# Investigating the Impact of Inhomogeneous Soil Parameters on Shear Strength in Biaxial Tests

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ABSTRACT

The creation of shear bands in soils is a complex and important geotechnical mechanism that is controlled by different factors, such as soil parameters, loading conditions and loading history. Understanding the development of shear bands is important in order to reliably predict the behaviour of soil in diverse engineering and geotechnical contexts, including slope stability, foundation design and earthworks. Therefore, this study simulates the effect of soil inhomogeneity on the peak shear strength in fine-meshed finite element biaxial tests by using the Hypoplasticity and Mohr-Coulomb constitutive model. For this purpose, hypoplastic calculations were carried out by distributing the initial void ratio with a normal distribution across all elements in a certain range. Similarly for Mohr-Coulomb models, equivalent cohesion, friction angle and elastic modulus were distributed among the elements. The results indicate that the presence of inhomogeneity leads to reaching the peaks strength at lower strain levels. Furthermore, increasing the inhomogeneity leads to a reduction in shear strength for both hypoplasticity and the Mohr-Coulomb model.

KEYWORDS

Inhomogeneous Soil; Shear Strength; Numerical Simulation; Constitutive models

## 1. INTRODUCTION

Obtaining a comprehensive understanding of the mechanical properties of soil under different loading conditions is essential in the field of geotechnical engineering. In the scientific field of soil behaviour, the presence of inhomogeneity within the soil mass arises as a crucial component that has a substantial impact on its shear strength and deformation patterns (Schneider-Muntau et al., 2021). The presence of inhomogeneity in soil, which is defined by variations in characteristics like density, moisture content, particle distribution, and void ratio, can result in complex stress-strain relationships, the occurrence of strain localization phenomena, and early shear band formation (e.g. Medicus & Schneider-Muntau, 2018; Desrues et al., 2017; 2018). These factors present difficulties for conventional geotechnical analysis and design principles.

Since producing identical samples with the same inhomogeneity patterns is not possible in standard laboratory tests, experimental methods cannot be used to accurately investigate the impact of heterogeneity on peak strength and shear band formation. Consequently, in this study numerical methods are used for the systematic investigations.

## 2. LITERATURE REVIEW

The impact of soil heterogeneity on shear parameters and the influence on results of finite element simulations has been widely studied in several fields, but mostly focusing und variation of undrained shear strength. Hicks and Samy (2002) employed finite element analysis to determine the effect of heterogeneity on the stability of a clay slope with a spatially varied undrained shear strength $c_u$. The findings suggested that, for a given factor of safety, reliability in the simulation results is optimum when $c_u$ is constant with depth. As $c_u$ increases linearly with depth, reliability reduces due to the wider variety of probable rupture surfaces. In this study, horizontal anisotropy (of heterogeneity) produces minor variations in reliability. Nevertheless, situations have been observed in which anisotropy is expected to have a substantial impact. Hicks and Spencer (2010) studied the effect of heterogeneity in undrained shear strength on the performance of a long slope cut in clay. Random field theory is used to model heterogeneity. It has been demonstrated that, while the selected slope geometry, loading conditions and mean strength profile are all two-dimensional, the presence of 3D material heterogeneity indicates that the slope is not genuinely 2D. Ching and Phoon's (2013) research primarily focused on studying simple and uniform stress situations. Undrained compression and shear were applied to a 12.8 m wide by 48 m high plane strain soil specimen using finite element analysis (FEA). The mobilized strength statistics are found to be closely related to the statistics generated by the minimum of line averages along possible slip curves. The orientations of critical slip curves are mostly influenced by mechanics rather than spatial variation. However, the vertical positions are somewhat random and depend on the realizations of random fields. The crucial slip curve's line average is the lowest value among the line averages for the potential slip curves. Li et al. (2014) studied the infinite slope reliability with spatially varying shear strength parameters that rise linearly with depth. The spatial variability of undrained shear strength and friction angle is described using random field theory. The influence of geographical heterogeneity on critical slip line depth and failure probability is examined using infinite slope instances. The results show that the clay slope stability is affected by the mean trend of shear strength parameters. Ignoring a linearly rising shear strength trend will overestimate failure probability. Critical slip lines at the slope bottom are less likely when the mean trend of undrained shear strength is included. The distribution of sandy slope critical failure depths is affected by the friction angle's linearly rising mean trend. Anoyatis et al. (2023) represented soil inhomogeneity by using a power law relationship to describe the variation of shear modulus with depth. Their study emphasized the impact of inhomogeneity on the interaction between soil and piles during vertical vibrations and provided a new analytical solution for piles that are subjected to axial loads in heterogeneous soils. Chidichimo et al. (2014) performed an investigation by using 1-g shaking table test results on the effect of soil inhomogeneity and nonlinear simulation on kinematic soil-pile interaction and its bending by passage of vertically propagating seismic shear waves. The inhomogeneity was applied by using two vertical layers of soil that have different strength parameters. The findings demonstrate that these parameters have a significant impact on both site response and kinematic pile bending. Therefore, nonlinear analysis is essential for accurately predicting the dynamic response of pile foundations. Schneider-Muntau et al. (2021) investigated the scattering in strength by comparing results of compression drained triaxial tests at different stress levels and with repeated test on samples with the same mean density. The impact of the number of stress levels on determination of the shear strength parameters, was assessed quantitively. Results demonstrated that repeatability of laboratory experiments are not given, even when same sample preparation procedure, same mean density and same stress level are investigated. This leads to a statistically wide range of potential shear parameters that can be obtained due to inhomogeneities in sample preparation.

