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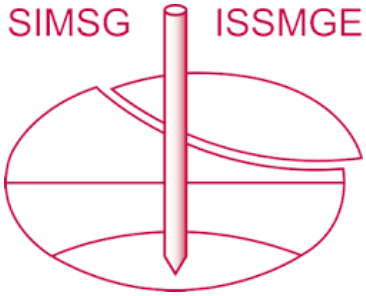

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Hambly's Paradox and the practicing geotechnical engineer: a study of the robustness of lateral soil loads

Le Paradoxe d'Hambly et l'Ingénieur Géotechnicien praticien: Étude de la robustesse des charges latérales du sol

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ABSTRACT: This paper presents two cases of simple lateral soil loading problems to assess how intuitive the results might be to a young geotechnical engineer. In particular, the paper examines if the level of robustness implied by a load factor, material factor or factor of safety may be misunderstood. It is demonstrated that for the results to be intuitively understood the designer needs to understand the methods and assumptions behind the analysis and the limitations this brings to the results. The differing levels of robustness imparted by a numerically identical load factor, material factor or factor of safety are explained for each case study.

The examples are inspired by Hambly's Paradox, a simple example comparing three and four legged chairs and eloquently demonstrating the concepts of ductility, robustness and redundancy. The examples are completed using limit equilibrium analysis and are insightful to this analysis methodology. The paper demonstrates how studies of simplified examples can be used to build understanding of an analysis, assess sensitivity of the results and develop the intuition of a young geotechnical professional.

1 INTRODUCTION.

Lateral soil loads are a basic element of geotechnical engineering. Assessment of active and passive earth pressures by the Coulomb theory (Coulomb, 1776) and the Rankine theory (Rankine, 1857) remain a key part of the education and practice of the young geotechnical engineer.

Given its simplicity one might expect a young engineer to be intuitively able to apply active earth pressures. However, John Burland in the ICE Manual of Geotechnical Engineering (Burland et. al., 2012) presents an equivalently simple example, Hambly's Paradox (Hambly, 1985). Whilst the Paradox comprises a very simple example, the results are only intuitive if the subtlety of the problem is understood and applied correctly.

Burland's presentation of Hambly's Paradox prompted the author to investigate several seemingly simple examples of lateral soil loading to assess how intuitive the results might be to a young engineer. This paper uses two examples to examine if the level of robustness implied by a load factor (LF), material factor (MF) or factor of safety (FoS) included in an earth pressures calculation may be misunderstood.

1.1 *Hambly's Paradox*

Hambly's Paradox (Hambly, 1985) considers how much load is carried by each leg of two different stools under a 60kg load. The three-legged stool is straightforward with 20kg in each leg. With the four-legged stool, one leg does not touch the ground and two legs each carry 30kg.

The paradox of the additional leg increasing the load carried per leg illustrates a number of concepts. The aspect of the paradox that prompted this paper was that it is not intuitive unless the subtlety of the problem is understood and applied correctly.

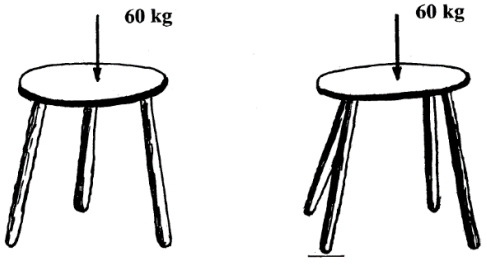

Figure 1. Three- and four-legged stools – Hambly's paradox

2 CASE A - DETERMINING ACTIVE EARTH PRESSURES USING LIMIT EQUILIBRIUM ANALYSIS

Case A examines if active earth pressures can be intuitively assessed using limit equilibrium analysis (LEA). LEA can be convenient to evaluate earth pressures when including irregular back slopes, surcharges and soil layering. The Factor of Safety (FoS) in the LEA is compared to a load factor on the active force.

A series of analyses were completed in which a vertical face of a single dry frictional soil unit has a horizontal point load equal to the Rankine active earth pressure, with or without a load factor, applied at one-third of the face height.

The following range of parameters were considered:

ϕ' = 30° or 35°
γ = 18 kN/m^3
Load Factor = 1.0 & 1.5 (for ϕ' = 30°)
= 1.0, 1.3, 1.5, 2.0 and 3.0 (for ϕ' = 35°)

A typical analysis is illustrated in Figure 2 and the results are presented in Table 1 below:

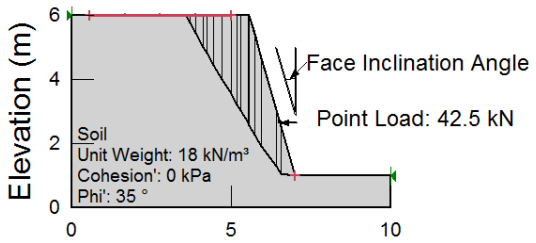

Figure 2. Example Analysis model. For Case A the face was vertical

Table 1. Case Study A results

ϕ'	Load Factor	Factor of Safety
30°	1.0	1.01
	1.5	1.65
35°	1.0	1.00
	1.3	1.30
	1.5	1.51
	2.0	2.26
	3.0	6.78

Case A confirms active earth pressures (as a force) can be determined using LEA by assessing the point load force that results in a FoS of 1.0. At a FoS of 1.0 the analysis is equivalent to the active limit state. The answer should be intuitive but

situations can be envisaged where a designer could misunderstand the analysis:

1. They may target a FoS greater than 1.0, as is typical in a usual stability analysis, resulting in a larger load.
2. They may not equate the force as being an active pressure, using it where an in-situ (K_o) pressure may be appropriate.

