The Influence of Boundary Conditions on the Results of Nonlinear Dynamic Numerical Analyses

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ABSTRACT

Up-to-date numerical software for the geotechnical analyses of dynamic problems offer different options for lateral boundary conditions. Ideally, the lateral boundary should not affect the movements or response spectrum at any point within the analysed domain. The choice of boundary conditions, distance to the boundary, FE mesh density and some other modelling details were analysed for a specific case and parametric analysis was performed. Plaxis 2D software was used with Hardening soil small strain stiffness model. Seismic excitation was applied at the bottom of the model. The results of FEM analyses were compared also with the results of 1D SHAKE software.

Keywords: Boundary conditions, dynamic soil response, Plaxis, SHAKE

1 Introduction

Besides static loading, buildings and other structures may occasionally be subject to much higher stresses due to earthquakes. In order to construct those entities safe during earthquake events many numerical software packages for dynamic analyses were developed. Main goal of this study is to compare the soil response of simple numerical models with different boundary conditions using equivalent linear programme SHAKE and 2D Plaxis. FEM software requires implementation of advanced material models (e.g. Hardening soil small strain stiffness) in order to describe nonlinear elastoplastic behaviour of the soil during dynamic loading. Initially, short description of numerical formulation and type of boundary conditions in Plaxis are presented, followed by a practical application and final findings on how boundary conditions influence response spectra.

2 A short overview of the dynamic formulation in SHAKE and Plaxis

A short review of the theoretical background of two software packages (SHAKE and Plaxis) for calculating site response analysis is highlighted in this chapter. SHAKE is one dimensional equivalent-linear program based on analytical solution with many assumptions. Plaxis, on the other hand, is the program based on a finite element method, capable of modelling one or multidimensional problems.

2.1 SHAKE

In the early 1970s, software SHAKE was developed with purpose to calculate the vertical propagation of shear waves through soil profile (Schnabel et al. 1972). Program is applicable to a horizontally layered ground profile with well known soil parameters - density ρ, shear modulus G, critical damping ratio ζ and thickness of each layer H as illustrated in Figure 1. The solution of wave equation for each layer can be expressed as the summation of upward and downward propagating waves. Transition between two adjacent layers is described with transfer function which ensure the continuity of stresses and displacements (Kolsky 1963). Thus, the input motion can be assigned to any layer within the model and SHAKE calculates motions in all other layers.

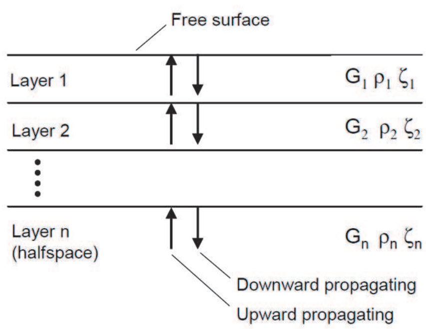

Fig. 1: Scheme of soil profile in SHAKE (Mejia & Dawson 2006).

SHAKE is equivalent linear program. This means that strain dependent modulus and damping of a soil layer is accounted for by iterative linear procedure. Dependencies of shear modulus and damping with shear strain are expressed with "S-shaped" reduction curves. SHAKE repeats iterations until the difference between shear strain in previous and present iteration is within acceptance criterion (Mejia & Dawson 2006).

Ground motion can be applied at the free surface ("outcrop" motion) or at the interface of two layers ("within" motion) as acceleration time history with equally spaced time steps. Considering a zero shear stress condition at the surface (upward and downward wave propagation should be equal at the surface) it is known that motion at the surface is two times multiple of the upward propagating wave. Consequently, a half of the measured motion at the rock outcrop (wave propagation through elastic medium) should be applied at the base of the finite element model with absorbing boundary condition for downward propagating waves to get the surface target motion.

Main advantages of SHAKE are its simplicity and wide correlation with collected data. Moreover, it is possible to perform a deconvolution to prepare input motion(s) for analyses in software with finite elements. It should be mentioned that in case of a strong earthquake with high frequencies, the results are less reliable and additional studies are required. With respect to our study, it is important to note that there are no lateral boundary conditions in 1D SHAKE program and can therefore serve as a reference tool when studying the effects of different boundary conditions on calculation results.

