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Consideration of the rheological properties of clay soils in the calculation of settlement of a single pile and pile foundation

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ABSTRACT

For many years, pile foundations have been one of the most popular types of foundations used in practice. Among the parameters considered in pile design, special attention is given to parameters such as settlement and bearing capacity of the pile. Since many buildings and structures (both in Russia and abroad) are erected on foundations composed of weak clay soils, the settlements of which are usually uneven and can develop over years and even decades, taking into account the rheological properties of clay soils when designing various types of foundations, including piles, is necessary. Taking into account the rheological properties of clay soils is necessary in order to subsequently avoid excessive development of settlements, tilts and displacements of various buildings and structures, as well as changes in the stress-strain state of the base-foundation-above-ground structures system, since these phenomena can lead to disruption of normal operating conditions of various buildings and structures up to their destruction. The paper presents the determination of the viscosity coefficient of clay soil based on experimental studies in a simple shear device, and also solves the problems of determining the settlement of a single pile and pile foundation (using a computational cell as an example) taking into account the rheological properties of clay soils.

KEYWORDS

Simple shear; Rheological properties; Single pile; Pile foundation.

1. INTRODUCTION

For many years, the pile foundation has been one of the most popular types of foundations used in practice due to its reliability, speed of construction, and the ability to be installed in virtually any soil conditions. Predicting the settlement of the pile as well as studying the distribution mechanism of the load transferred to it play an important role in the design of pile foundations. Among the parameters considered in the design of piles, particular attention is paid to parameters such as settlement and bearing capacity of the pile. Considering the fact that many buildings and structures are erected on foundations consisting of weak clayey soils, the settlement of which is usually uneven and can develop over years and even decades, it is necessary to take into account the rheological properties of clayey soils when calculating and designing various types of foundations, including pile foundations. Significant contributions to the development of soil rheology have been made by Tertzagi K. (Tertzagi, 1961), Vyalov S.S. (Vyalov, 1978), Maslov N.N. (Maslov, 1968), Florin V.A. (Florin, 1959, 1961), Tsytovich N.A. (Tsytovich, 1963), Meschyan S.R. (Meschyan, 2005), Ter-Martirosyan Z.G. (Ter-Martirosyan, 1990) and many others. A large number of scientific works are devoted to the interaction of a single pile and a pile foundation with surrounding and underlying soils, in which experimental, numerical, and theoretical research methods are presented with the derivation of basic formulas and various mathematical dependencies (Abbas et al., 2010; Feng et al., 2017; Liu et al., 2004; Ogranovich, 1963; Ter-Martirosyan and Akuletskiy, 2021; Ter-Martirosyan et al., 2021; Zhang et al., 2010).

2. MATERIALS AND METHODS

To study the rheological properties of clayey soil in the laboratory of the Research and Educational Center "Geotechnics" named after Z.G. Ter-Martirosyan at Moscow State University of Civil Engineering (MGSU), a series of experimental studies was conducted using a simple shear device (Figure 1).

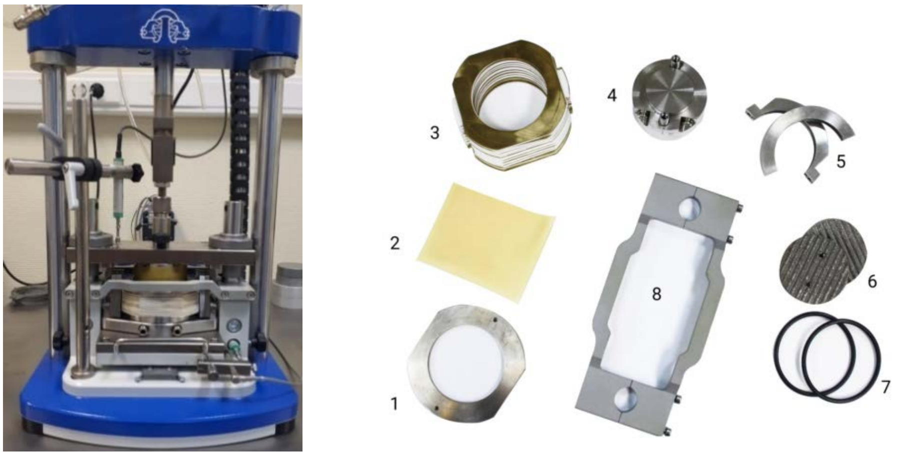

Figure 1. General view of the simple shear device and its main components: 1 - steel ring; 2 - rubber shell; 3 - stack of brass and fluoroplastic rings; 4 - upper punch; 5 - metal casing; 6 - porous discs with ribbed surface; 7 - rubber rings; 8 - half clamps

