On the Soil-Structure Interaction of Surface Strip Footings at Ultimate Limit State

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Abstract: Theoretical methods for estimating the bearing capacity of surface strip footings usually do not consider soil-structure interaction. A few publications deal with the subject, but all of these either do not apply rigorous methods or do not consider the general case of cohesive soils with internal friction.

In the first part of this contribution, the general solution of the bearing capacity of surface strip footings with limited flexural resistance and subjected to a centered vertical load is derived, on the basis of the upper bound theorem of the limit analysis method. The theoretical results are presented by fundamental dimensionless parameters.

In the second part a criterion for the assessment of the relevance of the proposed failure mechanism is presented, showing that the problem of bearing capacity of strip footings of limited resistance is not yet completely clarified.

Keywords: bearing capacity, combined failure mechanism, cracking bending moment.

1 Introduction

The design of foundations must satisfy two requirements, i.e. failure of the foundation must be avoided and total and differential settlements should not exceed threshold values. This article is only concerned with the former aspect within the framework of the soil-structure interaction and for the case of surface strip footings subjected to centered vertical load.

Analytical analysis of surface strip footings at ultimate limit state was generally addressed considering soil and foundation individually (perfectly rigid and infinitely resistant punch). A complete analysis of rigid punches with smooth base on weight-less soils was first made by Prandtl (1920) and then by Hill (1950). However, including soil self-weight greatly complicates the solution and the extension to the 3D case has never been done. Recently, numerical techniques have been employed to solve the issue relative to the self-weight (Veiskarami et al. 2014 and Han et al. 2016). From another side, Terzaghi (1943) applied the limit equilibrium method to compute bearing capacity of surface and shallow footings on cohesive soils with internal friction. His solutions were established by direct superposition of the different contribution to the resistance, only for perfectly smooth and perfectly rough interfaces and they were based on pre-defined failure mechanisms. A further improvement was made by Meyerhof (1951), who optimized failure mechanisms for each case and introduced solutions for base of any roughness (Meyerhof 1955). Nevertheless, the limit equilibrium method is not rigorous and one cannot know if the collapse load established is a lower or an upper bound of the actual collapse load. Chen (1975) was the first to apply the rigorous limit analysis method to the problem of bearing capacity of foundations.

Despite the fact that the hypothesis of a perfectly rigid and infinitely resistant foundation usually leads to reasonable estimation of the bearing capacity for practical purposes, a comprehensive understanding of the phenomenon needs considerations about soil-structure interaction, even at failure. Herzog (1987) and Landgraf & Quade (1993) investigated the behavior of foundations of limited resistance resting on elasto-plastic springs. A first attempt to derive analytical solutions for the bearing capacity of surface strip footings with limited flexural resistance by the upper bound theorem of the limit analysis method was made by Plumey et al. (2004). However, His solutions are limited to purely cohesive and cohesion-less soils and a rigorous validation of the proposed failure mechanism within the hypothesis of the theory of plasticity is missing.

In this study, the authors extend the solution of Plumey et al. (2004) and Plumey (2007) to the general case of cohesive soils with internal friction. The theoretical results are presented in graphical form by fundamental dimensionless parameters and for any value of base roughness. A criterion for the assessment of the relevance of the proposed failure mechanism is also presented, showing that the problem of bearing capacity of strip footings of limited resistance is not yet completely clarified.

2 Bearing Capacity of Surface Strip Footings with Finite Flexural Resistance on Cohesive Soils with Internal Friction

2.1 Combined Failure Mechanism

A kinematically admissible combined failure mechanism, equivalent to the one proposed by Plumey (2007) for surface strip footings resting on cohesion-less materials, is shown in figure 1. It is assumed that at collapse load (Q_R: collapse load per unit length), the system fails by development of a plastic hinge in the foundation and two log-spiral failure lines of a parameter φ' (effective soil shear strength angle) in the soil. The latters are defined by the distance between their center of rotation and the load axis (x), the angle of the initial radius θ_0 and the angle of the final radius θ_h. The velocity field υ is perpendicular to the log-spiral radii and it forms an angle φ' with the failure lines.

