The paper was published in the proceedings of the 25th European Young Geotechnical Engineers Conference and was edited by Ernest Olinic and Sanda Manea. The conference was held in Sibiu, Romania 21-24 June 2016.

Finite Deformation Modelling of Cyclic Sand Behaviour using Bounding Surface Plasticity

Christian Carow [1*]

[1] *Technische Universitaet Berlin,*
Chair of Soil Mechanics and Geotechnical Engineering, Berlin, GERMANY

ABSTRACT

Precise and efficient numerical simulation of contemporary geotechnical problems requires elaborate constitutive models for the stress-strain behaviour of sands. The models are supposed to incorporate the finite deformation theory as well as sophisticated mechanisms for cyclic loading. With regard to that, a short review of existing constitutive models is given. The bounding surface model of Li can be considered one of the most comprehensive approaches and further development of it seems appropriate. The model is presented thoroughly and two leverage points for improvements are identified. First, the model is reformulated in a manner consistent with finite deformation theory. The reformulation is outlined and the consequences for the practical application of the model. Second, a mechanism for cyclic compression is added to the model in order to improve simulation results. The general methodology of the additional mechanism is examined here. Further experimental investigation is required as a basis for the desired extensions of the model.

Keywords: constitutive modelling, sand, bounding surface plasticity, cyclic loading

1. INTRODUCTION

Analytical and numerical models of complex geotechnical processes have been developed at the soil mechanics division of Technische Universitaet Berlin for several decades. Recent work aims to design advanced instruments for numerical modelling. They are meant to predict displacement and maximum bearing capacity in situations where subsoil experiences cyclic loading as well as large inelastic deformations. This happens in offshore-areas and earthquake danger zones as well as during pile installation

In a first step the Arbitrary Lagrangian Euler Method (ALE-method) was adapted to meet geotechnical requirements by Aubram (2015). ALE allows to perform finite element calculations in cases where conventional Lagrangian FEM ceases to function due to severe element distortion.

On this basis, the Multi Material Arbitrary Lagrangian Euler Method (MMALE) method was developed by Aubram et al. (2015) and Savidis et al. (2015). In MMALE the material is able to move freely through the finite element

* presenting author

mesh. As a result, there are no limitations with respect to the magnitude of deformation that can be taken into account.

There are of course several other methods that could be used in geotechnics if large deformations are expected to occur, as shown in (Wang et al., 2015). Nevertheless, in all those methods the quality of the results that are obtained relies heavily on the constitutive model that is used for the soil skeleton.

The model to be employed must be consistent with the finite deformation theory. Furthermore, it should be able to capture most characteristics of the soil's stress-strain behaviour in order to account for the complex loading paths which are encountered in the aforementioned problems.

This paper makes an attempt to explain how to find such a model for sandy soils.

2. CONSTITUTIVE MODELS FOR SANDY SOILS

Constitutive models for cyclically loaded non-cohesive soils have been subject of myriads of scientific treatises. These days it seems that the most promising models either are of the bounding surface plasticity type or descend from the Karlsruhe school of hypoplasticity.

Hypoplastic models, as was first outlined in (Kolymbas, 1977), consist of a single tensorial equation. Its shape has been derived deductively on the basis of general axioms of material theory. Gathering from (Nübel & Cudmani, 2000; Tsegaye et al., 2000; von Wolffersdorff & Schwab, 2009), the hypoplastic model of (von Wolffersdorff, 1996) in conjunction with the intergranular strain extension of (Niemunis & Herle, 1997) can be used successfully for problems with cyclic loading. However, there is room for further improvements. Currently, volumetric strains and excess pore pressure associated with shearing through dilatancy are not predicted satisfactorily. This is an important issue since dilatancy is the cause of accumulation of settlements and liquefaction under cyclic loading.

Li & Dafalias (2000) revealed a new state-dependent dilatancy model in the context of elastoplasticity. It predicts shear-induced volumetric strains based on the state-parameter ψ. The scalar number ψ, which is attributed to (Been & Jefferies, 1985), quantifies the distance of the current material state from the critical state line (CSL). Since the CSL is a curve in pressure/void ratio space, the stress- as well as the density-dependency of the behaviour of sands are inherently captured through the concept of (Li & Dafalias, 2000). As a consequence, a single set of material constants can be used to achieve good simulation results over a wide range of stress-levels and densities.