Most of mentioned investigations tried to study the effect of inhomogeneity on the strength parameters, by applying inhomogeneity in different ways e.g. variation of water content on density, shear modulus inside the studied samples. Since producing a sample with controlled inhomogeneity is experimentally very difficult, this study aims in performing numerical simulations of fine meshed biaxial tests with two different constitutive models to investigate the influences observed by soil heterogeneity. In this study inhomogeneity was implemented by variation of void ratio which is a state variable in hypoplasticity.

## 3. METHODOLOGY

### 3.1 Numerical Simulation Framework

This work utilizes the finite element software ABAQUS 2019 for numerical modelling to investigate the effect of inhomogeneity on the soil behaviour. Due to the influence of element size and shape function on the precision of numerical models, especially in investigations related to strain localization, mesh dependency analysis was performed. This procedure entailed doing initial simulations to assess the impact of various element sizes on the outcomes. We have observed the expected influence of mesh dependency also in our results. However, to obtain comparable results, in this study all analyses were performed based on the same finite element mesh.

2D consolidated and drained biaxial tests with 5000 plane strain elements with a bilinear shape function were simulated. Boundary conditions and the geometry are shown in Figure 1. The sample is fixed at the middle node of the bottom plate and horizontal displacement of the bottom elements is allowed, while they are constrained in vertical direction. The side boundaries are free to move in vertical and horizontal direction. The displacement-controlled shearing is applied by a prescribed displacement $\delta$ of the top nodes. This indicates that the soil sample has the ability to undergo vertical and horizontal expansion or contraction. Samples were subjected to three different confining pressure levels. ($\sigma'_3$ = 50, 120, and 200 kPa)

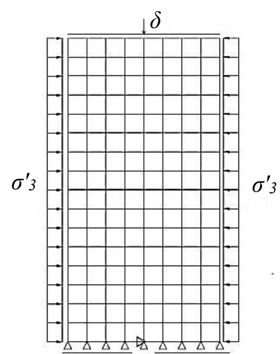



Figure 1. Schematic boundary and loading conditions

In our investigations, three different mean values for the void ratio distribution ($e_{mean}$) were considered: 0.51, 0.57, and 0.64. To achieve modelling inhomogeneous samples, standard deviation values (*SDV*) of 0.01, 0.05, and 0.08 from the mean values were taken into account, and the void ratio was normally distributed across the elements of soil sample.

### 3.2 Material

This study uses Weald clay as material (Weifner and Kolymbas, 2008; Mašín, 2013). The determination of the range of possible $e_{mean}$ was done based on critical state relationships, considering the loading range between 50 – 300 kPa. The hypoplastic parameters $\varphi_c = 24°$, $N = 0.8$, $\lambda^* = 0.059$ and $\kappa^* = 0.018$ were chosen according to Mašín (2013). Modulus. For the Mohr-Coulomb model dilatancy angle $\psi$ and Poisson's ratio $\nu$ assumed to be respectively 0 and 0.3.

### 3.3 Constitutive Models

For the first series of simulations hypoplasticity was chosen to simulate the soil behaviour. This model employs void ratio as one of the state variables ($\dot{\boldsymbol{\sigma}} = \mathbf{h}(\boldsymbol{\sigma}, \boldsymbol{D}, e)$) and enables us to apply variations in the void ratio to represent the inhomogeneity. This represents well what is expected to be relevant in real soil samples.

