Finite Element Analyses for fracture assessment of rocks through an energetic approach: application of the SED criterion

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1 Introduction

This study aims to analyze the fracture behavior of rocks, which is irretrievably linked to their heterogeneous nature both at macro- and micro-scales. This heterogeneity is defined, on the one hand, by the composition of the rock matrix itself, and on the other hand, by the presence of different elements such as pores, microcracks, discontinuities or grain boundaries that can be considered as defects from an integrity point of view. All these elements act as stress risers and play a key role in the analysis of the mechanical behavior of rocks. They generate stress concentrations around them, leading to crack initiation, propagation and, eventually, to brittle failure. Besides, the brittle condition of intact rocks, their high inherent strength, low toughness and relatively high stresses to which rocks are subjected both in nature or industrial exploitations make them very sensitive to their presence.

Rock fracture mechanics has traditionally tended to be over-conservative as a heritage of linear elastic fracture mechanics, where a crack behavior is usually assumed even for notch-type defects. Several practical situations can be found in literature assuming sharp crack behavior within different applications of rock cutting, hydraulic fracturing or underground excavations (e.g., Whittaker et al. 1992, Aliabadi 1999, Jaeger et al. 2007). However, notch-type defects generate less demanding stress fields than crack-like defects and, therefore, develop higher load-bearing capacity. This problem can be applicable to different fields within civil (e.g., slopes, foundations), mining (e.g., tunneling, drilling) and energy engineering (e.g., geothermal exploitations). For this reason, this article aims to offer a new accurate tool for rock fracture assessment. The strain energy density (SED) criterion will be used with this purpose, which implies an energetic continuum approach. A similar study using another method, namely the Theory of the Critical Distances (TCD), was made by the authors of this article at Justo et al. (2017).

With all this, Section 2 gathers a brief theoretical background on the basis of the SED criterion. Section 3 includes an explanation of the followed methodology both for the experimental program and the numerical analyses. And finally, Section 4 comprises the results and the conclusions of the study.

2 Theoretical background: SED criterion

The SED criterion comes from the combination of the elementary volume proposed by Neuber (1958) and the local Mode I concept proposed by Erdogan et al. (1963). It has been successfully applied to different materials like steels or polymers during the last few decades (e.g., Lazzarin et al. 2005, Berto et al. 2014), but its applicability on rocks still requires further research (e.g., Aliha et al. 2017, Justo et al. 2018).

Basically, the SED criterion states that failure will occur when the mean strain energy density ($\overline{W}$) over a given control volume is equal to a certain critical value (W_c). This latter value is represented by the area under the stress-strain curve of the material, which in the case of linear elastic and quasi-brittle materials like rocks has a very simple expression:

$$W_c = \frac{\sigma_u^2}{2E} \tag{1}$$

where σ_u is the tensile strength and E is the Young's modulus of the rock. On the other hand, the control volume over which the strain energy density is averaged becomes a control area (Ω) in plane strain conditions as considered in this study.

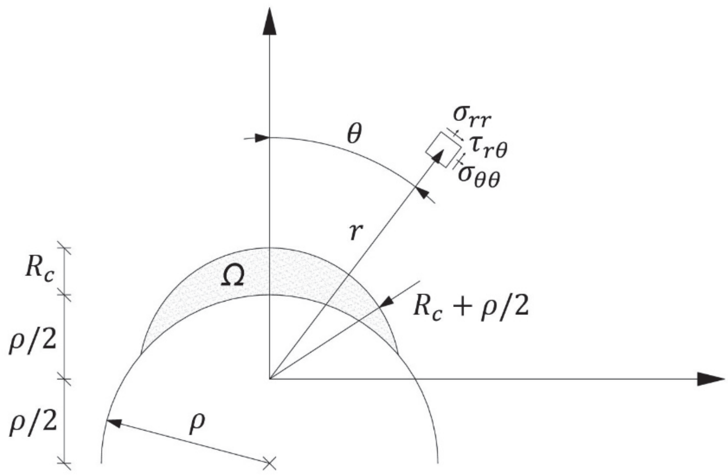

Fig. 1: Definition of the control area at the notch tip

The control area is defined by a radius R_c as shown in Fig. 1, and is obtained by the following expression proposed by Yosibash et al. (2004) for static loads and U-shaped notches, as those studied in this article:

$$R_c = \frac{(1+\nu)(5-8\nu)}{4\pi}\left(\frac{K_{IC}}{\sigma_u}\right)^2 \tag{2}$$

This expression only depends on the analyzed rock through its tensile strength (σ_u), the fracture toughness (K_{IC}) and the Poisson's ratio (ν).