The ideas of (Li & Dafalias, 2000) gained much positive resonance. Consequently they have been incorporated into the popular SANISAND-Model by Dafalias & Manzari (2004). SANISAND uses the concepts of bounding surface plasticity in a singular and very efficient manner to model hardening and softening of non-cohesive soils under complex loading/unloading-paths. Thereby, SANISAND is well-suited to be used in earthquake-simulations, as has been proven in (Taiebat et al., 2010).

Another popular model which employs the approach of (Li & Dafalias, 2000) is the one developed by Li (2002) on the base frame of (Wang et al., 1990). It will be called Critical State Sand (CSSA-) model throughout this paper. It has been validated successfully for the use in cases were cyclic loading is present by Ming & Li (2003) and matches SANISAND in various respects. One thing that makes the CSSA-Model unique is its ability to compute plastic strains caused by changes in average effective stress p'. This is accomplished by using an additional cap-shaped bounding surface besides the usual cone-surface. A similar mechanism has been incorporated into SANISAND by Taiebat & Dafalias (2008). Nevertheless, according to (Prada Sarmiento, 2009) it causes numerical instabilities there.

This (very) short review of the state-of-the-art in modelling the cyclic behaviour of sands may be summarized as follows. There are several highly developed models. All have certain advantages and drawbacks. The CSSA-Model is equipped with a reliable and powerful dilatancy model, which is of major importance when cyclic loading is being considered. Due to its cap-mechanism, the CSSA-Model seems to provide the most comprehensive formulation in the elastoplastic realm. Therefore, the model may be used in conjunction with the aforementioned high-class numerical methods to solve contemporary boundary value problems involving cyclic loading and large soil deformation. Hence, it seems expedient to take a closer look at the CSSA-model and outline strategies for removing it's still remaining deficiencies.

3. CRITICAL STATE MODEL FOR SANDS

The CSSA-Model designed by Li (2002) is of the elastoplastic rate-type and has been formulated within the geometrically linearized small-strain theory.

As a member of the bounding surface family set up by Dafalias (1986), the CSSA-Model is endowed with two bounding surfaces in principal stress space and corresponding loading surfaces. They are shown here in Figure 1. A cone-shaped bounding surface

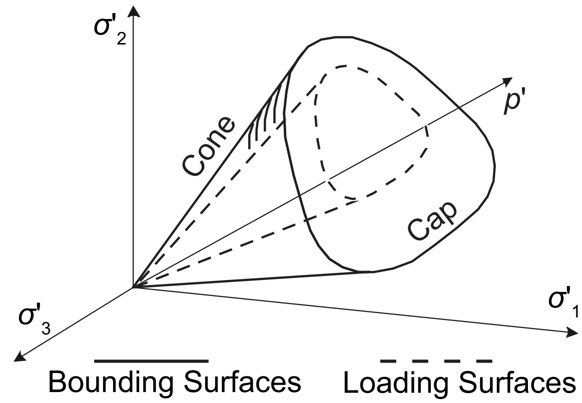

Figure 1: Bounding and loading surfaces of the CSSA-Model in principal stress space

processes changes of the so called stress ratio. In general, the stress ratio is represented by the second order tensor $\boldsymbol{r} = \boldsymbol{s} / p'$, with $\boldsymbol{s}$ being the deviatoric part of the effective stress $\boldsymbol{\sigma}'$. Under axisymmetric conditions, the stress ratio is represented by the scalar $\eta = q / p'$, in which q is the equivalent deviatoric stress.

The cone is being closed off by a flat cap. As has been indicated in the previous section, the cap is meant to model plastic strains caused by pure compression of the soil along a stress path with constant stress ratio.