The current analysis is based on the upper bound theorem of the limit analysis method, which requires the computation of the rates of external work and internal energy dissipation. The former is given by the vertical load acting on the foundation (P_Q) as well as the weight of the soil mass involved (P_γ), while the latter includes energy dissipation in the foundation plastic hinge (D_M), along the failure lines (D_c), and eventually at the soil-structure interface (D_δ).

Introducing the angular velocity Ω, as shown in figure 2, considering the flexural resistance of the footing per unit length M_R, soil self-weight γ, effective soil cohesion c' and interface shear strength angle δ between the footing and the ground surface, the different contributions can be computed:

$$P_Q = Q_R \cdot \Omega \cdot x \tag{1}$$

$$P_\gamma = 2 \cdot \Omega \cdot \gamma \cdot x^3 \cdot \left(\frac{f_1(\theta_0, \theta_h, \varphi')}{\cos^3 \theta_0} - \frac{\tan \theta_0}{6} + \frac{1}{6} \frac{\tan^3 \theta_0}{\tan^2 \theta_h} \right) \tag{2}$$

$$D_M = 2 \cdot \Omega \cdot M_R \tag{3}$$

$$D_c = \frac{c' \cdot \Omega \cdot x^2}{\tan\varphi' \cdot \cos^2\theta_0} \cdot \left\{ \exp\left[2 \cdot (\theta_h - \theta_0) \cdot \tan\varphi'\right] - 1 \right\} \tag{4}$$

$$D_\delta = Q_R \cdot \tan\delta \cdot \Omega \cdot x \cdot \tan\theta_0 \tag{5}$$

The application of the principle of virtual works leads to:

$$P_Q + P_\gamma = D_M + D_c + D_\delta \tag{6}$$

Substituting equations (1) to (5) into (6), introducing dimensionless parameters $\zeta = x/b$ (with b the footing width), $\mu = M_R / (b^2 c')$, $G = \gamma b / (2c')$ and rearranging leads to the expression of the dimensionless bearing capacity:

$$\frac{q_R}{c'} = \frac{1}{1 - \tan\delta \cdot \tan\theta_0} \cdot \left[\frac{2}{\zeta}\mu - 4G\zeta^2 K_1(\theta_0, \theta_h, \varphi') + \zeta K_2(\theta_0, \theta_h, \varphi') \right] \tag{7}$$

where $q_R = Q_R / b$ is the mean contact pressure under the footing at failure and parameters K_1 and K_2 are respectively:

$$K_1(\theta_0, \theta_h, \varphi') = \frac{f_1(\theta_0, \theta_h, \varphi')}{\cos^3\theta_0} - \frac{\tan\theta_0}{6} + \frac{1}{6}\frac{\tan^3\theta_0}{\tan^2\theta_h} \tag{8.a}$$

with function f_1 given by Chen (1975), and

$$K_2(\theta_0, \theta_h, \varphi') = \frac{\exp\left[2(\theta_h - \theta_0) \cdot \tan\varphi'\right] - 1}{\cos^2\theta_0 \cdot \tan\varphi'} \tag{8.b}$$

Since θ_0 and θ_h are related by the implicit equation (9), the dimensionless bearing capacity q_R / c' for a given foundation and a given soil is found by optimization of parameters ζ and θ_0 only.

$$\sin\theta_h \cdot \exp\left[(\theta_h - \theta_0) \cdot \tan\varphi'\right] = \sin\theta_0 \tag{9}$$

2.2 Determination of Optimum Parameters

Beside the implicit relation between θ_0 and θ_h, the analytical derivation of optimum values is complicated because it leads to unmanageable expressions. By way of example, the optimization with respect to ζ lads to the following expression:

$$\zeta = u + v \tag{10}$$

$$u = \left\{ \frac{1}{8GK_1} \cdot \left[\mu + \sqrt{\mu \cdot \left(\mu - \frac{K_2^3}{864GK_1} \right)} \right] \right\}^{1/3} \tag{10.a}$$

$$v = \left\{ \frac{1}{8GK_1} \cdot \left[\mu - \sqrt{\mu \cdot \left(\mu - \frac{K_2^3}{864GK_1} \right)} \right] \right\}^{1/3} \tag{10.b}$$

Further to what exposed above, a numerical approach was preferred. The selected numerical methods are the Newton-Raphson procedure for the resolution of the implicit equation (9) and the algorithm of simplex for the optimization of θ_0 and ζ because it does not need the direct determination of the derivative of the objective function.