2.2 Plaxis

Plaxis is one of the most widely used software in geotechnical engineering. Besides many applications (tunnelling, excavation, slope stability, etc.) dynamic analyses can also be performed.

Basic concept of solving dynamic problems is based on Newton`s second law of motion, where resultant force is equal to product of mass and acceleration. Equation bellow describes this relationship (on a matrix level) (Galavi 2013 and Plaxis):

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{C} \cdot \dot{\mathbf{u}} + \mathbf{K} \cdot \mathbf{u} = \mathbf{F} \qquad (1)$$

Where $\boldsymbol{M}$ is mass matrix, $\boldsymbol{C}$ is damping matrix, $\boldsymbol{K}$ is stiffness matrix and $\ddot{\boldsymbol{u}}$, $\dot{\boldsymbol{u}}$, $\boldsymbol{u}$ and $\boldsymbol{F}$ are acceleration, velocity, displacement and dynamic force vectors, respectively.

Matrix $\boldsymbol{C}$ stands for material damping, caused by inter particle interaction (e.g. friction, irreversible deformations and viscosity) in soil. Nowadays advanced material models can capture the majority of soil nonlinearity and plasticity. Plaxis is capable of solving dynamic problems with any material model in drained or undrained conditions. However, if necessary small amount of damping can be still applied through matrix $\boldsymbol{C}$ by Eq. (2).

$$\mathbf{C} = \alpha_R \cdot \mathbf{M} + \beta_R \cdot \mathbf{K} \qquad (2)$$

α_R and β_R are Rayleigh coefficients, where α_R controls the impact of mass on damping, while β_R determines the influence of stiffness (Zienkiewicz & Taylor 1991). To damp lower frequencies more, higher α_R is used and respectively higher frequencies can be damped with higher β_R. Rayleigh coefficients are estimated on the basis of the target damping ratio (ξ) in the range of two target frequencies. The first one is natural frequency of soil deposit and the second is a first odd integer of a quotient between dominant ground motion frequency and first natural frequency. More details are described in Hudson, Idriss & Beirkae (1994) and Hashash & Park (2002).

Implicit Newmark`s integration scheme is used to numerically solve Eq. (1) in Plaxis:

$$(c_0\mathbf{M} + c_1\mathbf{C} + \mathbf{K})\Delta u = \mathbf{F}_{ext}^{t+\Delta t} + \mathbf{M}(c_2\dot{u}^t + c_3\ddot{u}^t) + \mathbf{C}(c_4\dot{u}^t + c_5\ddot{u}^t) - \mathbf{F}_{int}^{t} \quad (3)$$

Dynamic integration coefficients c_0 to c_5 in Eq. (3) depend on critical time step and Newmark integration parameters α and β (Plaxis).

3 Dynamic boundary conditions in Plaxis

System of equations (Eq. 3) can be solved only when initial and boundary conditions are applied into the model. The main goal in dynamic analysis is to correctly simulate wave propagation through a soil profile. Therefore, to avoid analysing space and time consuming model, boundaries without wave reflection back into the model are required. In the following, four types of dynamic boundary conditions in Plaxis are described.

3.1 Tied degrees of freedom

This type of boundary condition is only appropriate for simulating the shear wave propagation through a 1D soil column with a symmetrical mesh distribution. With this condition, nodes at the same level at both lateral sides of the model are connected and characterized with same vertical and horizontal movement. Wave with its source inside the mesh or those reflected back into the model because of geometry anomaly or structural elements cannot be absorbed by tied degrees of freedom boundary condition (Galavi et al 2013 and Plaxis).

3.2 Viscous boundary

Beginnings of viscous boundaries stretch to late 1960s when Lysmer and Kuhlmeyer (1969) implemented the idea to absorb outgoing waves. This absorbing boundary is capable to nullify reflected stresses caused by dynamic input. Usefulness of viscous boundary is limited on dynamic source that needs to be inside the mesh. Moreover, it is not compatible with structural elements (Galavi et al 2013 and Plaxis).