The major benefit of the bounding surface concept appears when repeated loading and unloading is to be rendered. Then, the distance between the current stress point and its image on the bounding surfaces serves as a measure for the influence of the loading history on the

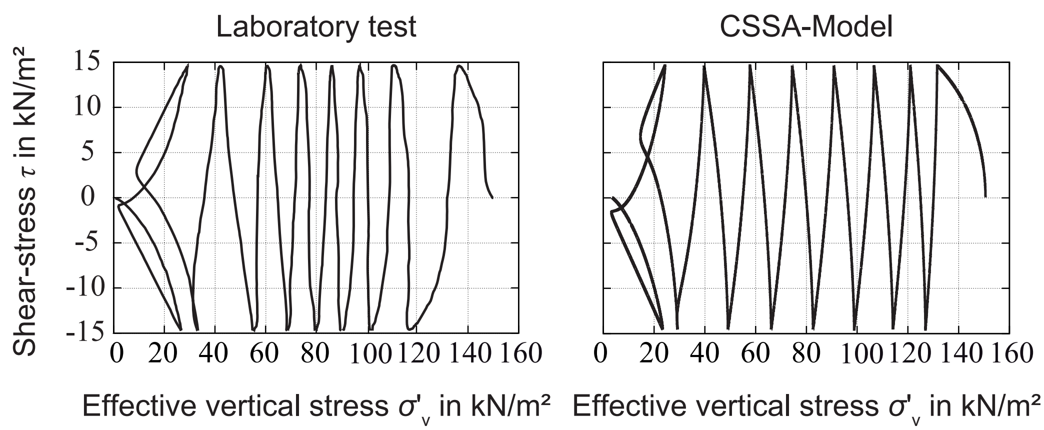

Figure 2 Stress path of cyclic simple shear test on Berlin Sand ($I_{D0} = 65\,\%$)

current stiffness. That has been proven to be quite efficient if cyclic loading is being considered.

Both bounding surfaces are equipped with dilatancy functions which originate in the work of Li & Dafalias (2000). As mentioned before, their most important input is the state parameter ψ. It provides information about the amount of dilatancy to be computed with regard to the stress level as well as to the soil's density.

In cases where cyclic shearing is predominant, the functional interaction of the dilatancy model and the bounding surface concept works out rather well. This is being illustrated by back-calculating a cyclic simple shear test, the result of which is presented in Figure 2.

In the following, the application of the CSSA-Model to the solution of a boundary value problem will be demonstrated. The problem to be investigated is a reservoir dam which encounters seismic loading by the 1971 San Fernando Earthquake. The finite element model, shown in Figure 3, utilizes the fully coupled two-phase-formulation of Zienkiewicz & Shiomi (1984). It captures the influence of the soil skeleton's deformation on the pore pressure, which is of major importance here because the sandy soil that forms the dam's body is expected to compact due to the shear loading of the earthquake. This may, under certain circumstances, result in considerable pore-pressure-buildup and in turn reduce effective stress and shear strength considerably.

Selected results of the simulations are shown in Figure 4 and Figure 5. The sand of the fill material that forms the main dam body was initially assumed to be of medium relative density. This leads to a moderate buildup of excess pore pressure. The crest settles only slightly.

Compared to that are results for loosely deposited fill material. Herein, significant horizontal displacements occur in the upstream slope. This is supposed to be an indicator for approaching slope-failure.

The observed behaviour in the second case is naturally attributed to the fact that stiffness and shear strength of loose sands are lower than those of medium dense sands. In addition to that, the lower density induces a stronger tendency of the soil to compact when being sheared. This results in excess pore-pressure if cycling loading is applied rapidly, as during earthquakes. As a consequence, the effective stress level decreases and the shear strength is reduced rapidly, as can be observed in Figure 4. This explains the comparatively large displacements in second case.

Excess pore pressure in loose, non-cohesive dam fill material may even reduce the effective stress to zero in a state of liquefaction. This happened at the Lower San Fernando Dam during the 1971 earthquake, as has been documented in (Seed et al., 1975).

Bottom line, the simulation results show qualitatively that the model of Li (2002) is able to describe the influence of soil density on the cyclic behaviour of sands quite well. Still, the model has two weak spots that prevent it from being used in large deformation cyclic analyses out of stock. First, the model is not compatible with finite deformation theory. Second,

