The examination of different soil parameters' coefficient of variation values and types of distributions

L'examen du coefficient de variation et de la répartition des différents paramètres du sol

Istvan Kadar & Laszlo Nagy
Department of Engineering Geology and Geotechnics, Budapest University of Technology and Economics, Hungary, kadar.istvan@epito.bme.hu

ABSTRACT: The use of Normal and Lognormal Distributions for geotechnical parameters is dominant, however, in many cases the measured data follows different tendency. The characteristic values of soil parameters – as input data for design – are generally calculated using Normal Distribution. When the true distribution of a variable is not known, a Normal Distribution is often assumed. In most cases, very limited amount of data is available, on which to decide what statistical distribution to use. In this paper based on different sized set of data from laboratory testing and field investigation distribution functions and coefficient of variation values are calculated and compared. The distribution of the input parameters of the design has an effect on the distribution of the output parameters as factor of safety values. The main objective of this paper is to examine how the number of samples and the vol ume of the project influence the used coefficient of varation values.

1 INTRODUCTION

Eurocode 7 standard defines the characteristic value of a geotechnical parameter as 'a cautious estimate of the value affecting the occurance of the limit state'. For limit states that depend on the strength of the ground (typical of many ultimate limit states), it is the mean strength mobilized along the failure surface that is required – Eurocode 7 suggests that this should be selected with 95% confidence (Bond, 2011).

The characteristic value of a parameter depends on its coefficient of variation (COV). The main objective of this paper is to examine how the number of samples and the volume of the project influence the used COV values. Therefore 3 different sets of data were investigated.

2 COEFFICIENT OF VARIATION

While the standard deviation is a useful indicator of the amount of scatter in a variable, the degree of dispersion is easier to see in context if it is expressed in terms of the coefficient of variation (COV), which is the ratio of the standard deviation (δ) divided by the average (X_{mean}).

$$COV = \delta / X_{mean} \quad (1)$$

The COV is a dimensionless measure of the amount of scatter (Rétháti, 1985). In Hungarian and especially international literature many recommendations can be found for COV values. The coefficient of variation is useful because the standard deviation of data must always be understood in the context of the mean of the data. In contrast, the actual value of the COV is independent of the unit in which the measurement has been taken, so it is a dimensionless number. For comparison between data sets with different units or widely different means, one should use the coefficient of variation instead of the standard deviation.

In Eurocode 7 the material properties, effects and resistance are introduced into design as characteristic values with a prescribed probability of not being exceeded in a hypothetically unlimited test series. (Bond & Harris, 2008). The characteristic value can be determined with the help of coefficient of variation. The well-known formula for calculating the characteristic value is shown in Eq. (3).

$$X_k = X_{mean} \cdot (1 + k_n \cdot COV) \quad (2)$$

where X_k is the characteristic value, X_{mean} is the mean value (normal distribution is assumed), k_n is a statistical coefficient that depends on the number of the samples (n) and the value which is relevant for the examination.

3 EXAMINED SOILS

The examined soils can be divided into 3 main groups. The soils' place of origin and the number of the experiments carried out make the difference between the groups.

The first series contains 5 different soils (fine sand, silty sand, sandy silt, high plasticity clay and fly ash). During the targeted laboratory experiments at least 30 direct shear tests were carried out on each type of soil.

The second series' samples are from a medium-sized project. Four different soils were investigated (fly ash, clayey silty sand, gracel and high plasticity clay). In this order 20, 11, 12 and 19 direct shear test experiments were made.

The third series is part of a large-scale project. Numerous soil parameters were examined. In the followings two layers' (sand/silty sand, sand/gravelly sand) results are given from 38 and 21 direct shear test measurements. A few values of COV of grain size distribution, phase composition and CPT sounding parameters are also given.

4 RESULTS

4 .1 *Shear strength parameters*

The COV values for the different sets of data are summarized in the following tables.

Table 1. Coefficient of variation values (1st series)

Soil	φ	c	τ_{100kN}	τ_{200kN}
Sand	0.069	0.154	0.046	0.037
Silty sand	0.072	0.257	0.030	0.031
Sandy silt	0.106	0.205	0.036	0.033
Clay	0.309	0.521	0.120	0.105
Fly ash	0.119	(1.864)	0.078	0.066

Table 2. Coefficient of variation values (2nd series)

Soil	φ	c
Fly ash	0.42	0.79

Clayey silty sand	0.43	1.06
Gravel	0.08	(3.46)
High plasticity clay	0.72	0.57

Table 3. Coefficient of variation values (3^{rd} series)

Soil	**φ**	**c**
sand / silty sand	0.11	(0.78)
sand / gravelly sand	0.03	(0.84)

4 .2 *Grain size distribution and phase composition parameters*

The COV values for the different sets of data are summarized in the following tables.