3.3 Free field and compliant base

Free field boundary condition consists of an extra column next to the vertical boundary of the main model to simulate far field with minimum reflection (Figure 2). Same material properties are adopted for this new artificial column as inside the mesh. During dynamic loading, normal and shear stresses are computed separately in the free field boundary and applied to the main domain as equivalent stresses (Eq. (4) and (5)). Free field boundary considers absorption of reflected waves from internal anomalies (Galavi et al 2013 and Plaxis).

$$\sigma_n^m = -\rho v_p \left(\frac{\delta u^m}{\delta t} - \frac{\delta u^{ff}}{\delta t}\right) + \sigma_n^0 \tag{4}$$

$$\tau^m = -\rho v_s \left(\frac{\delta v^m}{\delta t} - \frac{\delta v^{ff}}{\delta t}\right) + \tau^0 \tag{5}$$

Where ρ is soil density, v_p and v_s are compressional and shear wave velocity, σ_n^0 and τ^0 are normal and shear stresses in static condition, u^m (v^m) and u^{ff} (v^{ff}) are normal (tangential) displacements in main domain and free field boundary, respectively.

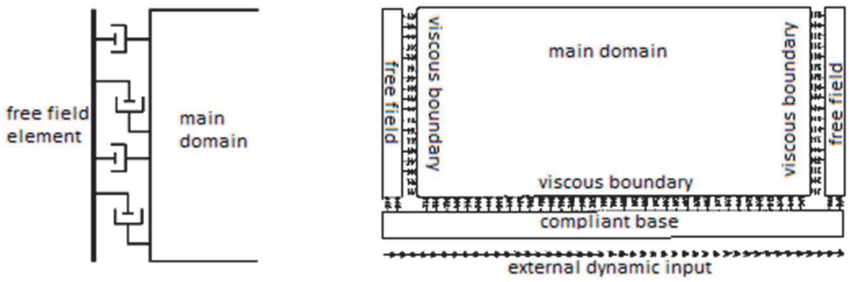

Fig. 2: Scheme of free field element with compliant base (Galavi et al 2013).

Free field boundary can be applied only at the lateral sides of the model. At the base of the model, similar condition to the free field is available in Plaxis i.e. compliant base. Compliant base boundary is based on the same principle of absorbing outgoing waves into the infinite space bellow model base with additional potential to transfer prescribed dynamic load up into the model. Firstly, input ground motion is multiplied with a factor of 2, because half of the input energy ("downward propagating waves") is absorbed by viscous part of compliant base boundary. After ground motion history is transformed into stress time history, it is applied to the main domain (Galavi et al 2013 and Plaxis).

Both boundary conditions can absorb perpendicular outgoing waves better than those who are coming under small angle. In order to correctly convert stresses from artificial columns or artificial base to the main domain interface elements should be applied at the same places to create node pairs.

4 Practical application

4.1 Numerical model and soil parameters

All models in our 2D (and “1D”) analyses were constructed with intention to get essential details for parallel modelling of a 3D model of an underground silo located in SE Slovenia. At the site, extensive borehole drilling program was carried out as well as other in-situ and laboratory tests. Estimation of geometry data and soil parameters are based on the geological and geotechnical investigation report.

Soil deposit consists of four representative horizontal layers. Under gravel to sandy gravel layer (11 meters thick), overconsolidated silty layer of Miocene age can be found. Upper 19 meters of the Miocene layer represents transitional zone with sandy silt to silt. At depths bellow 89 meters shear wave velocity exceeds 800 m/s and consequently linear elastic material was assigned to the model bellow this elevation.

In order to model soil response with hysteretic nonlinear stress-strain relation and soil stiffness variation due to increase/decrease of strain magnitude, the Hardening soil small strain stiffness model (HSS) was adopted for the upper three layers. We assume elastic behaviour of bottom layer and simpler Linear Elastic material model was chosen for it. Soil properties and 2D model in Plaxis are illustrated in Figures 3 and 4.

Ground water level in the model was at 0.7 meter under the surface.