Samples of the tested soil were placed in a rubber shell surrounded by 25 separate rings with special "ears" designed to ensure the joint operation of all rings during the test. To reduce friction, fluoroplastic and brass rings were alternated with each other. Porous discs were installed in the upper and lower parts of the sample to provide drainage during compaction. Porous discs with a ribbed surface were used for better adhesion to the sample of the tested soil Characteristics of the tested soil samples: diameter of the sample d=71.4 mm; height of the sample h=23.0 mm; density ρ=2.2 g/cm^3; natural moisture w=22.0%; plasticity index I_P=10.6; liquidity index I_L=0.8.

3. RESULTS

3.1. Experience in determining the viscosity coefficient of clayey soil

A series of experimental studies were conducted using a simple shear device in the kinematic mode of loading ($\dot{\gamma} = const$) at four different shear displacement rates ($\dot{u}$=0.005 mm/min, $\dot{u}$=0.05 mm/min, $\dot{u}$=0.5 mm/min, $\dot{u}$=5 mm/min) and at three different values of vertical stress (σ_{n_1}=200 kPa, σ_{n_2}=400 kPa, σ_{n_3}=600 kPa) (Figure 2) to determine the viscosity coefficient of clayey soil. Considering that the obtained graphs of dependence $(\tau - \gamma)$ had a clearly pronounced bilinear character, the viscosity coefficients (Table 2) were determined for two characteristic sections of the graph based on the simplest Newton's equation using the formula (1):

$$\eta = \frac{\tau}{\dot{\gamma}} \quad (1)$$

where τ – shear stresses, $\dot{\gamma}$ – shear rate.

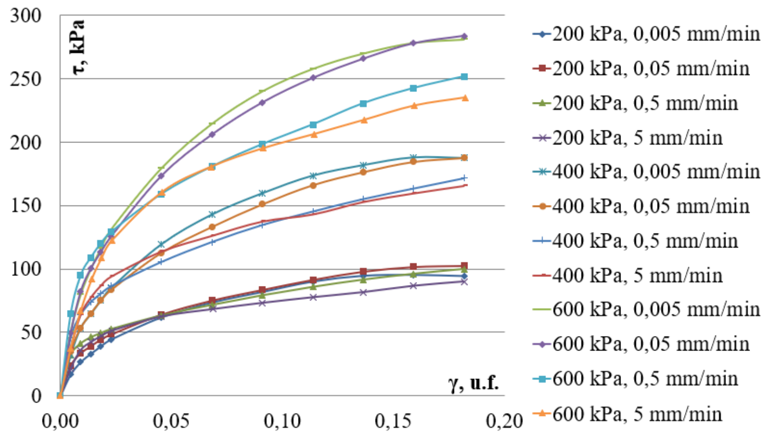

Figure 2. Graph of dependence of shear stresses on shear strains $(\tau - \gamma)$

Table 2. Viscosity coefficients of clayey soil

Vertical stress, kPa	Shear displacement rate, mm/min	Soil viscosity in the first section, kPa*min	Soil viscosity in the second section, kPa*min
σ_n	$\dot{u}$	η_1	η_2
200	5	226	133
	0,5	2310	1707
	0,05	21296	21780
	0,005	194920	221320
400	5	414	259
	0,5	3802	3023
	0,05	36740	40876
	0,005	374000	426800
600	5	539	418
	0,5	5680	4466
	0,05	55264	61732
	0,005	576840	610720

