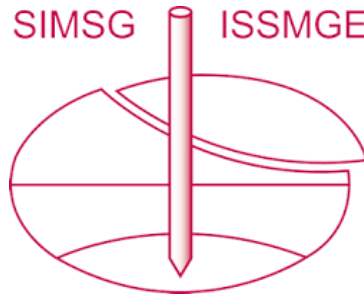


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Determination of characteristic values of geotechnical parameters

Monica LÖFMAN^{1*}

¹Aalto University School of Engineering, Department of Civil Engineering, Espoo, FINLAND

ABSTRACT

According to Eurocode, statistical methods may be used in the determination of characteristic values of geotechnical parameters. In this paper, different statistical methods represented by Eurocode and Finnish norm are described and compared using undrained shear strength data from Perniö test site. Both the current version of Eurocode and the suggested new Annex HS are considered. The effect of number of observations to the 'cautious mean' characteristic value of each method is studied via simulations. Both COV-unknown and COV-known -methods of Eurocode are considered. In addition, factors in Annex HS affecting the derived characteristic value are studied using sensitivity analysis. Based on the results, it is clear that when the number of observations is low, all these methods must be treated with cautiousness. The COV-known – method seems to be the most reliable one, assuming that appropriate 'known' COV is adopted.

Keywords: characteristic value, COV, coefficient of variation, Eurocode

1. INTRODUCTION

According to Eurocode, statistical methods may be used in determination of characteristic values of geotechnical parameters. Even so, in practical geotechnical design, deterministic approach is commonly used instead due to its simplicity and traditions in design (Lee et al. 1983, p. 58). However, deterministic approach leads to characteristic values that are highly subjective and thus uncertain as the selection of cautious estimate is based on engineering judgement (Phoon 2008, pp. 3-8). In statistical methods on the other

hand, the process is systematic and the uncertainty in the soil property, all the available prior information considered, is defined quantitatively.

In this paper, different statistical methods represented by Eurocode and Finnish RIL-guideline are described and compared based on undrained shear strength results from Perniö test site. Both the current version of Eurocode and the suggested new Annex HS are considered.

Any complementary information and *a priori* knowledge can be taken into account in the determination of characteristic values. In Eurocode, this can be done by assuming that the

* presenting author

coefficient of variation (COV) of the soil property is known (so called COV-known - method) (EN 1990 2005, Annex D). Even though not covered in Eurocode, another possibility would be to use Bayesian approach, which is a very powerful tool especially in reliability-based design (RBD).

In the analysis, the effect of number of observation to the 'cautious mean' characteristic value of each method is studied via simulations. Both COV-unknown and COV-known -methods of Eurocode are studied and compared. In addition, factors in Annex HS affecting the derived characteristic value are studied using sensitivity analysis.

2. DETERMINATION OF CHARACTERISTIC VALUE

2.1. Definition according to Eurocode

EN-1997-1 defines the characteristic value as being "selected as a cautious estimate of the value affecting the occurrence of the limit state". In the definition "selected" emphasizes the importance of engineering judgement, and "cautious estimate" means that some conservatism is required and finally the selected value must relate to the limit state (EN 1997-1 2004, Frank et al. 2004, pp. 24-28).

According to Frank et al. (2004), when selecting the characteristic value, two major aspects are (i) the amount of knowledge of the parameter values and the degree of confidence in the knowledge and (ii) the soil volume involved in the limit state considered and ability of the structure to transfer loads from weak to strong zones in the ground (Frank et al. 2004, pp. 24-28).

The amount and degree of confidence in the information depends on (i) the amount of information (local test results and other relevant information such as *a priori* knowledge and (ii) the scatter of the results, which is caused by the variability of soil (Frank et al. 2004, pp. 24-28).

The scatter in the test results is caused by both inherent variability of soil and measurement error. (Lee et al. 1983, pp. 57-58, Phoon & Kulhawy 1999). The uncertainty caused by inherent variability, measurement error or other factors can be quantified by using coefficient of variation or COV:

$$COV_x = \frac{\sigma_x}{\mu_x} \approx \frac{SD_x}{x_m} \quad (1)$$

where σ_x is the standard deviation of the random variable x ; μ_x is the expected value; SD_x is the standard variable of the sample and; x_m is the mean of the sample. High COV implies high uncertainty.