Material model	Identification	$\gamma_{sat/unsat}$ [kN/m³]	E [MPa]	ν	G [MPa]	v_s [m/s]
LE	Elastic-rock	21	5140	0,2	2140	1000

Material model	Identification	$\gamma_{sat/unsat}$ [kN/m³]	E_{50}^{ref} [MPa]	E_{ur}^{ref} [MPa]	p^{ref} [kPa]	c^{ref} [kPa]	φ^{ref} [°]	$\gamma_{0.7}$	G_0^{ref} [MPa]	Rayleigh α	Rayleigh β
HSSmall	Gravel	21	35	105	25	10*	38	0,0001	220	0,09425	0,0007958
HSSmall	Miocen Upper	20	23	70	100	30	34	0,0002	200		
HSSmall	Miocen Lower	20	23	70	100	30	34	0,0004	250		

Comment: *10 kPa of cohesion was applied to gravel layer in order to avoid numerical instability at the surface of the model.

Fig. 3: Soil properties for Plaxis analyses.

For dynamic loading three earthquakes (Tolmezzo, Hercegnovi and Ulcinj) recorded on rock outcrop were used in both 1D and 2D response analyses. Ground motions were modified to target peak ground acceleration of 0.43g and adjusted to uniform hazard spectrum on rock with return period of 2500 years. Analyses were performed for both perpendicular directions in horizontal plain (N-S and E-W) and the results were then combined into an average spectrum of all 6 simulations.

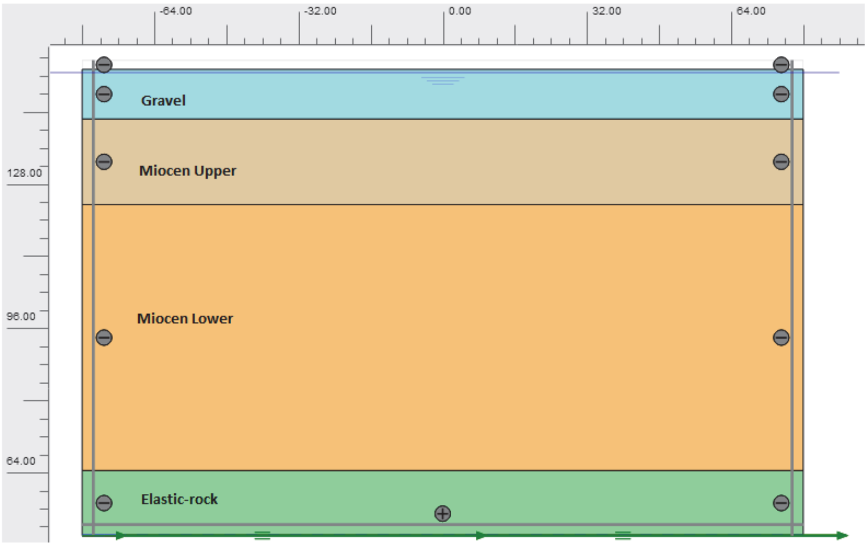

Fig. 4: 2D model in Plaxis.

Due to SHAKE`s simplicity, increase of soil stiffness with depth was achieved with layered model in SHAKE. The model was constructed of 16 layers (upper 7 layers were 5 meters and the rest 6 meters thick). Shear modulus and shear wave velocity profile, as well as reduction curves are shown in Fig. 5. Moreover, cross-hole tests were carried out at the site and good matching with calculated shear velocities was achieved. Soil density varies between 18.5 and 20.6 g/cm^3.