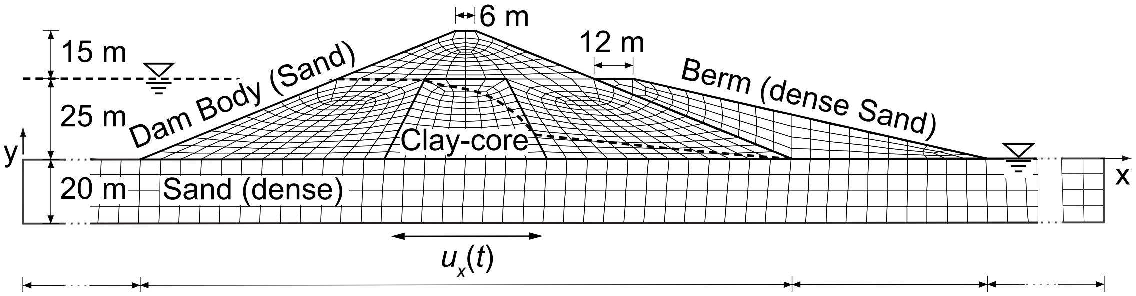

Figure 3: Finite Element Model of the reservoir dam

there is one aspect of cyclic sand behaviour which is not captured well. These issues are treated in the following two sections.

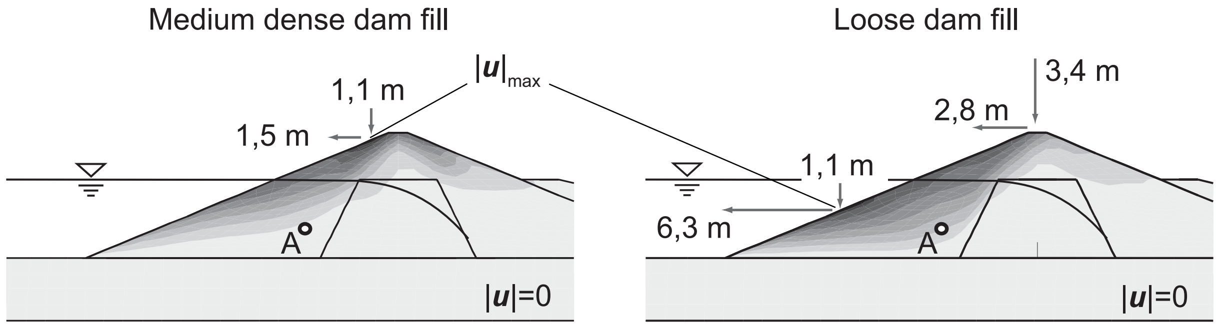

Figure 5: Contour plot of displacement caused by the applied earthquake

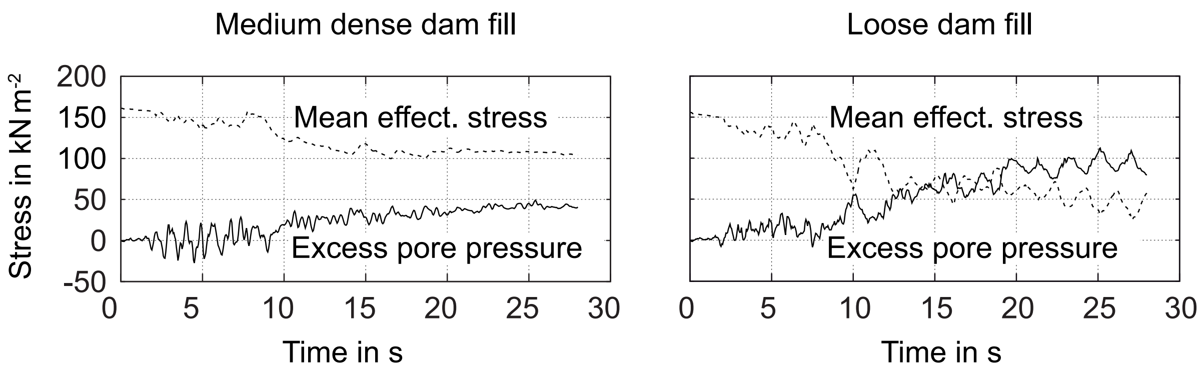

Figure 4: Evolution of mean effective stress and excess pore pressure during the earthquake

4. INCORPORATION OF FINITE DEFORMATION THEORY

As explained in the first section, an elaborate constitutive model is needed which is compatible with the finite deformation theory. The CSSA-Model however, in accordance with the general practice in traditional soil mechanics, has been formulated within the geometrically linearized infinitesimal theory. Therefore, the model equations have to be reformulated in order to be theoretically compatible with MMALE & Co.