Inherent soil variability is represented in Figure 1. The actual value of the soil property $\xi(z)$ varies through depth, but one can determine the trend function $t(z)$. The fluctuating component $w(z)$ represents the inherent soil variability. This spatial variability is one of the main reasons for the need of conservatism in the definition of characteristic value. In the figure, correlation length θ is the distance within which points are significantly correlated (Fenton & Griffiths 2008, p. 103, Phoon & Kulhawy 1999).

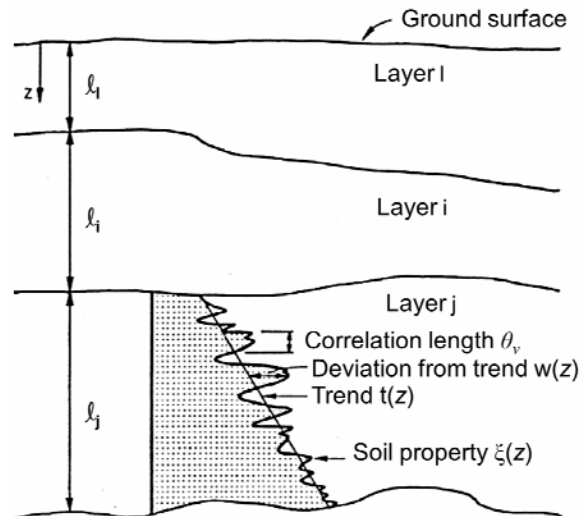


Figure 1. Inherent soil variability (Phoon & Kulhawy 1999).

2.2. Eurocode 7

The soil volume involved in the limit state affects the definition of characteristic value, which can be either (a) cautious mean, which is the estimated mean value corresponding to a 95 % confidence level or (b) local low value, which is the estimated 5 % fractile (EN 1997-1 2004, Frank et al. 2004, pp. 46-49).

Thus according to definition, there is a probability of 95 % that the (unknown) mean value governing the occurrence of a limit state in the ground is more favourable than the selected mean characteristic value. 5 % fractile on the other hand means that there will be only 5 % probability that somewhere in the layer considered there is an element of soil having property values lower than the characteristic value (Frank et al. 2004, pp. 46-49).

So when is it required to use a cautious mean (large soil volume) and when a local low value (small soil volume)? The difference between these two cases is related to correlation length θ : When θ is small compared with the dimensions of the soil volume, low and high local values compensate, and cautious mean is adopted. This phenomenon is often referred to as “averaging” of weak and strong soil layers. If, on the other hand, θ is larger compared with the dimensions of the soil volume involved, the local low value might affect the occurrence of the limit state instead. Thus 5 % fractile or a value somewhere between 5 % fractile and cautious mean must be used (Frank et al. 2004, pp. 46-49).

Suggested values for both horizontal and vertical correlation lengths can be found from the literature for different soil properties (Phoon & Kulhawy 1999). However, it is usually much more challenging to estimate the dimensions of the soil volume involved in limit state. Especially when it comes to the stability of an embankment, one rarely knows where the failure surface would occur. However, if brittle failure or strain softening behaviour can be expected, usage of the

5 % fractile is recommended (Frank et al. 2004, pp. 46-49).

In Eurocode, if statistical methods are used, characteristic value is defined as (EN 1990 2005, Annex D, Frank et al. 2004, p. 46):

$$X_k = x_m(1 - k_n \text{COV}_x) \quad (2)$$

where COV_x is either the coefficient of variation calculated for the sample or tabulated “known” COV for the whole population and; k_n is a statistical coefficient.

The value of statistical coefficient depends on the number n of test results (observations), on the ‘type’ of characteristic value (mean or fractile), the statistical level of confidence required, and *a priori* knowledge about the COV (known or unknown) (Frank et al. 2004, p. 29).

In the case of 95 % reliable mean value, k_n is defined as:

$$k_{n,\text{COVknown}} = 1.645 \sqrt{\frac{1}{n}} \quad (3)$$

$$k_{n,\text{COVunknown}} = t_{n-1}^{0.95} \sqrt{\frac{1}{n}} \quad (4)$$

where n is number of observations; $t_{n-1}^{0.95}$ is the value of the t factor of Student’s distribution (with degree of freedom being $n - 1$) corresponding to a probability of 95 %. If COV is known, normal distribution can be used and the corresponding 95 % value is 1.645.