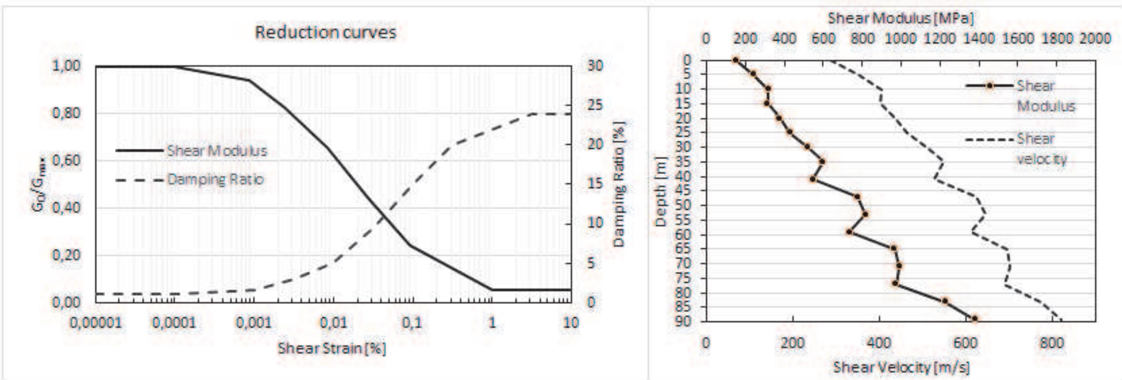

Fig. 5: Reduction curves (left) and shear modulus profile (right) - SHAKE.

4.2 A short overview of all simulations

In order to compare the results from SHAKE, Plaxis equivalent 1D analysis with tied degrees of freedom boundary condition was initially carried out. Furthermore, 2D analyses with different model properties, distance to the lateral boundary and mesh density were examined. At the beginning, the effect of Rayleigh damping was also analysed.

The compliant base boundary condition was used in all Plaxis analyses. Consequently, half of outcrop motion was applied at the model base.

The initial 2D analyses were performed in order to check the numerical stability of the model and the matching of the spectra on the surface and at the contact between the linear elastic rock and lower Miocene layer. Initially, a smaller width of the model (B = 160 m) and some model variations such as including a 5 meter wide column with higher strength along the lateral edges and/or applying average shear modulus independent on minor principal stress (σ'_3) in gravel were checked. The reason for the latter variation was to avoid often and considerable changes of stiffness during dynamic loading due to the rotation of principal stresses σ'_1 and σ'_3. Lastly, wider models (200, 240, 280, 320 and 360 m) and different mesh densities for 280 meters wide model were analysed.

Tab. 1: List of all Plaxis analyses.

No.	**Type of analysis**	**Model width [m]**	**Type of boundary condition**	**Rayleigh damping**	**Other changes**
1, 2	1D	2	TDF	Yes & No	
3, 4	2D	160	FF	Yes & No	
5	2D	160	FF	Yes	Constant moduli in gravel
6	2D	160	FF	Yes	Constant moduli in gravel and soil column with higher strength at model edges
7, 8, 9, 10, 11	2D	200, 240, 280, 320 and 360	FF	Yes	Constant moduli in gravel
12…	2D	280	FF	Yes	5 different mesh densities
Comment: TDF-tied degrees of freedom; FF-free field					

4.3 Comparison between SHAKE and "1D" Plaxis

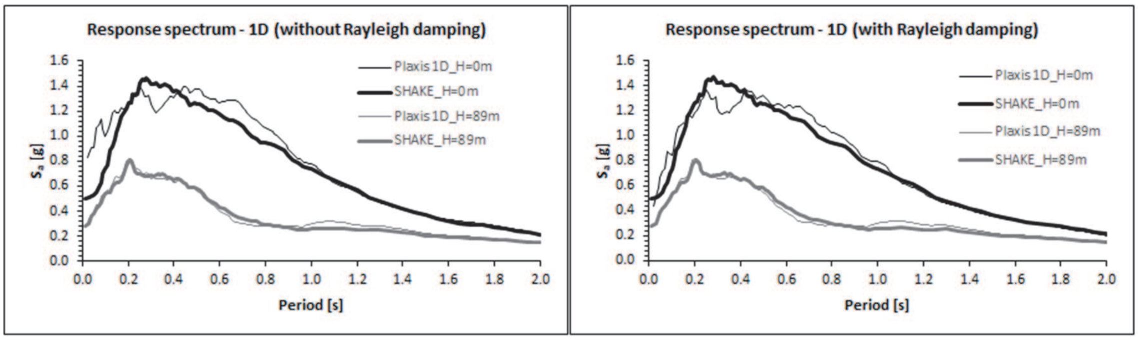

Fig. 6: Comparison of spectra without (left) and with (right) Rayleigh damping - SHAKE vs 1D Plaxis.