In the second step a series of modelling was performed using the Mohr-Coulomb constitutive model. To establish comparability between the two sets of simulations, equivalent values for cohesion *c*, friction angle *φ*, and elastic modulus *E* were chosen based on the corresponding void ratio value *e* in the hypoplastic model. As the void ratio scatters in the fine meshed sample, a wide range of void ratios had to be considered: *e* = 0.400-0.799.

In order to calculate the equivalent parameters a series of hypoplastic single element triaxial compression tests (CD) with three different confining pressures (50, 120 and 200 kPa) were simulated. Cohesion, friction angle and elastic modulus were derived from the triaxial stress-strain relationships (Equation (1), (2)) obtained from the simulation with hypoplasticity for each of the relevant void ratios by using the following formulations where $\varepsilon_{ii}$, $\gamma_{ij}$, $\sigma_{ii}$, $\tau_{ij}$ represent normal strains, shear strains, normal stresses and shear stresses, M is the slope of trend line drawn between peak deviatoric stresses ($q_{peak}$), $p'$ shows the effective mean stress, $q_{50} = 0.5$. $q_{peak}$, $\varepsilon_{q50}$ would be the value of deviatoric strain corresponding to $q_{50}$ and $E_{50}$ is considered as elastic modulus for the Mohr-Coulomb model.

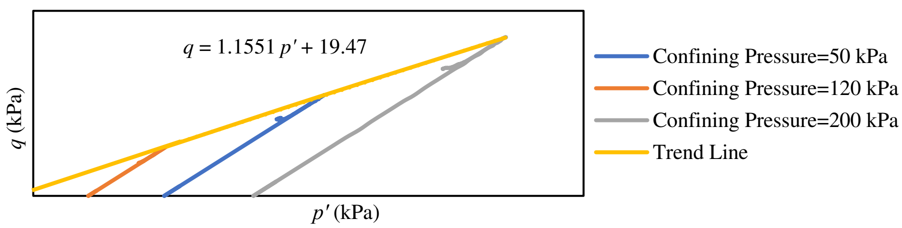



Figure 2. A sample of $p'$-$q$ diagram for determining equivalent Mohr-Coulomb parameters for *e* =0.450

$$\varepsilon_q = \sqrt{\frac{2}{9}[(\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2 + (\varepsilon_{22} - \varepsilon_{11})^2] + \frac{1}{3}({\gamma_{23}}^2 + {\gamma_{31}}^2 + {\gamma_{13}}^2)} \quad (1)$$

$$q = \sqrt{\frac{1}{2}[(\sigma'_{22} - \sigma'_{33})^2 + (\sigma'_{33} - \sigma'_{11})^2 + (\sigma'_{22} - \sigma'_{11})^2] + 3(\tau_{23}^2 + \tau_{31}^2 + \tau_{12}^2)} \quad (2)$$

$$M = \frac{6 \sin\varphi}{3 - \sin\varphi} \quad (3)$$

$$q = Mp' + M\frac{c}{\tan\varphi} \quad (4)$$

$$E_{50} = \frac{q_{50}}{\varepsilon_{q50}} \quad (5)$$

With this procedure, we obtain a parameter set of cohesion, friction angle and young's modulus for each single void ratio. In the last step a series of fine meshed simulations were performed by using the linear elastic - perfectly plastic Mohr-Coulomb constitutive model and scattered parameters to observe the soil behaviour in case of heterogeneity presence in soil sample. In order to have comparable results, according to the void ratio *e* of each element in the hypoplastic fine meshed simulation, the equivalent cohesion *c*, friction angle *φ*, and elastic modulus *E* were assigned to the corresponding element. As it was expected the same inhomogeneity pattern was produced for all shear parameters as can be observed in the plots (Figure 3). Hence, we would have samples with identical inhomogeneity distribution to investigate the soil behaviour with two different constitutive models.