However, this method is valid only for cases where the soil is relatively homogeneous and there is no significant trend in the soil property. In addition, normal distribution is assumed. For some geotechnical parameters, such as undrained shear strength, lognormal distribution is a better fit (Lacasse & Nadim 1996). Furthermore, lognormal distribution is non-negative whereas normal distribution can contain unrealistically low or even negative values. If lognormal distribution is used, before applying the formulae represented above,

the parameter value X must be transformed to its logarithm $Y = \ln X$ (Frank et al. 2004, p. 29). In Annex D of Eurocode, determination of X_k for log-normally distributed properties is provided in further detail (EN 1990 2005, Annex D). However, if $COV < 30 \%$, there is not a significant difference between normal and lognormal distributions.

2.3. Finnish RIL-guideline

As for statistical methods, Finnish RIL-guideline only covers the determination of cautious estimate, but it is based in Eurocode otherwise. In this method however, *a priori* knowledge cannot be taken into account as in Eurocode 7. In RIL 207-2009 the characteristic value X_k is suggested to be calculated as follows (RIL 207-2009 2009):

$$X_k = x_m - 1.645 \cdot \frac{SD_x}{\sqrt{n}} \quad (5)$$

where n is the amount of test results and others are as earlier defined.

In the equation, 1.645 is yet again the value of normal cumulative distribution function corresponding to a probability of 95 %.

In this method, it is assumed, that the calculated standard deviation SD represents the distribution of the whole population; if the standard deviation of the population is unknown, Student t -distributions should be used instead of normal distribution. If $n = \infty$, normal and Student's distribution actually yield the same value of 1.645. As a matter of fact, this method produces the same result as COV_{known} -method of Eurocode, if one uses the calculated sample COV instead of tabulated 'known' COV .

2.4. Suggested Annex HS

'TC250/SC7/EG11: Characterization' is a project, which aims to provide a user friendly and consistent guidance on determining characteristic values of ground parameters, using both the traditional approach and statistics. Proposed Annex HS represents a simplified method which is based on

statistics and which considers level of experience, the amount and quality of test results and the zone of influence. In this method, no statistical terms are introduced at all, which supposedly makes the method easier to use in practice. The method fulfills all relevant aspects of the requirements of Eurocode 7 and is also open to be adjusted (via factor a) according to different national experiences (EG11 2015).

In Annex HS, characteristic value is defined as (EG11 2015):

$$X_k \cong x_m - a \cdot (x_m - x_{extr}) \cdot \sqrt{\frac{1}{L_v}} \quad (6)$$

where x_m is the mean of the derived values based on field or lab tests and/or estimated mean value from comparable experience and/or estimated mean value from tabulated soil properties; x_{extr} is the extreme soil value recorded or estimated corresponding to an expected extreme (unfavourable) value for the hypothetical case of large number of tests; L_v represents the zone of ground governing the behaviour of a geotechnical structure at a limit state. As such, L_v is the vertical dimension of the zone of influence. Inside the square root 1 stands for a typical vertical correlation length of 1 m, and as such, the term is dimensionless (EG11 2015).

In the equation a is a factor to account for extent and quality of field and laboratory investigations or estimation method, type of tests for selecting derived values, sampling methods and level of experience. The suggested range of a is 0.5-1.0. Smallest value of 0.5 is proposed to be used in the case of several high quality test values and reliable, good local site information based on excellent comparable experiences. Value of $a = 0.75$ could be used for average quality. For example, in the calculation example of Annex HS, $a = 0.7$ for field vane is proposed. The most conservative $a = 1.0$ is suggested to be used when the derived values are estimated from general

experience or tabulated values (no local site investigation) (EG11 2015).

Suggested Annex HS states that derived values (x_m and x_{extr}) should be corrected from uncertainties of the testing methods as well from the transformation model used to arrive at derived values from raw data of lab or field tests. x_{extr} can also be estimated or confirmed by using tabulated values of COV (EG11 2015).

3. COMPARISON OF THE METHODS

3.1. The effect of the number of observations

In the first analysis, fall cone test results from 8 sampling points near each other were studied. The studied site is located in Perniö, Finland. In this site, full-scale embankment failure test was conducted in 2009 (Lehtonen et al. 2015). The data used in this study was provided and studied by Igor Mataic (Mataic 2016).

All the test results are represented in Figure 2. In the analysis, only the soft layer at depth of 2.5...5 m is studied. There is overall $n = 28$ test results. For this layer, 'cautious mean' characteristic value of undrained shear strength is determined using all the methods described above, and the results are compared and further analysed.