Calculating the strain energy density at a certain point (W) in the neighborhood of the notch requires a previous characterization of the stress field, which can be obtained numerically. For the selected isotropic rocks, for example, assuming they obey a linear elastic law and using the polar coordinate system defined in Fig. 1, the expression for the strain energy density is as follows:

$$\begin{aligned} W(r,\theta,z) = \frac{1}{2E}\{&\sigma_{\theta\theta}^2 + \sigma_{rr}^2 + \sigma_{zz}^2 + 2\tau_{r\theta}^2 \\ &- 2\nu(\sigma_{\theta\theta}\sigma_{rr} + \sigma_{\theta\theta}\sigma_{zz} + \sigma_{rr}\sigma_{zz} - \tau_{r\theta}^2)\}\end{aligned} \tag{3}$$

According to the stated failure criterion, the strain energy density must be averaged in the control area (Ω) defined by Eq. (2).

$$\overline{W} = \frac{\int W \cdot d\Omega}{\Omega} \tag{4}$$

The obtained average value is to be compared with the critical one calculated by Eq. (1). When $\overline{\mathrm{W}} = \mathrm{W_c}$ fracture will occur. To avoid a direct and costly calculation of $\overline{W}$ for each individual case, Lazzarin et al. (2005) developed Eq. (4) and expressed the mean strain energy density ($\overline{W}$) in a simpler form, which was particularized by the authors of this article (Justo et al. 2018) for the case study:

$$\overline{W} = 0.785 \cdot H(R_c/\rho,\nu) \cdot \sigma_{\max}^2/E \tag{5}$$

where σ_{max} and E are the maximum stress at the notch tip and the Young's modulus, respectively. The constant is derived from the geometry of the defect, which, for U-shaped notches, is equal to 0.785. Finally, the H function depends on the ratio R_c/ρ (ρ being the notch radius) and the Poisson's ratio ν, and can be obtained from tabulated values like those offered by Lazzarin et al. (2005). However, the range of tabulated values of R_c/ρ they propose only reaches 1 as the maximum value, which may be sufficient for materials like steels or polymers (e.g., Lazzarin et al. 2005, Berto et al. 2014) but is not enough for rocks. These ratios are of the order of several tens in the case of rocks (Justo et al. 2018).

Therefore, this study makes use of Finite Element Analyses in order to obtain the new values of the H function corresponding to the appropriate range of R_c/ρ values for the analyzed rocks. Once the H is known, it is straightforward to apply Eq. (5) and to obtain the predicted fracture load.

3 Methodology

3.1 Experimental program

Two different rocks have been selected for this analysis: a Moleano Limestone (C) and a Floresta Sandstone (F). They are both fully described by the authors at Justo et al. (2017), and their characterization required a rigorous laboratory campaign including 96 four-point bending tests with U-notched specimens and with notch radii (ρ) varying from 0.15 mm up to 15 mm (See Fig. 2), 24 uniaxial compression tests and 24 splitting tensile (Brazilian) tests for both rocks.

