaxial tests use external Linear Variable Differential Transformers (LVDTs) to measure large strains (e.g., 0.01%–10%). However, these LVDTs measure the global strain applied and not the local strain developed in the triaxial soil sample during shearing. Accurate determination of soil small-strain stiffness is difficult to achieve using global LVDTs attached to the actuator of automated triaxial system in routine laboratory testing. In this study, Hall effect local strain transducers are used to investigate the small-strain stiffness behavior of unreinforced, fiber, cemented, and fiber-reinforced cemented Toyoura sand specimens in triaxial tests.

Figure 3. Hall effect local strain transducer mounted on a typical sample.

All of the specimens were saturated with de-aired water and CO_2 until a B-value of at least 0.96 was reached, before starting the consolidation stage. Higher B-values were possible in the cemented samples due to the application of higher back pressures (e.g., 320 kPa), short curing duration (e.g., 3 days), and lower cement contents (0-3%). All of specimens for the consolidated drained (CD) tests were isotropically consolidated to the desired mean effective stress (e.g., 100 kPa) under computer control. The consolidation stage was continued until 100% primary consolidation was reached. The rate of axial displacement used to shear all of the specimens was 0.06 mm/min (Head, 1986; Schmidt, 2015; Safdar, 2018) to eliminate any concerns over rate effects, when comparing the results.

Table 1. Testing program for local strain measurements.

Test No.	Test ID	Mean Effective Stress, p', kPa	Cement Content, %	Fiber Content, %
		Sand Only		
1.	LSM-C0F0M0	100	0	0
		Fiber Only		
2.	LSM-C0F0.5M0	100	0	0.5
3.	LSM-C0F1M0	100	0	1
4.	LSM-C0F2M0	100	0	2
		Cement Only		
5.	LSM-C1F0M0	100	1	0
6.	LSM-C2F0M0	100	2	0
7.	LSM-C3F0M0	100	3	0
8.	LSM-C4F0M0	100	4	0
		Fiber + Cement		
9.	LSM-C3F1M0	100	3	1
10.	LSM-C3F2M0	100	3	2
11.	LSM-C3F3M0	100	3	3
12.	LSM-C2F1M0	100	2	1

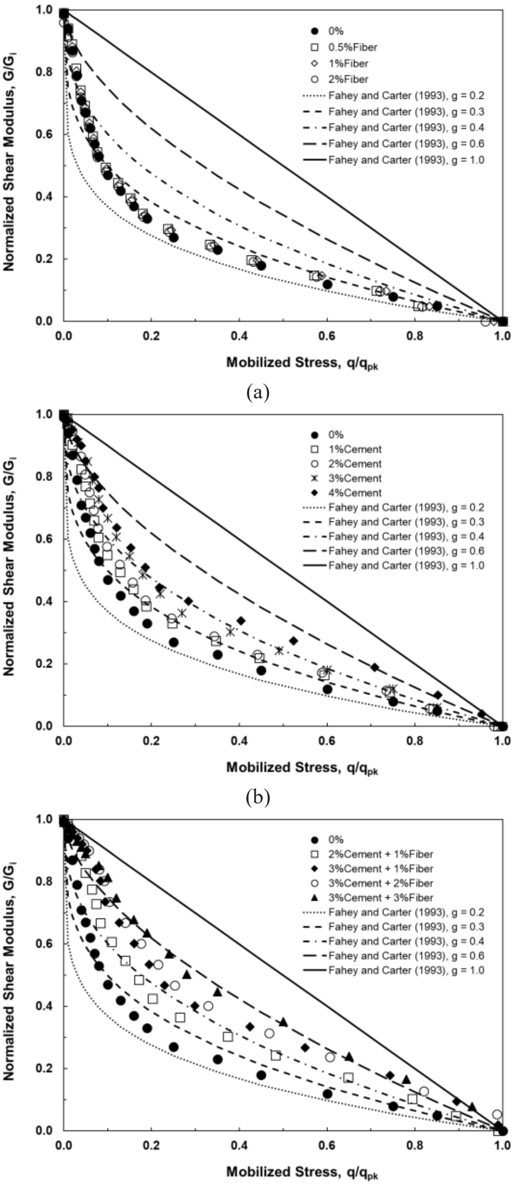
3 RESULTS AND DISCUSSION

Figure 4 shows the normalized shear modulus reduction (G/G_i) versus mobilized stress ($^{q}/_{q_{pk}}$) curves. Where, G = shear modulus at any shear strain level and Gi = initial shear modulus. The value of initial shear modulus (G_i) is obtained from the range of local strain measurements (e.g., reference strain, $\gamma_r =$ 0.00013% to 0.00024%). Fahey and Carter (1993) presented similar test results in terms of modulus reduction versus mobilized stress for uncemented granular soils. They proposed a simple hyperbolic relationship (Equation 3) for clean sands with a limited range of exponents (0.2–0.4) as shown below:

$$\frac{G}{G_0} = \left[1 - (q/q_{pk}\right]^{g} \qquad (3)$$

where G/G_0 = shear modulus reduction, $^{q}/_{q_{pk}}$ = mobilized stress and g = an exponent to encompass laboratory test data.