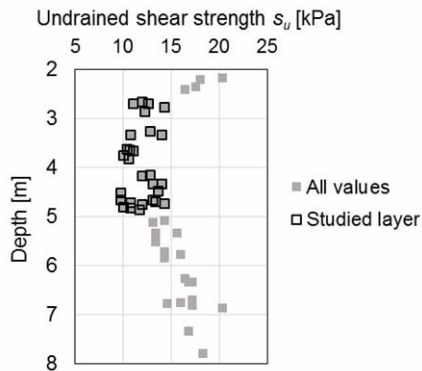


Figure 2. Fall cone test results at Perniö site.

First, the effect of number of observations n is studied. From the studied layer, samples consisting of $n = 3...27$ results are pulled from the data using random number generator in Excel. For each sample, characteristic value is determined using each method. In this

analysis, the following parameters are kept constant: (i) 'known' COV is 30 %, which is the suggested standard value for undrained shear strength (Müller 2013) (ii) $a = 0.75$. Since in this analysis the number of test results varies from $n = 3...28$, factor a for average test quality is the most suitable (iii) $L_v = 2.5$, which is the thickness of the layer. Thus it is assumed, that the whole layer affects the occurrence of the ultimate limit state. In addition, normal distribution is assumed.

The determined characteristic values X_k and the calculated mean values in each case of n is represented in Figure 3.

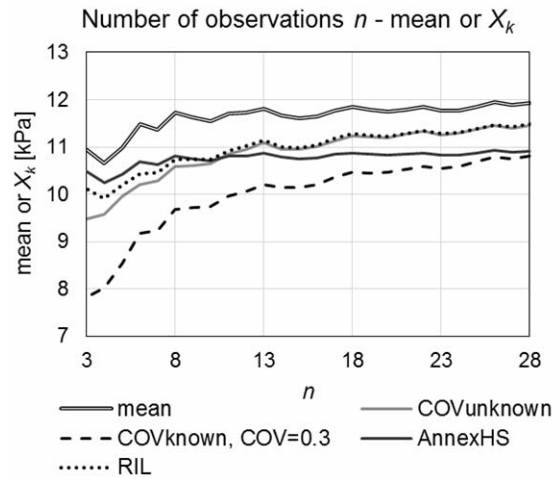


Figure 3. The effect of number of observations to the calculated mean and X_k .

When the number of observations is less than 8, there is great amount of scatter which is mostly caused by changes in the calculated mean. However, when it comes to $COV_{unknown}$ - and RIL -methods, the scatter is also partly caused by changes in the standard deviation SD and COV (see Figure 4).

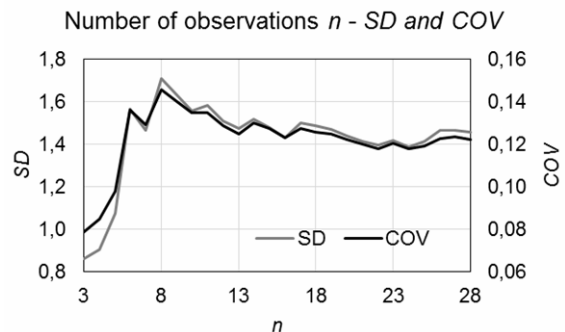


Figure 4. The effect of number of observations to the calculated SD and COV .

Most of the characteristic values and mean stabilize after $n > 13$, when SD and COV stabilize as well. Annex HS yields the highest value when $n < 10$, but as the number of observations increase, the value remains approximately constant and intersects with COV_{known} -value. All in all, Annex HS is the most stable because the results does not depend on the number of observations at all (factor a being constant).

After $n > 10$, RIL and $COV_{unknown}$ yield approximately the same values, because the value of factor t of Student's distribution gets closer to the value of 1.645 as the number of observations increases.

Overall, the COV_{known} -method yields the lowest value since the assumed COV (0.30) is much larger than the one calculated from the samples. Note that the calculated SD and COV are abnormally low when $n < 10$. Indeed, according to Schneider (1999), statistical methods can be applied successfully only if $n > 10$. Thus it is highly recommended to use COV_{known} -method if there is not enough data (Frank et al. 2004, pp. 46-47). In this analysis, $COV_{unknown}$ yielded higher values than COV_{known} -method, but this is only due to the uncertainty brought by small n values and because the selected 'known' COV was too conservative.