Graphs (Fig. 6) above show the difference between response spectrum from SHAKE and 1D Plaxis analysis. It can be seen that better matching is achieved with additional Rayleigh damping, especially at lower periods. Furthermore, Plaxis spectrum deviates from SHAKE spectrum at periods 0.25 to 0.4 seconds, which is around natural frequency of the ground.

4.4 Comparison between "1D" and 2D Plaxis

This chapter summarizes comparisons between 1D and 2D analyses in Plaxis. The graphs bellow summarize results of all performed analyses. Comparable differences between spectra with or without damping were also obtained with 2D analyses.

Even bigger drop of the spectral values were obtained near the ground natural frequency with 2D analyses. Therefore, some parameter alternations (average stiffness with m = 0 in gravel, HS – strengthened column at edges) were implemented into the model, but neither one drastically improved matching spectra in the case of 160 meters wide model (Fig. 8 (left)). Additional analyses with wider FE models (200 to 360 m) were performed. Figure 7 shows the effect of the model width on the response spectrum. When model width is larger than 280 meters, only negligible mismatching can be observed from 1D analysis.

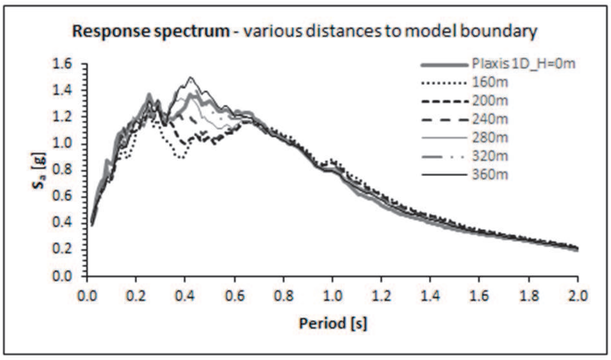

Fig. 7: Effect of distance to the model boundary on response spectra.

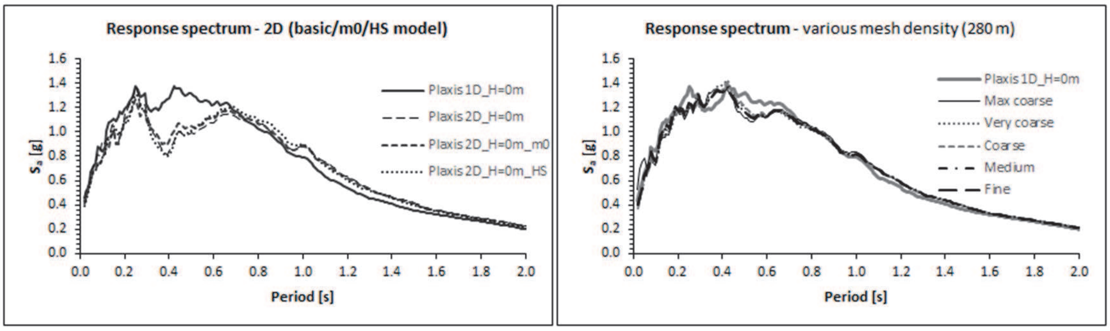

Fig. 8: Effect of some modelling details (left) and mesh density (right).

When the appropriate width of model was found (B = 280 m), the influence of standard mesh densities in Plaxis was examined (Fig 8 (right)). In the previous 2D analyses mesh density satisfied the condition, where element size is controlled by maximum frequency of the input ground motions and minimum shear wave velocity in the soil profile by equation: $\mathrm{ElementSize} \leq v_{s,min}/(8 \cdot f_{max})$ (Kuhlmeyer and Lysmer 1973). Nevertheless, negligible differences between the results with various Plaxis standard mesh densities were observed, due to the simple model with horizontal layers (similar soil properties) and due to free field surface.

5 Conclusions

This paper provides the comparisons of soil response to seismic ground motion between widely used SHAKE and Plaxis software packages. It was found out that good agreement with spectrum from 1D analysis can be achieved when the model is width enough (over 280 meters in this particular case). It is important to notice that free field boundary conditions alone do not assure small enough influence on the results of ground response in case of seismic excitations. Moreover, the impact of Rayleigh damping on lower periods is shown, where hysteretic damping through HSS material model alone cannot match the 1D response from SHAKE with adopted modulus and damping reduction curves.

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