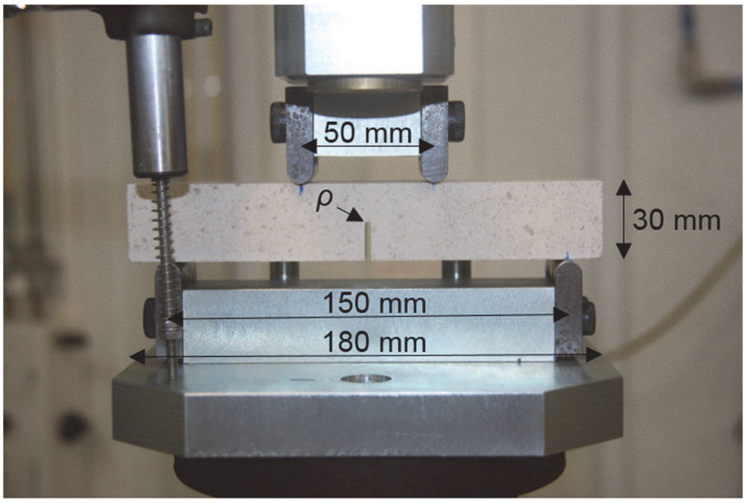

Fig. 2: Experimental setup in four-point bending tests

The four-point bending tests with the smallest notch radius (ρ = 0.15 mm, which can be assimilated to a crack), allow obtaining the fracture toughness (K_{IC}) of the analyzed rocks. The rest of the experimental results obtained from the four-point bending tests will be compared with the predictions based on the SED criterion. Likewise, the Young's modulus (E) and the Poisson's ratio (v) are derived from the uniaxial compression tests, in which both the longitudinal and transverse deformations are measured using strain gauges. Finally, the tensile strength (σ_u) of the rocks is indirectly obtained from the Brazilian tests. These are the only four required material parameters for the correct application of the SED criterion. The experimental procedure is deeply detailed at Justo et al. (2017) and Justo et al. (2018). Table 1 gathers the obtained mechanical properties for the Moleano Limestone (C) and the Floresta Sandstone (F).

Tab. 1: Main mechanical properties of each rock

	Moleano Limestone (C)	Floresta Sandstone (F)
Fracture toughness, K_{IC} (MPa·m$^{1/2}$)	0.73	0.37
Young's modulus, E_{50} (GPa)	38.4	19.6
Poisson's ratio, v	0.27	0.24
Tensile strength, σ_u (MPa)	7.08	3.38

3.2 Numerical evaluation of function *H*

Once the corresponding material parameters are known, the *H* function can be determined numerically. In this work, a finite element code called PLAXIS 2D (Brinkgreve et al. 2015) has been used. A linear elastic model is considered to simulate the rocks, so only two parameters are needed: the Young's modulus (E) and the Poisson's ratio (v). It is important to emphasize that *H* is theoretically independent of the considered notch radius in the calculation model (ρ_{FE}). For the particular case of U-shaped notches, the *H* function only depends on v and on the ratio R_c/ρ. Any combination of R_c and ρ offering the same ratio will provide the same value of *H* for any material with a certain v. Thus, from a strict point of view, it is sufficient to conduct a single numerical model with a specific notch radius (ρ_{FE}) for each material, only varying the value of R_c. However, this assertion is limited, since R_c defines the size of the control area (*Ω*) over which the strain energy density is averaged (see Fig. 1), and this size is obviously restricted. First, the lower bound of *Ω* is delimited by the refinement of the mesh and, by contrast, the upper bound is physically limited by the geometry of the numerical model. Besides, the closed form expressions are only valid in the vicinity of the notch, so large control areas would distort the results.

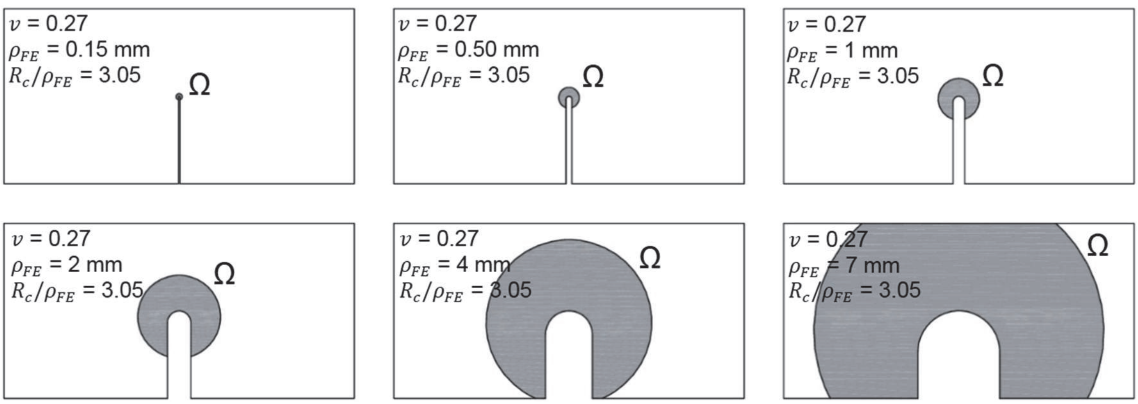

Fig. 3: Control areas (*Ω*) of the Moleano Limestone (C) for R_c/ρ_{FE} = 3.05 and for models with different notch radii (ρ_{FE}).

As shown in Fig. 3, for a certain value of R_c/ρ_{FE}, the size of the control area increases as higher notch radii are modeled, even exceeding the geometric limits as in the case ρ_{FE} = 7 mm in that example. For this reason, a sensitivity analysis has been carried out trying to determine the most suitable notch radius to be implemented. The results corresponding to a numerical model with ρ_{FE} = 0.15 mm, ρ_{FE} = 0.50 mm and ρ_{FE} = 1 mm will be compared in this article, since larger values of ρ_{FE} offer the poorest predictions.