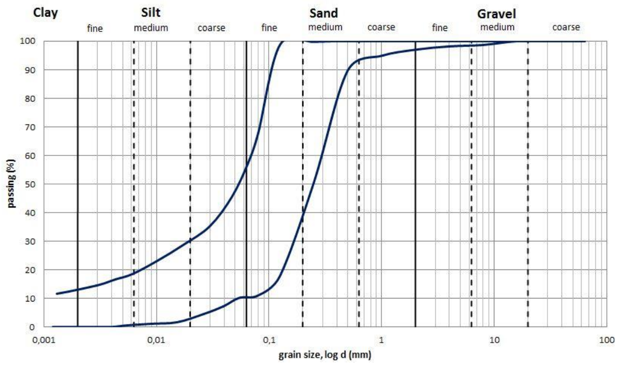

Figure 1. Particle size distribution curve of sand / silty sand (3^{rd} series)

Table 4. Coefficient of variation values of sand / silty sand (3rd series)

Parameter	**Mean**	**St. dev.**	**COV**
Sand percentage	89.4	11.0	0.12
d_{60}	0.19	0.05	0.26
Porosity (n)	39.70	2.00	0.05
Void ratio (e)	0.66	0.05	0.08
Dry bulk density (ρ_d)	1.60	0.05	0.03

4 .3 *CPT tip resistance and friction ratio values*

The measured tip resistance and calculated friction ratio values' histogram can be seen on Fig 2.

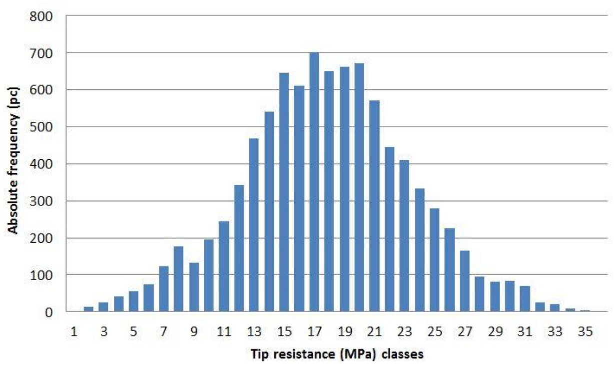

Figure 2. Histogram of sand / silty sand (3^{rd} series)

Table 5. Coefficient of variation values of sand / silty sand (3rd series)

Parameter	**Mean**	**St. dev.**	**COV**
Tip resistance (q_c)	17,36	5,52	0,32
Friction ratio (R_f)	0,86	0,21	0,25

Table 6. Coefficient of variation values of sand / gravelly sand (3rd series)

Parameter	**Mean**	**St. dev.**	**COV**
Tip resistance (q_c)	16.40	4.48	0.27
Friction ratio (R_f)	0.72	0.16	0.22

5 SUMMARY

Based on the results of the measurements and the data found in the literature, the following facts can be stated in connection with the reliability of shear strength and other parameters:

- All types of soils cannot be characterized with one coefficient of variation neither for the angle of internal friction nor for cohesion.
- The value of the coefficient of variation grows towards cohesive soil in both cases of shear strength statistical parameters.
- Living up to the expectations the coefficients of variation of angles of internal friction is less than that of cohesion which has wider ranges (see Figure 3, 4).
- It is important to notice, that the coefficient of variation at the cohesion and angles of internal friction is higher than the coefficient of variation at the shear strength. So the unreliability of the shear strength parameters does not arise from the unreliability of the experimental results.
- With the introduction of Eurocode 7 we can use COV(φ) = 0.04-0.07 for coefficients of variation of angles of friction of granular soils and COV(c) = 0.5-0.6 for coefficients of variation of cohesion of cohesive soils without hesitation.
- For each soil parameter based on the size of the project a suggestion can be made on the minimally required number of experiments.

It is important to handle the coefficients of variation of shear strength parameters separately to meet the contradictory requirements of safety and thrift (Takács 2012). The results of the laboratory testing suggest that using the recommended COV(φ) = 0.1 and COV(c) = 0.4 values by Schneider (1999) in Eurocode 7 (for every type of soil) cannot be done without critical overview.

6 REFERENCES

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