A downstream scheme has been implemented in a Python code in which the main simplex algorithm, in charge of the optimization of θ_0, contains two sub-routines, namely a Newton-Raphson procedure for the determination of θ_h and a simplex procedure for ζ. Basically, that implies the optimization of parameters θ_h and ζ for each trial of θ_0.

2.3 Bearing Capacity

Evolution of dimensionless bearing capacity q_R/c' with increasing dimensionless footing flexural resistance μ is shown in figure 3 for the case of $\varphi' = 30°$, two values of G and different values of δ. It is worth mentioning that each point of each curve corresponds to an optimal geometry of the combined failure mechanism (set of parameters θ_0, θ_h and ζ).

Beyond a certain value of μ, the collapse load obtained through the proposed solution exceeds that of a rigid punch, which becomes then determinant (plateau in figure 3). This corresponds to a failure within the soil without development of a plastic hinge in the foundation. Such solutions according to the upper bound theorem of the limit analysis method are given by Chen (1975) and correspond in turn to the minimum between a Prandtl type mechanism and a Hill type mechanism (as reported in figure 4). In these solutions energy dissipation occurs only along soil failure lines and eventually at the soil-structure interface.

For sake of comparison, the solutions obtained by Terzaghi (1943) with the limit equilibrium method for foundations with smooth interface (dashed light gray lines) and perfectly rough interface (dashed black lines) are also reported in figure 3. His solutions are based on a Prandtl type mechanism with fixed geometry.

The optimum value of flexural bending resistance μ_{opt}, as defined by Plumey et al. (2004), corresponds to the point where the solution of the combined failure mechanism and that of a rigid punch are the same. This because lower values of μ would avoid exploiting the full capacity of the soil, while higher values would do the same for the foundation. Figure 3 shows clearly that in the case of a cohesive soil with friction, such a value is not only affected by δ, but also by G.

According to figure 3, Terzaghi's solution for rigid punches with smooth interface seems to overestimate bearing capacity. While in the case of rough surface it overestimates collapse load for $G = 1$ and gives lower values for $G = 10$. This could be explained by the fact that limit equilibrium method is not a rigorous method and solutions can be either an upper or a lower bound. Moreover, in the solutions of Terzaghi the geometry of the failure mechanism is defined a priory.

3 Relevance of the Combined Failure Mechanism with Plastic Hinge in the Foundation

To authors' opinion, it is necessary to investigate the relevance of the combined failure mechanism with development of plastic hinge in the foundation. The applicability of the hypothesis of the limit analysis method should be checked in the range where the combined failure mechanism governs the bearing capacity.

Given the fact that foundations are concrete or reinforced concrete structures, a ductile behavior must be ensured by a minimum reinforcement ratio. In other words, the combined failure mechanism can only develop if the resisting dimensionless moment provided by the minimum reinforcement is lower than μ_{opt}, or equivalently if the footing dimensionless cracking moment per unit length μ_r (equation 11) is lower than μ_{opt} (equation 12).

$$\mu_r = \frac{M_r}{b^2 c'} = \frac{h^2}{6} f_{ct} \cdot \frac{1}{b^2 c'} = \frac{1}{6} \cdot \frac{1}{\lambda^2} \cdot \frac{f_{ct}}{c'} \tag{11}$$

$$\mu_r < \mu_{opt} \tag{12}$$

Where M_r is the bending cracking moment of the footing per unit length, h the footing thickness, f_{ct} the concrete tensile strength and $\lambda = b/h$ the slenderness ratio of the footing cross-section.