The only required outputs from the model are the maximum stress at the notch tip (σ_{max}) and the stress field in the neighborhood of the notch. The strain energy density (W) is calculated at any point from Eq. (3) once the stress field is known, and W is averaged through the control area (Ω) in order to obtain the mean strain energy density ($\overline{W}$) according to Eq. (4). The applied load in the model is not determinant, since H does not depend on σ_{max} and is proportional to $\overline{W}$. Finally, the values of the function H are directly obtained by inverting Eq. (5).

This procedure allows obtaining the seeked H function corresponding to the range of R_c/ρ values suitable for the studied rocks. Thus, maintaining the obtained H values and considering the failure criterion defined by $\overline{\mathrm{W}} = \mathrm{W_c}$, the maximum stress at the notch tip (σ_{max}) that leads to the fracture of the rock can be calculated from Eq. (5). The resultant stress should be consistent with the expression proposed by Creager & Paris (1967) to define the stress distribution as a function of distance from the notch tip (r). This stress will be maximum at the notch tip for r = 0:

$$\sigma(r=0)=\sigma_{\max}=\frac{K_I}{\sqrt{\pi}}\cdot\frac{2\rho}{\rho^{3/2}} \tag{6}$$

Where K_I is the stress intensity factor and can be calculated inverting Eq. (6), considering the value of σ_{max} that corresponds to the failure situation.

On the other hand, there is much bibliography with analytical solutions of K_I for common test specimens (e.g., Tada et al. 1985), for which:

$$K_I=\frac{P_{SED}}{B\sqrt{h}}Y \tag{7}$$

B and h are, respectively, the thickness and the height of the used specimens, and Y is another geometrical factor that, in the case of the tested samples, is equal to 10.162 (Justo et al. 2017). Considering the value of K_I calculated by Eq. (6), the predicted fracture load (P_{SED}) is derived from Eq. (7).

4 Results and conclusions

Fig. 4 collects the results corresponding to the two analyzed rocks: the Moleano Limestone (Fig. 4a) and the Floresta Sandstone (Fig. 4b). The horizontal axis represents the notch radii of the tested specimens and the vertical axis stands for the ratio between the predicted and the experimental fracture load of the rocks. Therefore, the horizontal solid line corresponding to $P_{SED}/P_{EXP} = 1$ indicates the theoretically exact prediction. Likewise, the horizontal dashed lines form an envelope of $\pm 20\%$, which tries to encompass the intrinsic uncertainties of the performed laboratory tests and the variability of the tested materials as a consequence of their heterogeneous nature. It is a common practice in fracture mechanics to consider a strip of $\pm 20\%$ as the limits of a reasonable accuracy.

a)

b)