3.2. Settlement and long-term bearing capacity of a single pile

Consider the solution to the problem of a long non-compressible pile of a given diameter $2a_1$ and length l_1, that is embedded in the soil and supported at its lower end by an underlying, relatively dense layer of soil with a modulus of deformation $E > 40$ MPa, penetrating it by an amount $\triangle l \ll l_1$ (Figure 3). The computational domain of the massif containing the pile is a two-layer soil cylinder of finite dimensions $(L, 2b_1)$. Let's write the equilibrium condition:

$$\pi \cdot a_1^2 \cdot \sigma_N = 2\pi \cdot a_1 \cdot l_1 \cdot \tau_a + \pi \cdot a_1^2 \cdot \sigma_R \tag{2}$$

where σ_N - normal stress acting on the pile head; σ_R - normal stress acting under the pile bottom; τ_a - shear stress acting on the lateral surface of the pile.

From equation (2) it follows:

$$\tau_a = \frac{a_1}{2l_1} \cdot (\sigma_N - \sigma_R) \tag{3}$$

Moreover $\tau_a \cdot 2\pi \cdot a_1 = \tau_r \cdot 2\pi \cdot r$. From this equation it follows:

$$\tau_r = \tau_a \cdot \left(\frac{a_1}{r}\right) \tag{4}$$

where τ_r - shear stress at point r; r - horizontal coordinate.

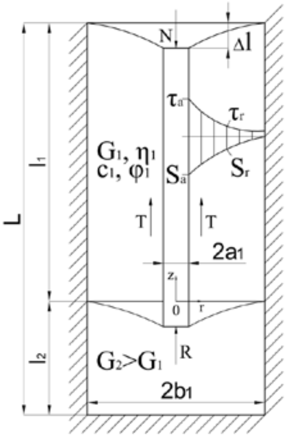

Figure 3. Calculation scheme of interaction of a long and incompressible pile with a thick-walled soil cylinder and underlying soil layer proposed by Ter-Martirosyan Z.G.

The settlement of the soil under the pile heel can be determined using the formula for calculating the settlement of a circular rigid stamp, taking into account K_l:

$$S_R = \sigma_R \cdot \frac{\pi \cdot a_1 \cdot (1-\nu_2) \cdot \omega \cdot K_l}{4 \cdot G_2} \tag{5}$$

where G_2 - shear modulus of the underlying soil; ν_2 - Poisson's ratio of the underlying soil; K_l - coefficient considering the influence of the depth of the pile heel (accounts for the depth of load application on the stamp) ($K_l < 1$)); ω - coefficient considering the shape and rigidity of the stamp.

Let's determine the displacements of the soil around the pile according to the telescopic mechanism of mutual displacements of concentric soil cylinders with a thickness $\Delta r = const$:

$$S_r(r) = \int \gamma(r)\, dr + C \tag{6}$$

$$\gamma(r) = -\frac{\tau_r}{G_1} \tag{7}$$

where $\gamma(r)$ - angular deformation at point r; G_1 - shear modulus of the surrounding soil.

Solution of the problem in linear formulation. Taking into account formulas (6), (7) and (4) and integrating the obtained equation taking into account the condition $S_r(r) = 0$ at $r = b_1$ we obtain:

$$S_r(r) = \frac{\tau_a \cdot a_1}{G_1} \cdot ln(b_1/r) \tag{8}$$

At $r = a_1$ we obtain the maximum displacement of the soils around the pile:

$$S_a = \frac{\tau_a \cdot a_1}{G_1} \cdot ln(b_1/a_1) \tag{9}$$

From the condition of equality of soil and pile displacement $S_r(0) = S_a = S_R$, considering τ_a from (3) we obtain:

$$\sigma_R \cdot \left\{\frac{\pi \cdot (1-\nu_2) \cdot \omega \cdot K_l \cdot G_1 \cdot l_1}{2 \cdot G_2 \cdot a_1 \cdot ln(b_1/a_1)} + 1\right\} = \sigma_N,\ \sigma_R = \sigma_N / A_1 \tag{10}$$

$$A_1 = \frac{\pi \cdot (1-\nu_2) \cdot \omega \cdot K_l \cdot G_1 \cdot l_1}{2 \cdot G_2 \cdot a_1 \cdot ln(b_1/a_1)} + 1 \tag{11}$$