For pure Toyoura sand and fiber-only reinforced sand, it can be seen that the results agree well with the hyperbolic model (Equation (3)) employing an exponent value of 0.2–0.3. For purely cemented sand, the results show close agreement adopting values in the range of 0.3–0.4. However, a slightly greater value of exponent (e.g., 0.4–0.6) is required to fit the results of the fiber-reinforced cemented sand. A range of exponent, g = 0.2–0.4 was suggested by Fahey and Carter (1993) for uncemented granular soils. In contrast, it can be seen that for cemented and fiber-reinforced sands, the range of exponent lies between 0.3 and 0.6, showing a more intense decay of stiffness with straining.



(a)

(b)

(c)

Figure 4. Shear modulus reduction (G/G_i) versus mobilized stress ($\frac{q}{q_{pk}}$) curves from drained compression tests for various Toyoura sand specimens ($\sigma'_c = 100$ kPa). (a) Pure Sand, and 0–2% Fibers; (b) Pure Sand, and 0–4%; (c) Pure Sand, 0–3% Cement and 0–3% Fibers.

Oztoprak and Bolton (2013) proposed a generic relationship (Equation 4) for the G/G_0 versus shear strain (ε_q) curves based on a database of 454 tests from the literature. Three curve fitting parameters control the shape of the curve (see Equation (4)): (1) an elastic threshold strain (γ_e), up to which the elastic shear modulus is constant at G_0, and which enables the expression to cover cementation and interlocking effects at small-strains; (2) a reference strain (γ_r), the shear strain at which the secant modulus reduces to 0.5 G_0—the two characteristic strains were found to vary with sand type (e.g., uniformity coefficient), state of the soil (e.g., void ratio, relative density), and mean effective stress; and lastly, (3) a curvature parameter (a), which controls the rate of modulus reduction. An average value of curvature parameter, a = 0.88, was employed for a database of 379 tests on uncemented sands.

$$\frac{G}{G_0} = \frac{1}{1+\left[\frac{\gamma-\gamma_e}{\gamma_r}\right]^{a}} \qquad (4)$$

Where γ_e = elastic threshold strain, γ_r = reference strain, and a = curvature parameter

Figures 5–7 show G/G_i versus shear strain (ε_q) curves from similar drained triaxial tests at varying cement (1%–4%) and fiber (0.5%–3%) contents. Table 2 shows the values of best-fit parameters proposed by Oztoprak and Bolton (2013) and for unreinforced and reinforced Toyoura sand.

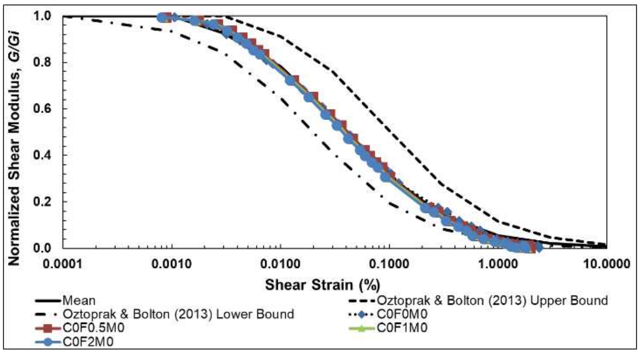


Figure 5. Pure Sand, and 0.5–2% Fibers. G/G_i versus shear strain (ε_q) curves from CD compression tests for unreinforced and fiber-reinforced Toyoura sand specimens consolidated to 100 kPa mean effective stress at different fiber contents (0–2%)

Table 2. Comparison of curve-fitting parameters for unreinforced and reinforced Toyoura Sand with Oztoprak and Bolton (2013).

Sample ID	Elastic Threshold Strain, γ_e	Reference Strain, γ_r	Curvature Parameters, a
C0F0M0	0.001	0.043	0.88
C0F0.5M0	0.0008	0.042	0.88
C0F1M0	0.0007	0.040	0.88
C0F2M0	0.0007	0.039	0.88
C1F0M0	0.0009	0.048	1
C2F0M0	0.001	0.050	1
C3F0M0	0.0012	0.052	1
C4F0M0	0.0014	0.056	1
C2F1M0	0.0015	0.065	1
C3F1M0	0.0018	0.074	1
C3F2M0	0.0020	0.076	1
C3F3M0	0.0022	0.080	1
Oztoprak and Bolton (2013)			
Lower Bound	0	0.02	0.88
Mean	0.0007	0.044	0.88
Upper Bound	0.003	0.1	0.88