The second part of the example considers if a load factor and FoS are equivalent in LEA. The results are contradictory and not intuitive without understanding of the analysis method. For example in the 35° friction angle case the load factor and FoS are equivalent up to a value of 1.5. At a load factor of 2.0 and 3.0 respectively the FoS increases to 2.26 and 6.78 respectively.

These results can be understood with a simple conceptual understanding of how the factored active force (point load) is being incorporated into the limit equilibrium equations, as shown in Equation 1 below:

$$FoS = \frac{F_{Resisting} / MF}{F_{Driving} + F_{Point\ Load} \times LF} \qquad (1)$$

As the point load, negative due to direction, approaches the value of the driving force the bottom line of Equation 1 approaches zero and the resulting FoS will tend to infinity. The bottom line will be zero when the factored point load is equal to the at-rest condition. For a 30° friction angle the at-rest earth pressure is 1.5 times the active earth pressure, hence the FoS of the result with a load factor of 1.5 is infinity for the theoretical active wedge. The analysis still returns a result by finding a slip surface encompassing a larger mass in which the modelled force does not result in a zero or negative bottom line in Equation 1. This results in a FoS greater than the load factor.

The ratio of the active to at-rest earth pressure coefficients ranges from 1.34 to 1.64 for soil friction angles of 20° to 40° respectively with no backslope. Below this ratio, the FoS and load factor are equal, above this ratio the FoS is higher and implies an incorrect degree of robustness.

3. CASE B – DETERMINING ACTIVE EARTH PRESSURES FOR AN INCLINED FACE RETAINING WALL

Case B considers a 5m high retaining wall with the face inclined from vertical and ϕ'=35° as shown in Figure 2. Intuitively the active force is less for an inclined wall than for a vertical wall. It is less intuitive if the robustness imparted by a load factor and a material factor is equivalent despite being numerically equal. A series of LEA analyses with varying face inclinations examine the following:

1. The active force vs inclination.
2. The FoS with a load factor of 1.5 on the active force.
3. The factored active force determined by applying a material factor of 1.5 to the tangent of the soil friction angle.

Analysis 1 intuitively shows the active force reducing with increasing face inclination. The trend is linear and the active force is nil when the slope angle (90° - face inclination) equals the friction angle.

The results of Analysis 2 are shown in Figure 3 below:

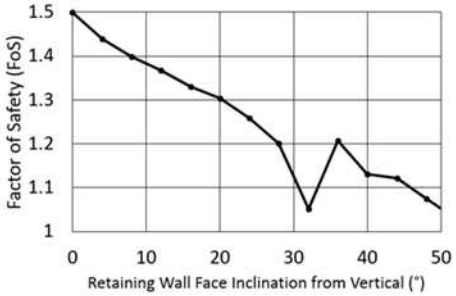

Figure 3. Plot of FoS vs face inclination for a load factor of 1.5

The FoS decreases with face inclination angle despite a constant load factor of 1.5 being applied. The robustness given by the load factor varies and maybe less than is intuitive.

Consider Equation 1 again, as the retaining wall face inclination increases the proportion of the total resistance provided by the soil strength (top line of Equation 1) increases and that provided by the active force (point load on the bottom line but negative) decreases. As only the active force (point load) is factored by the load factor and the point load is providing a reducing proportion of the resisting forces, the FoS decreases with face inclination.

Analysis 3 considers the comparative level of robustness provided by a load factor applied to the active force (point load) and a material factor applied to the resisting force via the soil strength respectively. The results are given in Figure 4 below. It shows the robustness imparted by the material factor is not sensitive to face inclination as the full top line in equation 2 above is factored. Accordingly, a material factor provides more intuitive results in this situation.

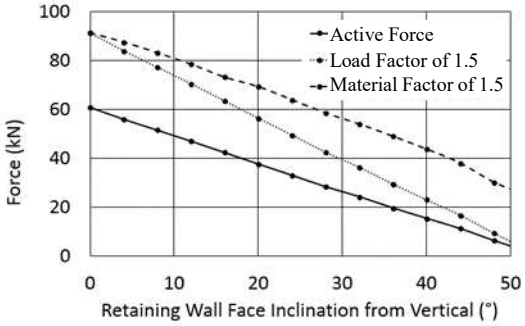

Figure 4. Plot of force vs face inclination for all cases

4 CONCLUSION

Both Case A and Case B have an element like the four-legged chair presented in Hambly's Paradox. The robustness implied by the results may not be intuitive unless the methods and assumptions behind the analysis are understood.

In the examples it is seen that a FoS and load factor may or may not imply the same level of robustness. Likewise a load factor applied to a force and a material factor on soil strength may result in differing levels of robustness.

For young geotechnical engineers the examples demonstrate the need to ensure models and assumptions are understood to provide an appropriate, but not excessive, level of robustness in design. The paper demonstrate how studies of simple models can build understanding of an analysis and develop intuition.

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