Substituting σ_R from (10) into (5) and neglecting unity, we obtain:

$$S_R = S_{\text{св.}} = \sigma_N \cdot \frac{a_1^2 \cdot ln(b_1/a_1)}{2 \cdot G_1 \cdot l_1} \qquad (12)$$

Solution of the problem in an elastic-viscous formulation based on the Maxwell model. If we take into account the viscous resistance of the soil surrounding the pile, based on the Maxwell model at $\eta = \eta_1(t)$ we obtain:

$$\dot{\gamma} = \frac{\dot{\tau}_a}{G_1} + \frac{\tau_a}{\eta_1(t)} \qquad (13)$$

where $\dot{\tau}_a$ - rate of change of shear stress acting on the lateral surface of the pile; $\eta_1(t)$ – varying weighted average soil viscosity.

By equating $\dot{\sigma}_N = 0$ in equation (3), we obtain:

$$\dot{\tau}_a = -\dot{\sigma}_R \cdot \frac{a_1}{2l_1} \qquad (14)$$

The rate of settlement of the soil surrounding the pile from the action of shear stresses $\tau_r = \tau_a \cdot \left(\frac{a_1}{r}\right)$ when solving the problem in the elastic-viscous formulation is equal:

$$\dot{S}_a = \frac{a_1 \cdot \tau_a}{\eta_1(t)} \cdot ln(b_1/a_1) + \frac{a_1 \cdot \dot{\tau}_a}{G_1} \cdot ln(b_1/a_1) \qquad (15)$$

The settlement rate of the pile under the pile heel can be determined from (5) by replacing σ_R by $\dot{\sigma}_R$, then we obtain:

$$\dot{S}_R = \dot{\sigma}_R \cdot \frac{\pi \cdot a_1 \cdot (1-\nu_2) \cdot \omega \cdot K_l}{4 \cdot G_2} \qquad (16)$$

By equating $\dot{S}_a$ to $\dot{S}_R$, assuming that $E_c \gg E_2$, and considering conditions (3) and (14), we obtain a differential equation of the following form:

$$\dot{\sigma}_R + \frac{\sigma_R}{\eta_1(t) \cdot A} = \frac{\sigma_N}{\eta_1(t) \cdot A} \qquad (17)$$

$$A = \frac{\pi \cdot (1-\nu_2) \cdot \omega \cdot K_l \cdot l_1}{2 \cdot G_2 \cdot a_1 \cdot ln(b_1/a_1)} + \frac{1}{G_1} \qquad (18)$$

The general solution of equation (17) at $\eta_1(t) = \eta_0 = const$ is:

$$\sigma_R(t) = \sigma_N + (\sigma_R(0) - \sigma_N) \cdot e^{-\frac{t}{\eta_0 \cdot A}} \qquad (19)$$

Substituting $\sigma_R(0)$ from equation (10) into equation (19), we obtain:

$$\sigma_R(t) = \sigma_N \cdot \left(1 + \frac{e^{-\frac{t}{\eta_0 \cdot A}}}{A_1} - e^{-\frac{t}{\eta_0 \cdot A}}\right) \qquad (20)$$

Substituting the obtained value of $\sigma_R(t)$ from (20) into the original equation (5), we determine the settlement of the pile:

$$S_{\text{св.}} = S_R = \sigma_R(t) \cdot \frac{\pi \cdot a_1 \cdot (1-\nu_2) \cdot \omega \cdot K_l}{4 \cdot G_2} \qquad (21)$$

Figure 4 shows graphs of pile settlement versus time $(S_R - t)$, obtained at various values of soil viscosity $\eta_0 = const$.

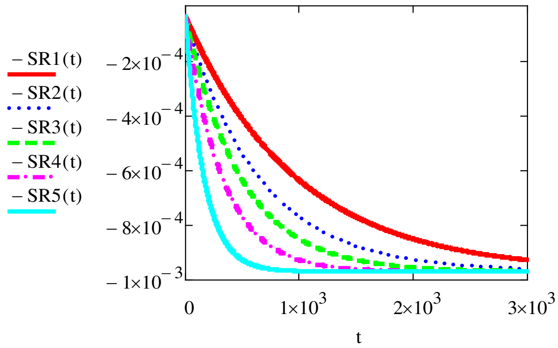

Figure 4. Graphs of pile settlement versus time $(S_R - t)$, obtained at various values of soil viscosity $\eta_0 = const$ $(\eta_1 > \eta_2 > \eta_3 > \eta_4 > \eta_5)$

3.3. Settlement of a pile foundation taking into account linear and rheological properties of soils

Since $S = \varepsilon \cdot l$ the problem of determining the settlement of the pile foundation cell can be solved with known length (l) and deformation (ε). Since the length is an input parameter, further tasks will be considered to determine the deformations of the cell.