3.2. The uncertainty caused by small amount of observations

In the second analysis, the effect of small number of observation is studied. The parameters are the same as in previous analysis. In this analysis however, five random samples are collected from the data for cases $n = 3, 5, 7, 10$, and 20 . The characteristic values and means are calculated for each sample and compared. For reference, results for $n = 28$ are represented as well.

When the values of the calculated COV in each case of n are plotted, it can be observed that if $n < 8$, COV is highly uncertain (Figure 5).

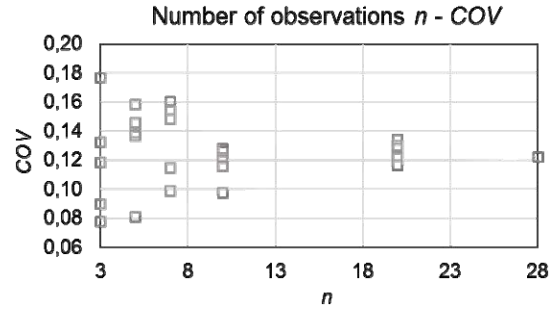


Figure 5. Calculated COV -values for each sample.

Thus at low values of n there is a considerable risk that $COV_{unknown}$ - and RIL-methods either under- or overestimate X_k .

When the determined highest and lowest characteristic values X_k and means are compared (max-min) within each case of n (Figure 6), it is clear, that $COV_{unknown}$ and RIL yield the most uncertain values at low values of n . COV_{known} , on the other hand, is the most stable in almost every case. Annex HS is relatively stable as well, since X_k is only affected by calculated mean and observed minimum value in this analysis.

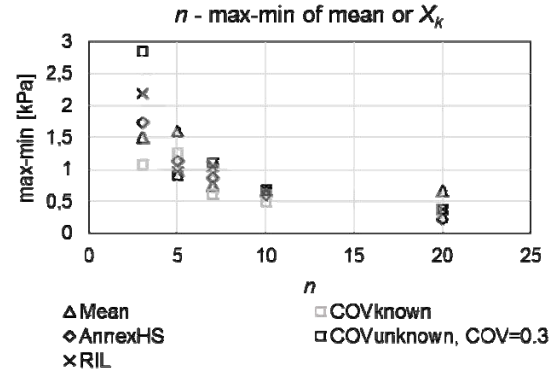


Figure 6. The difference between min and max values of mean s_u and X_k at different cases of n .

The calculated mean and $mean \pm SD$ values of the mean s_u and characteristic values X_k are represented in Figure 7 (above). The data contains results of five samples from cases $n = 5, 7, 10$ and 20 (overall 20 values of mean s_u and X_k). In Figure 7 (below) the same results are represented, the only difference being that in COV_{known} -method the 'known' COV is 15 %.

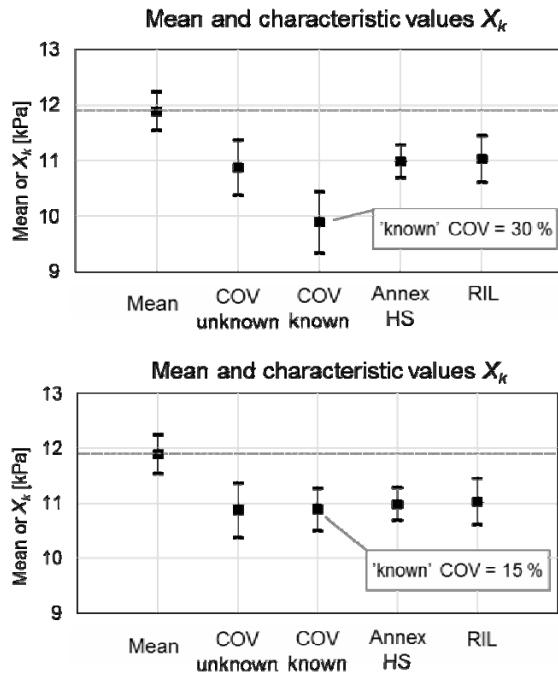


Figure 7. Mean and mean $\pm$ SD values of mean s_u and X_k 'known' COV being 30% or 15%.

The means of characteristic values are approximately the same in all methods expect COV_{known} when 'known' COV is 30 %, which is much larger than the COV calculated for the whole sample of 28 results (12%, Figure 4). If the 'known' COV is adjusted to 15 %, COV_{known} yields higher mean and smaller SD. Thus if a suitable 'known' COV is selected, usage of COV_{known} leads to more reliable X_k than $COV_{unknown}$.