Obviously, μ_r is proportional to f_{ct}, since this is the stress needed to crack the concrete. On the other hand, the inverse proportionality with respect to λ^2 translates the fact that slender cross-sections approach a beam flexural behavior.

In figure 5 the evolution of μ_r relative to f_{ct}/c' and λ is shown.

By comparing figures 5 and 3 it can be seen that actually equation (12) can theoretically be satisfied. In fact in the upper left side of figure 5 values of μ_r are lower than 30.

It is interesting to note that if G is small the soil behaves essentially as a cohesive material, while if G is large, soil weight rather than cohesion is the principal source of bearing capacity (Chen 1975). This implies that the ratio f_{ct}/c' increases with G. Therefore, higher μ_{opt} for large G does not imply greater chance to satisfy inequality (12).

To avoid determining the optimum dimensionless bending moment for each possible value of G, it has been calculated for the limiting cases of purely cohesive and cohesion-less soils (figure 6). The former is given as a function of $\beta = a/c$ where a is the adhesion at soil-structure interface, while the latter as a function of δ for different values of φ'. According to Plumey (2007), the dimensionless resisting moment for cohesion-less soils is defined as $\eta = M_R/(b^3\gamma)$. The dimension-less cracking moment becomes then

$$\eta_r = \frac{M_r}{b^3\gamma} = \frac{h^2}{6} f_{ct} \cdot \frac{1}{b^3\gamma} = \frac{1}{6} \cdot \frac{1}{\lambda^2} \cdot \frac{f_{ct}}{b\gamma} \tag{13}$$

meaning that the evolution of η_r can be read from figure 5 simply by substituting c' with $b\gamma$.

For clarification purposes it should be noted that Plumey's solution of the combined failure mechanism for cohesion-less soils has been reviewed by the authors in view of a mistake in the derivation of the velocity field at the soil-structure interface. The bearing capacity can be computed by neglecting the term D_c in equation (6) and it is given in equation (14).

$$\frac{q_R}{b\gamma} = \frac{-3.78}{1 - \tan\delta \cdot \tan\theta_0} \cdot K_1(\theta_0, \theta_h, \varphi') \cdot \eta^{2/3} \tag{14}$$

The present section showed that the combined failure mechanism proposed by Plumey et al. (2004) with development of a plastic hinge in the foundation is justified and can be used to target an optimum design in some circumstances. On the other hand, it is worth mentioning that within the range of most common practical values f_{ct}/c' (equivalently $f_{ct}/(b\gamma)$) and λ, inequality (12) would probably not be satisfied. Moreover, below μ_r (equivalently η_r) such a combined failure mechanism cannot develop. Therefore, in these situations the combined failure should occur by another mechanism. This is supported by the fact that in compact cross-sections shear deformations could be of relevance and shear failure may govern. It might be that in some cases combined flexural failure mechanism would do the transition between a combined shear failure mechanism and the rigid foundation failure mechanism, but this requires further studies.

4 Conclusion

A general solution of bearing capacity for surface strip footings with finite flexural resistance has been developed, on the basis of the upper bound theorem of the limit analysis method, by extending the previous analysis to cohesive soils with internal friction. Theoretical results can be expressed by the fundamental dimensionless parameters μ, G, φ' and for any degree of roughness of the base. The relevance of the studied combined failure mechanism has been critically analyzed through the introduction of the dimensionless cracking bending moment of the footing.

The analysis indicate that normalized bearing capacity increases with footing dimensionless flexural resistance, parameter G, shear strength angle and base roughness. It is in any case bounded by the solution of a rough rigid foundation. The combined failure mechanism with plastic hinge is theoretically possible, but it is also shown that in a number of practical situations its solution is either totally or partly not possible, leading to the conclusion that other failure mechanisms are needed to describe the complex behavior of the surface footing soil-structure interaction system at ultimate limit state. The susceptibility of shear failure has been qualitatively highlighted through the slenderness ratio of the footing cross-section.

This study may be of encouragement for further investigations on the optimization of soil-structure interaction systems.

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