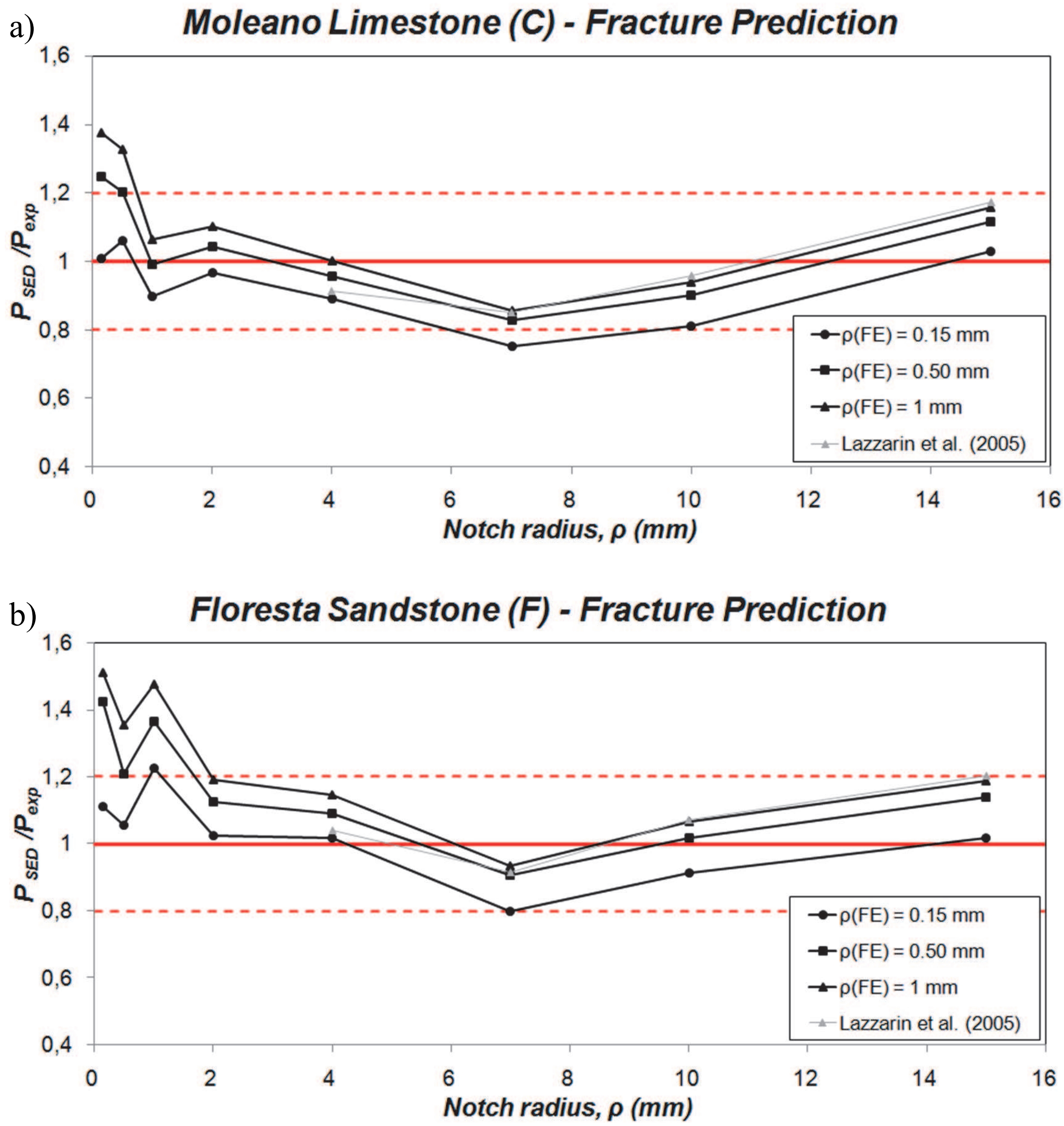

Fig. 4: Results of the predicted failure loads for both (a) the Moleano Limestone and (b) the Floresta Sandstone applying the SED criterion.

The represented results correspond to the predicted fracture loads of the four-point bending tests with different notch radii (ρ). The legend refers to the ρ_{FE} values that have been implemented in the numerical model to obtain the required H functions.

The predictions represented by the grey curves are derived from the tabulated values of H proposed by Lazzarin et al. (2005), which were, in origin, calculated numerically with ρ = 1 mm. Thus, these values are to be compared with the those corresponding to the numerical model with ρ_{FE} = 1 mm. They show a considerably good agreement, so the used model can be considered validated and suitable for the numerical analyses.

In general, relatively accurate predictions are observed for both rocks, even beyond the application range of Creager & Paris (1967), which is supposed to be only valid for long and narrow notches. The best results seem to correspond to the predictions coming from the model with the smallest notch radius (ρ = 0.15 mm), specially for the test specimens with the smallest notches. As higher radii are implemented in the model, a progressive increase in the predicted values is observed, leading to significant overestimations in the case of the test specimens of the Floresta Sandstone (Fig. 4b) with the smallest notch radii, for example. The values of H obtained from the numerical models with notch radii larger than 2 mm offered poor predictions, probably due to the large control areas (Ω) that even exceeded the geometric limits of the model. By contrast, the greater dispersion observed in the predictions of the samples with the smallest notch radii could be due to the small size of the area over which the strain energy density is averaged in these case, which show a significant sensitivity to the size of the mesh of the numerical model.

All in all, the SED criterion has proven to provide reasonable fracture predictions within a range of $\pm$20% with respect to the theoretical failure load. After characterizing the necessary material parameters (E, v, K_{IC}, σ_u), it is straightforward to obtain the required H function for the fracture assessment by the use of a very simple numerical model for each material. Once the values of H are known, these are valid for any material with the same Poisson's ratio (v) as long as the same range of R_c/ρ values are considered.

5 Awknowledgements

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