The scatter in Annex HS is yet again small, since factor a and L_v where not varied. Next, the effect of varying 'known' COV and parameters of Annex HS are studied.

3.3. Sensitivity analysis – 'known' COV and parameters in Annex HS

In this analysis, the data from the first analysis is adopted again and the 'known' COV is varied based on reported values for undrained shear strength (Müller 2013). The calculated COV_{known} -based values of X_k are shown in Figure 8. For comparison, values calculated using $COV_{unknown}$ are represented as well, containing both the results from the first

analysis and the minimum values from 5 samples in studied cases of n .

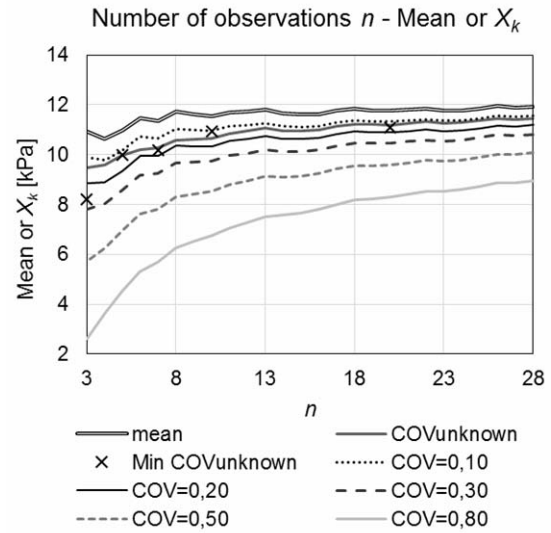


Figure 8. The effect of n to the calculated mean and X_k with varying 'known' COV.

If COV-values greater than 0.30 are used, COV_{known} yields unrealistically low values of X_k at low values of n . In addition, 'known' COV must be over 0.20 in order to reach the same values as in $COV_{unknown}$ – method at higher values of n since the calculated COV for the data is 0.12 (12%) as discussed in the previous section.

Next, the effect of the parameters in Annex HS are studied via simulations and sensitivity analyses. For each uncertain parameter, a probability density function (PDF) is defined based on both results from the simulations and assumptions.

PDF for mean undrained shear strength is determined by fitting a normal distribution to the results of the second analysis (overall 20 results). The parameters of the PDF are expected value $\mu = 11.8929$ and standard deviation $\sigma = 0.3506$. The fitted normal distribution is represented in Figure 9. In Simulations 1 and 2, this PDF for mean is used, but in Simulation 3 the input mean s_u is kept constant at its expected value.

In the distribution figures, there are two bars with percentiles and two values. The gray bar states that based on the input, the studied variable is between these two values at a probability of 90 %. Respectively, these two values taken, the

corresponding probability based on the fitted distribution is typed in the middle of the black bar.

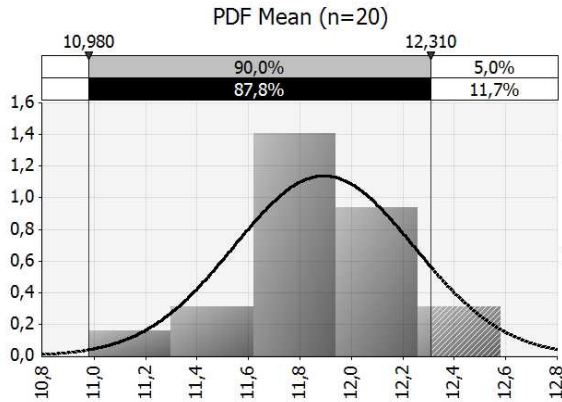


Figure 9. Histogram and fitted normal distribution of the mean s_u .

In all simulations, for factor a , a triangular distribution with minimum of 0.5, expected value of 0.75 and maximum of 1.0 is assumed. In Simulation 1, for L_v , a uniform distribution with a minimum of 1.0 and a maximum of 2.5 (the thickness of the studied layer) is assumed. According to Annex HS, $L_v = 1.0$ is suggested for shallow foundations (EG11 2015). Depending on the problem, L_v can be as large as 10 m (for example for pile foundations), and this case is studied in Simulations 2 and 3. Minimum value x_{extr} is kept constant at the observed minimum 9.8 kPa.