Solution of the problem in a linear formulation using the "pile column" scheme. Let a long pile of a given diameter $2a_1$ and length l_1 as part of a pile-slab foundation be embedded in the soil and supported at the lower end by an underlying relatively dense soil layer $(G_2 \gg G_1)$. The computational domain of the massif containing the pile and the foundation is a thick-walled soil cylinder of finite dimensions $(L, 2b_1)$. When a static uniformly distributed load $(\sigma_N = const)$ is applied to the pile grating, it will be distributed between the piles and the surrounding soil within each design cell. Considering the compression of the pile shaft and the surrounding soil mass, we can write the equilibrium equation:

$$\sigma_N = \sigma_c \cdot \omega + \sigma_г \cdot (1 - \omega) \tag{22}$$

where σ_N – uniformly distributed load applied to the pile grating; σ_c - stress acting in the pile shaft at the head level; $\sigma_г$ - stress acting in the soil mass at the contact with the pile grating; ω - dimensionless coefficient equal to the ratio of the cross-sectional area of the pile shaft to the cross-sectional area of the entire design cell, i.e. $\omega = a_1^2/b_1^2$; a_1 и b_1 – radius of the pile and radius of the design cell.

Let's write the condition of equality of settlements of the pile grating S_N, the pile S_c and the surrounding soil $S_г$:

$$m_p \cdot \sigma_N = m_c \cdot \sigma_c = m_г \cdot \sigma_г \tag{23}$$

where m_c and $m_г$ – coefficients of relative compressibility of the pile and soil.

The joint solution of equations (22) and (23) will allow us to determine σ_c and $\sigma_г$:

$$\sigma_c = \sigma_N \cdot \frac{E_c}{E_c \cdot \omega + E_г \cdot (1 - \omega)}; \ \sigma_г = \sigma_N \cdot \frac{E_г}{E_c \cdot \omega + E_г \cdot (1 - \omega)} \tag{24}$$

The reduced modulus of deformation of the design soil cell as a whole is found from condition (22):

$$E_{пр} = E_c \cdot \omega + E_г \cdot (1 - \omega) \tag{25}$$

The settlement of the pile-slab foundation is determined by the following formula:

$$S_ф = \frac{\sigma_N}{E_{пр}} \cdot 0{,}8 \cdot l_1 \tag{26}$$

Stress-strain state of the cell in an elastic-viscous formulation according to the "pile column" scheme based on the Kelvin-Voigt model and the Newton elastic model (Figure 13). Under the conditions of the one-dimensional problem, we write down the dependence of the deformations of the cell as a whole on the stresses $\varepsilon(t) - \sigma$ in the following form:

$$\sigma = \varepsilon \cdot E + \dot{\varepsilon} \cdot \eta \quad (27)$$

where $\dot{\varepsilon}$ – rate of development of deformations; η – soil viscosity coefficient.

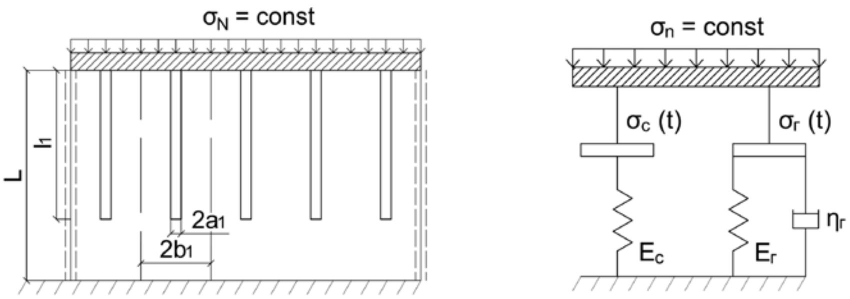

Figure 5. Delineation of design cell boundaries in a pile-slab foundation and design model of interaction between a pile in a pile-slab foundation and the surrounding soil mass under compressive compression conditions based on the Kelvin-Voigt model