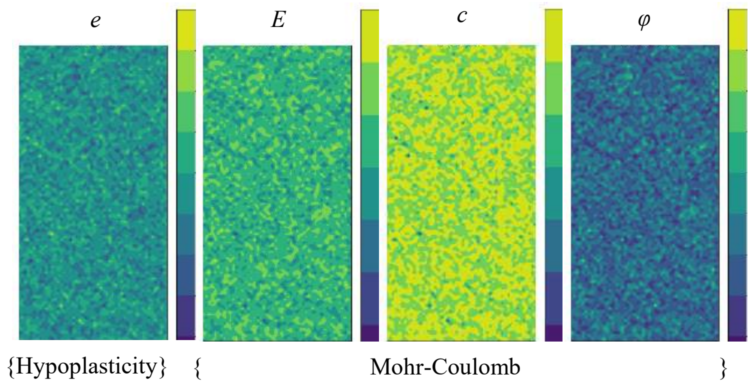



Figure 3. Contour plot of soil sample with scattered void ratio (hypoplasticity), elastic modulus, cohesion and friction angle respectively from left to right (Mohr-Coulomb)

## 4. RESULTS

The results in Figure 4 show the relationship between the average value of deviatoric stress $q$ and the deviatoric strain $\varepsilon_q$ for all elements of soil samples modelled with the hypoplasticity constitutive model (HP) for confining pressure $\sigma'_3$ = 120 kPa. Each curve represents a soil sample with a certain mean void ratio ($e_{mean}$) which is normally distributed in a range represented by the standard deviation value *SDV*. As mentioned before 0.51, 0.57, and 0.64 were considered as $e_{mean}$ values and 0.01, 0.05, and 0.08 as *SDV* in the simulations. It should be noted that in the following text the term void ratio $e$ refers to the mean value of this parameter in a soil sample.

The highest influence on peak strength is given by $e_{mean}$, that shows reduction of shear strength due to decrease in $e_{mean}$. This can be explained with the peak strength for denser samples. The charts further illustrate that by increasing the *SDV* from the mean void ratio $e_{mean}$, there is a tendency for the peak deviatoric stress $q_{peak}$ of the soil to decrease. These findings show that a higher scattering in the soil sample leads to a drop in shear strength. The influence of *SDV* on both shear strength and stress-strain behaviour is visible in all values of $e_{mean}$. Nevertheless, the impact is more prominent when $e_{mean}$ is smaller. Within a given sample, it is observed that the deviatoric strain $\varepsilon_q$ corresponding to $q_{peak}$ tends to decrease as the *SDV* increases. This suggests that soil samples with more heterogeneity have decreased brittleness. After the peak state, the curves exhibit a tendency to reduce the $q$ value, particularly in samples with higher $e_{mean}$ values. This suggests that samples with $e_{mean}$, indicating a denser soil, show a more noticeable $q$ reduction behaviour. The $q$ reduction rate and extent seem to be affected by the variability in void ratio, since more homogeneous samples (with lower *SDV*) exhibit a sharper decrease after reaching the peak, suggesting a more brittle failure.

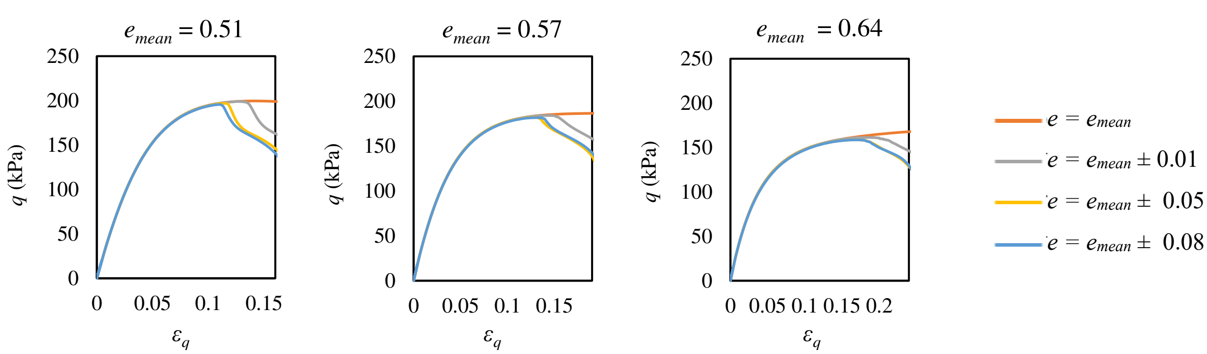



Figure 4. $q$ (kPa) - $\varepsilon_q$ results for Simulation of CD biaxial test using HP ($\sigma'_3$ = 120 kPa)

Although the Mohr-Coulomb model may not reflect soil's non-linear behaviour, it can still offer valuable observations into the general trends of the inhomogeneity effect on the shear strength of soil.