The simulations consists of 1000 iterations, and the resulted histogram and fitted normal distribution from Simulations 1 and 2 of X_k are represented in Figures 10 and 11.

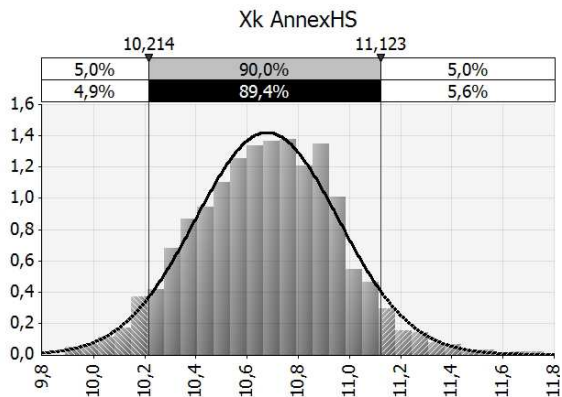


Figure 10. Simulation 1: Annex HS X_k ($L_v = 1.0 \dots 2.5$ m).

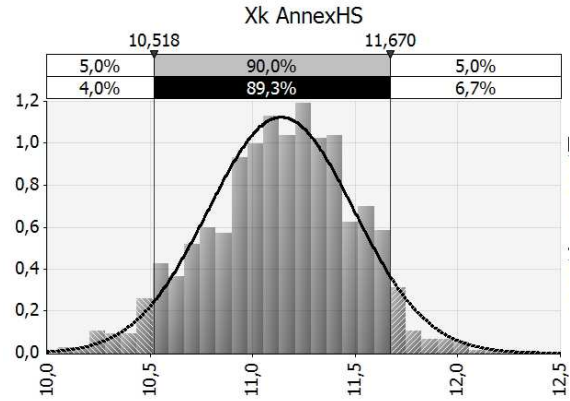


Figure 11. Simulation 2: Annex HS X_k ($L_v = 1.0 \dots 10$ m).

As can be seen from the histograms, the highest values of X_k (Figures 10 and 11) are even higher than the smallest values of the mean (Figure 9).

When the effect of each parameter on the X_k is studied via sensitivity analysis, the tornado graphs based on Simulations 1 and 2 are acquired (Figures 12 and 13).

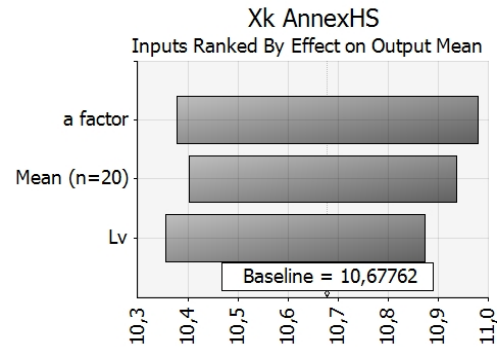


Figure 12. Simulation 1: Tornado graph based on the sensitivity analysis of Annex HS X_k ($L_v = 1.0 \dots 2.5$ m).

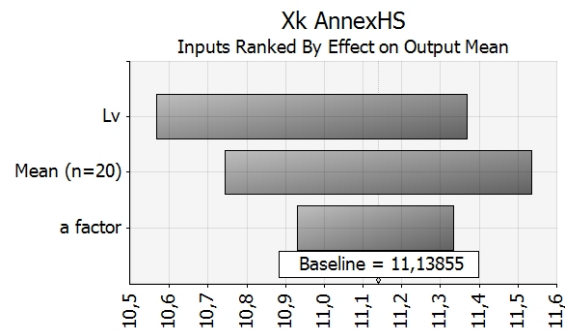


Figure 13. Simulation 2: Tornado graph based on the sensitivity analysis of Annex HS X_k ($L_v = 1.0 \dots 10$ m).

As can be seen from the tornado diagram of Simulation 1 (Figure 12), factor a affects the derived X_k the most. Interestingly, the selection of L_v affects the results almost as much as factor a and mean. In Simulation 2 L_v has the biggest influence whereas factor a affects the derived X_k the least (Figure 13).

In Simulation 3, mean is kept constant at the expected value of 11.89 kPa. Other parameters are the same as in Simulation 2. This approach enables more accurate comparison between the effects of factor a and L_v .

The histogram and fitted triangular distribution of X_k is shown in Figure 14 and the corresponding tornado graph in Figure 15. The distribution is heavily skewed to the left, implying that the highest values of X_k are more probable (Figure 14).