It can be seen in Figure 5 that the elastic threshold strain (γ_e) ranges from 0.0007% to 0.001% for unreinforced and fiber-reinforced sand. This range slightly increases to 0.0009%–0.0014% for cemented sand, shown in Figure 6. For the fiber-reinforced cemented sand shown in Figure 7, the threshold strain increases to a range of 0.0015%–0.0022%. The ranges for the reference strain (γ_r), for unreinforced and fiber-reinforced sand (0.039%–0.043%), cemented sand (0.048%–0.056%), and fiber-reinforced cemented sand (0.065%–0.08%) are also shown in Table 2. In addition, it can be seen that the curvature parameter (a) for unreinforced and fiber-reinforced sand was 0.88, and 1.0 for cemented and fiber-reinforced cemented sand. The range of curvature parameter from 0.88 to 1.0 provides a good comparison with the results of local strain. Overall, the comparison of the results and model shows that the small-strain results obtained using local strain transducers fall within the range of model upper and lower bound curves. The results of the unreinforced, fiber-reinforced, and cemented sand show a close agreement with the model mean curve, but fiber-reinforced cemented sand shows a good comparison with model upper bound.

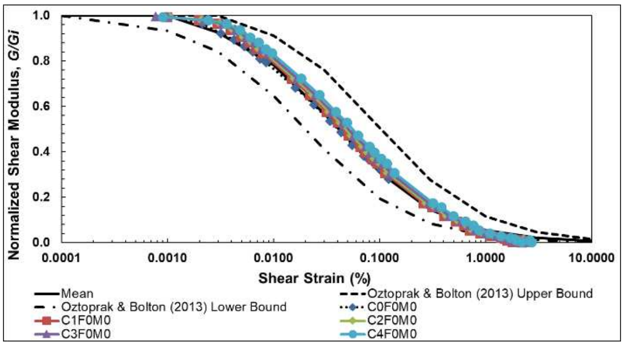


Figure 6. Pure Sand, and 1–4% Cement. G/G_i versus shear strain (ε_q) curves from CD compression tests for unreinforced and cemented Toyoura sand specimens consolidated to 100 kPa mean effective stress at different cement contents (0%–4%).

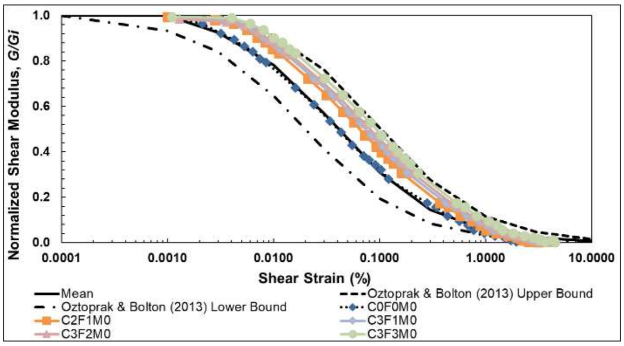


Figure 7. Pure Sand, 2–3% Cement and 1–3% Fibers. G/G_i versus shear strain (ε_q) curves from CD compression tests for unreinforced and fiber-reinforced cemented Toyoura sand specimens consolidated to 100 kPa mean effective stress at different cement (0%–3%) and fiber (0%–3%) contents.

4 SUMMARY AND CONCLUSIONS

In this study, a series of local strain measurements were obtained on unreinforced, fiber, cemented, and fiber reinforced cemented Toyoura sand specimens. It is shown that small-strain stiffness slightly reduces with the addition of fibers. The small strain stiffness of the fiber reinforced and unreinforced specimens are approximately similar or even slight reduction may be seen in fiber reinforced sands. Accordingly, the soil skeleton appears to resist most of the applied load at small strain levels (in case of fiber reinforced sand a slight increase in skeleton void ratio may occur), while the load resisted by the fibers is more substantial at higher strain levels. The larger strain corresponding to the peak deviator stress displayed by the fiber reinforced specimens suggests that fibers increase the ductility of the reinforced soil specimen. Similar results and conclusions were found in a few recent studies (Nakamichi and Sato, 2013; Schmidt, 2015; Safdar et al., 2021). In contrast, addition of cement enhances the small-strain stiffness properties of pure Toyoura sand specimens. The results highlighted that the weak cementation level (e.g. 3 days curing) induced by chemical treatment was sufficient to moderately increase the small-strain stiffness. In addition, fiber reinforced cemented sand specimens showed increases in small-strain stiffness compared to unreinforced specimens. The fiber used in this study vary in diameter from 110-120 μm, with striation widths of 5 μm to less than 1 μm along the 12 mm length. These micro-striations have small filaments protruding from them; likely a result of the extrusion process used in their fabrication. These striations and filaments give the fibers a rough surface, and with the existing angularity of the Toyoura sand, provide an ideal medium for cementitious bonding (Schmidt, 2015; Safdar, 2018). Results of the modulus degradation and mobilized stress curves show good agreement with the hyperbolic relation proposed by Fahey and Carter (1993). The comparison of the results with Oztoprak and Bolton (2013) model shows that the results of the local strain transducers fall within the range of model upper and lower bound curves.

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