Let's write the equilibrium condition, characterizing the process of development of settlement of the pile grating in time, assuming in the first approximation that it coincides with the equilibrium equation (22):

$$\sigma_N = \sigma_c(t) \cdot \omega + \sigma_\Gamma(t) \cdot (1 - \omega) \quad (28)$$

The stresses acting in the pile shaft at the level of the head σ_c and in the soil mass at the contact with the pile grating σ_Γ are represented in the following form:

$$\sigma_c(t) = \varepsilon_c \cdot E_c;\ \sigma_\Gamma(t) = \varepsilon_\Gamma \cdot E_\Gamma + \dot{\varepsilon}_\Gamma \cdot \eta_\Gamma \quad (29)$$

Based on the condition of equality of deformations $\varepsilon_c = \varepsilon_\Gamma = \varepsilon$, and taking into account equation (28):

$$\sigma_N = \varepsilon \cdot (E_c \cdot \omega + E_\Gamma \cdot (1 - \omega)) + \dot{\varepsilon} \cdot \eta_\Gamma \cdot (1 - \omega) \quad (30)$$

The solution of equation (30) is known and has the form:

$$\varepsilon(t) = \frac{Q}{P} \cdot (1 - e^{-P \cdot t});\ P = \frac{E_c \cdot \omega + E_\Gamma \cdot (1 - \omega)}{\eta_\Gamma \cdot (1 - \omega)};\ Q = \frac{\sigma_N}{\eta_\Gamma \cdot (1 - \omega)} \quad (31)$$

Figure 6 shows graphs of the dependence of cell deformations on time $(\varepsilon - t)$, obtained at different pile spacing and at the soil viscosity coefficient $\eta_\Gamma = 10^9$ Poise.

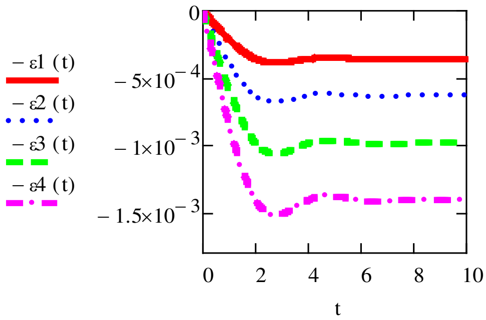

Figure 6. Graphs of the dependence of cell deformations on time $(\varepsilon - t)$, obtained at different pile spacing $(b_1 < b_2 < b_3 < b_4)$ and at the soil viscosity coefficient $\eta_\Gamma = 10^9$ Poise

4. CONCLUSIONS

Based on the results of experimental studies conducted in a simple shear device, it was found that the shear rate significantly influences the viscosity of clayey soil (the soil viscosity coefficient increases directly proportional to the decrease in shear rate).

In solving the problem in a linear formulation, the stabilized settlement of a single pile was determined, which directly depends on the geometric parameters of the pile (diameter, length), as well as on the shear modulus of the soil surrounding the pile. In solving the problem in an elastic-viscous formulation based on the Maxwell model, the unstabilized (long-term) settlement of a single pile was determined, depending on the time-varying weighted average viscosity of the surrounding soil, and the long-term bearing capacity of the single pile was determined.

The problem of the interaction between the pile, the pile grating and the surrounding soil mass was solved in a linear formulation using the "pile column" scheme, a formula for calculating the reduced modulus of deformation of the design soil cell was obtained, as well as a formula for calculating the settlement of the pile-slab foundation. In solving the problem in the elastic-viscous formulation of the "pile column" scheme on the basis of the Kelvin-Voigt model, it was obtained that the time to reach the ultimate vertical deformations is directly proportional to the value of the viscosity coefficient of the surrounding soil. At low values of the soil viscosity coefficient, comparable to a fluid medium, a peak value of vertical strains exceeding the residual vertical strains occurs, which is not observed in more viscous media.

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