The graphs in Figure 5 global behaviour shows a continuous reduction in peak deviatoric stress as *SDV* increases. The observed pattern is consistent with the hypoplasticity studies, indicating that greater inhomogeneity reduces soil shear strength. When comparing with the results from simulations with hypoplasticity, it is clear that the strain at peak stress shows smaller values and remains rather similar across varied *SDV*. However, there is a clear trend in which samples with smaller strength and stiffness parameters (which correlate to higher void ratios) exhibit peak stress at higher strains.
Given the findings on hypoplasticity, the curves generated using the Mohr-Coulomb model (MC) show a drop after the peak stress, indicating a global $q$ reduction reaction. This behaviour indicates that the soil experiences a decrease in shear strength, which is due to the localization of the deformation in shear bands.

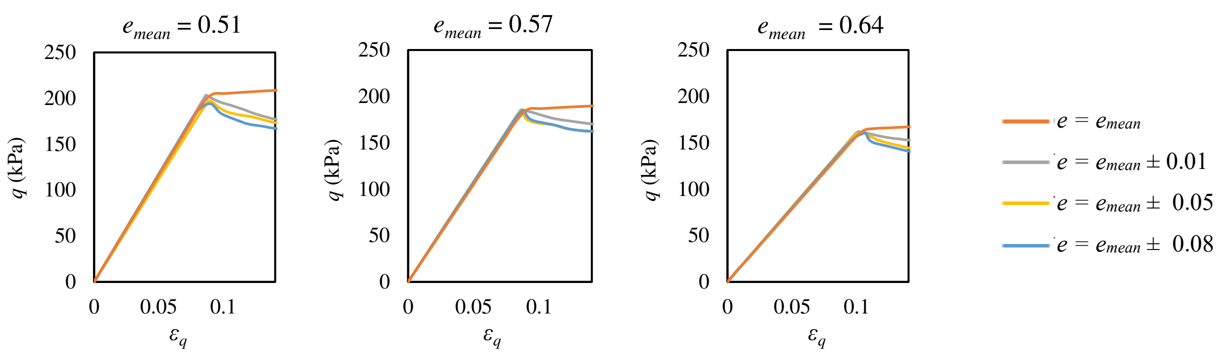
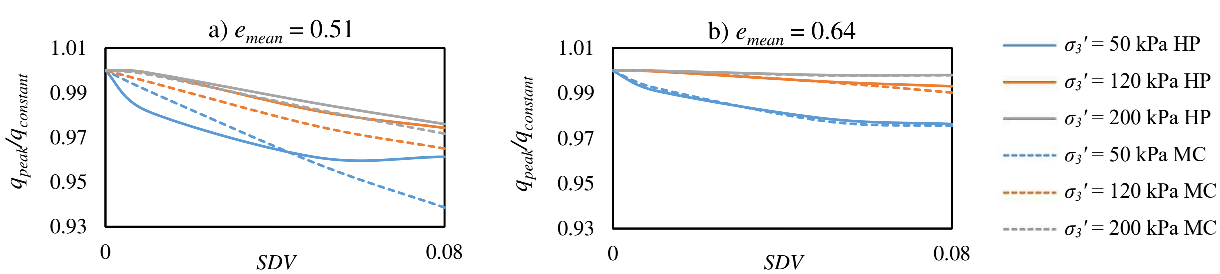



Figure 5. $q$ (kPa) - $\varepsilon_q$ results for Simulation of CD biaxial test using MC constitutive model ($\sigma'_3$ = 120 kPa)

In Figure 6 results of samples respectively simulated by using hypoplasticity and Mohr-Coulomb as constitutive model, demonstrate the effect of mean void ratio $e_{mean}$ and confining pressure on the shear strength of soil samples. $q_{peak}$, representing the maximum deviatoric stress of a soil sample with scattered parameters is normalized by $q_{constant}$, which is the maximum deviatoric stress for a sample with constant void ratio $e_{mean}$. This value is displayed in dependency of the *SDV* of the void ratio. The ratio of $q_{peak} / q_{constant}$ is always $\leq 1$, when the $SDV \geq 0$. This means scattering the void ratio leads to a reduction in shear strength.

By comparing Figure 6 (a and b) it can be noted that the effect of *SDV* on shear strength becomes less considerable in higher $e_{mean}$. In general, it can be observed that Mohr-Coulomb gives lower $q_{peak} / q_{constant}$ in samples that are heterogeneous and therefore is even more sensitive to inhomogeneity in comparison to hypoplasticity. It can be concluded that weak elements within the soil sample play the role in failure mechanism.

Figure 6 (c and d) illustrate the effect of confining pressure on the shear strength of soil. For both cases increasing the confining pressure reduces the effect of inhomogeneity. This influence is more obvious in looser samples with lower $e_{mean}$.