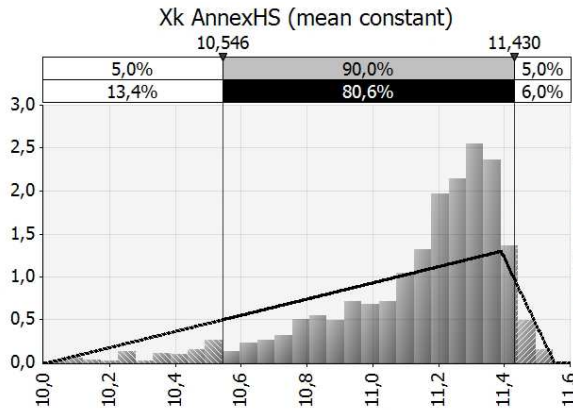


Figure 14. Simulation 3: X_k Annex HS (input mean constant, $L_v = 1.0 \dots 10$ m).

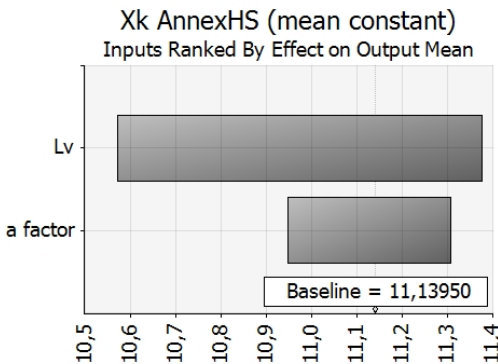


Figure 15. Simulation 3: Tornado graph based on the sensitivity analysis of Annex HS X_k (input mean constant, $L_v = 1.0 \dots 10$ m).

Yet again L_v has greater influence than factor a . In addition, the highest values of X_k are the result of high L_v (Figure 15).

The statistics of all the simulations are listed in Table 1.

Table 1. Statistics of the simulations

Simulation	Mean	SD	min	max
(Mean s_u)	11.89	0.351	10.98	12.58
Sim. 1: X_k ($L_v = 1.0-2.5$)	10.68	0.281	9.890	11.76
Sim. 2: X_k ($L_v = 1.0-10$)	11.14	0.355	10.05	12.29
Sim. 3: X_k (mean constant, $L_v = 1.0-10$)	11.14	0.270	10.02	11.55

According to the statistics, increase in L_v will result in higher mean values of X_k . Note that in the Simulation 3 in which the mean is kept constant the mean and maximum values of X_k are extremely close to the observed mean.

4. CONCLUSIONS

When the number of observations n is less than 8, all the described methods should be treated with cautiousness. Because calculated standard deviation and COV are extremely uncertain at $n < 8$, COV_{unknown}- and RIL -method should not be used at all in these conditions. Not only is there a risk of underestimating the characteristic value X_k , but also a risk of overestimation.

Frank et al. (2004, pp. 46-47) recommend the usage of COV_{known} - method if *a priori* information is available due to the fact that n is usually low in geotechnical problems. However, the results indicate that the selected 'known' COV must be suitable for the studied soil property and regional characteristics. As such, the authors recommend further investing in the research on COV in order to provide reliable *a priori* information for designers. Furthermore, research on soil variability provides tools for RBD as well. In RBD, distributions of the soil properties

are used instead of fixed value of X_k , which enables qualitative estimation of the reliability of the design (Phoon 2008, pp. 7-8).

Unlike other discussed methods, Annex HS -method does not directly depend on n . Results show that high values of L_v lead to excessively high values of X_k which hardly are 'cautious means' anymore. Furthermore, the effect of L_v to the X_k is the same or even greater than of factor a , which the authors consider problematic. Why would a larger zone of influence increase the reliability more than the number of results and the quality of the testing (factor a)?

The method of Annex HS should be modified so that L_v has less influence on the derived value of X_k . Alternatively, Annex HS could provide a maximum value of L_v in order to ensure that factor a has greater influence in all conditions.

To conclude, if n is low, the authors recommend using COV_{known} -method. Since COV_{known} -method yields extremely conservative values if $n < 8$, a feasible option would be to promptly assume $n > 8$ while using an appropriate COV . Nonetheless, if n is low, the best option would be to apply *a priori* knowledge on a distribution based on typical values and use Bayesian updating in order to acquire the most reliable distribution for the soil property for RBD (Phoon & Kulhawy 1999).

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