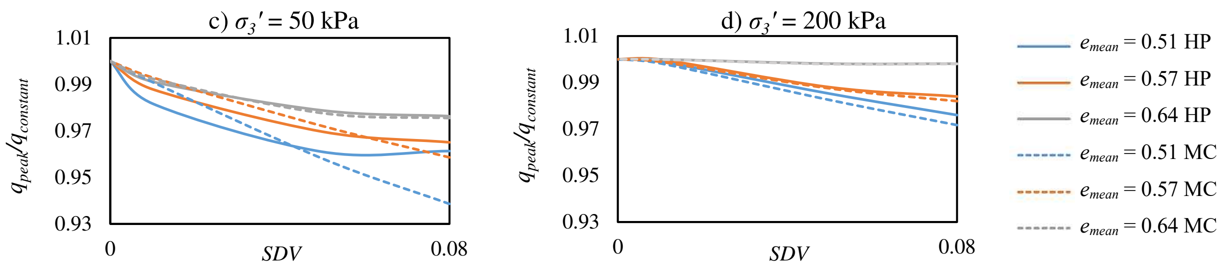


Figure 6. Effect of mean void ratio (a, b) and confining pressure on shear strength variation (c, d)

To summarize the results, it can be concluded that inhomogeneity causes reduction in shear strength and as the inhomogeneity level increases this effect would be more obvious and significant. This effect can be controlled by the confining pressure where in higher pressures inhomogeneity would have less effects on shear strength.

## 5. DISCUSSION

It is predicted by both hypoplastic and Mohr-Coulomb models that soil heterogeneity, as demonstrated by the use of non-uniform void ratio in hypoplasticity and non-uniform $c$, $\varphi$, and $E$ in Mohr-Coulomb model, significantly impacts shear strength of biaxial tests in numerical simulations. Under the same circumstances, the Mohr-Coulomb model showed a more substantial reduction in shear strength than the hypoplasticity model, with increase in void ratio variability. As hypoplasticity incorporates a stress dependent stiffness, simulations with hypoplasticity show in general higher deviatoric strain at peak. For a given void ratio, both models show that deviatoric strain $\varepsilon_q$ in $q_{peak}$ decreases as *SDV* grows, the sample behaves less stiff.

One major difference between the two constitutive models chosen for this simulation, lies in the fact that for hypoplasticity, the void ratio is a state variable. This means the void ratio changes within the elements during the simulation procedure and with this localization effects can be better reproduced. In the Mohr-Coulomb model, the assigned strength and stiffness parameters are not updated during the simulation. This means, the original pattern of parameters remains constant during the simulation.

Given how inhomogeneity affects soil shear strength and strain, this behaviour of the soil promotes the significance of considering it in geotechnical design. Since all constitutive models have their advantages and shortcomings, it is important to select a constitutive model that fits the needs of a given geotechnical study. The results show how important it is to characterize soil properly and choose constitutive models in numerical models suitably. When it comes to numerical analysis and design, the apparent impact of soil inhomogeneity on shear strength is crucial. The findings are applicable in constructions where soil shear strength plays the main role in the failure process and soil inhomogeneity is a determining factor.

## 6. CONCLUSION

The study conducted numerical simulations to obtain a qualitative insight on how soil inhomogeneity affects shear strength and strain localization in biaxial tests. In this study hypoplasticity and Mohr-Coulomb constitutive models were used. The investigation yields the following conclusions:

- The results showed that the variability in shear parameters of soil within a sample has a substantial impact on determining the soil's shear strength. Both constitutive models indicated a reduction in shear strength as the variability range of the parameters increased, highlighting the importance of soil inhomogeneity assessment in analysis.
- It was observed that the effect of variability in shear parameters is more obvious in lower ranges of confining pressure and for looser soil samples. This observation shows the remarkable influence of

loading condition on soil behaviour. Increasing the confining pressure leads to a higher shear strength, independent of the *SDV* and $e_{mean}$ value.

- The strain corresponding to the peak strength showed higher values in the hypoplastic model than in the Mohr-Coulomb model. The scattering influences the stiffness of the sample, the higher the inhomogeneity, the less stiff the sample behaves.

As a conclusion, the study demonstrated the significance of selecting an appropriate constitutive model and accurate shear parameters on the outcomes of the simulation. The approach of having the void ratio as a state variable in the constitutive models is advantageous when simulating soils with inhomogeneous density in laboratory experiments.

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