There is another important reason to do so. During calibration of the CSSA-model, triaxial compression tests featuring axial strains of 20 % and more have to be back-analyzed in order to determine critical state parameters. The assumption of infinitesimal deformation induces a significant error if such magnitudes of strain are encountered.

To extend a conventional plasticity model into the finite deformation range, the following issues have to be considered:

- The finite deformation theory distinguishes different placements of the material body in space-time. Which one should the model refer to?
- How to treat superposed rigid body motions properly?
- How can the deformation be split into elastic and inelastic parts?
- How can the elastic stress-strain relationship be put up?

Those issues were solved in plasticity theory long ago, but the solutions are not common in soil mechanics. The general concepts of finite deformation plasticity are for example described in (Belytschko et al., 2000). By making use of them, the reformulation of the CSSA-Model can be outlined as follows.

The reformulated CSSA-Model refers to the material placement at current time t. That is called a spatial or Eularian formulation. Superposed rigid body motions are accounted for by employing objective tensors as stress and strain measures only. The usual Cauchy-Stress is an example of an objective tensor. Objectivity means here that rigid body motion may alter the components of the stress tensor with respect to a fixed coordinate system, but does not induce additional stress.

The symmetric part $\boldsymbol{d}$ of the velocity gradient serves as a means to measure deformation in the reformulated CSSA-Modell. It is objective, too and called rate-of-deformation. Its additive decomposition

$$\boldsymbol{d} = \boldsymbol{d}^{\mathrm{e}} + \boldsymbol{d}^{\mathrm{p}}$$

into an elastic part $\boldsymbol{d}^{\mathrm{e}}$ and a plastic part $\boldsymbol{d}^{\mathrm{p}}$ is the starting point for the reformulation. With respect to $\boldsymbol{d}^{\mathrm{e}}$, the elastic stress-strain-relationship is postulated. It has the shape of the hypoelastic rate-model

$$\overset{\mathrm{O}}{\boldsymbol{\sigma}}{}' = \boldsymbol{c}^{\mathrm{e}}(\boldsymbol{\sigma}, e) : \boldsymbol{d}^{\mathrm{e}}, \quad (\overset{\mathrm{O}}{\sigma}{}')^{ij} = (c^{\mathrm{e}})^{ijkl} d^{\mathrm{e}}_{kl}.$$

The elastic moduli $\boldsymbol{c}^{\mathrm{e}}$ may use the same dependency on stress and void ratio e as in the infinitesimal model. However, since the material time derivative of the Cauchy-stress $\dot{\boldsymbol{\sigma}}$ is not an objective tensor, an appropriate objective stress rate $\overset{\mathrm{O}}{\boldsymbol{\sigma}}$ has to be chosen. Here, we pick the corotating rate $\overset{\mathrm{J}}{\boldsymbol{\sigma}} = \dot{\boldsymbol{\sigma}} + \boldsymbol{\sigma} \cdot \boldsymbol{\omega} - \boldsymbol{\omega} \cdot \boldsymbol{\sigma}$ proposed by Jaumann (1911). According to (Guo, 1963), it has some important theoretical advantages over other rates. It is based on the skew-symmetric part $\boldsymbol{\omega}$ of the rate-of-deformation.

The rate-of-deformation $\boldsymbol{d}$ resembles the infinitesimal strain-rate $\dot{\boldsymbol{\varepsilon}}$ in many ways. Therefore, the plastic mechanisms of the CSSA-model may be transferred into the reformulated model to compute $\boldsymbol{d}^{\mathrm{p}}$, which results in

$$\boldsymbol{d}^{\mathrm{p}} = \frac{p' \bar{\boldsymbol{n}} : \overset{\mathrm{J}}{\boldsymbol{r}}}{K_{\mathrm{p1}}} \left(\bar{\boldsymbol{n}} + \sqrt{\frac{2}{27}} D_1 \boldsymbol{I} \right) + \frac{\dot{p}'}{K_{\mathrm{p2}}} \left(\bar{\boldsymbol{m}} + \sqrt{\frac{2}{27}} D_2 \boldsymbol{I} \right)$$

In this equation, the hardening modulus K_{p1} controls the evolution of the cone-shaped bounding surface, the modulus K_{p2} that of the cap. The flow direction $\bar{\boldsymbol{n}}$ is normal to the cone. The flow direction $\bar{\boldsymbol{m}}$ of the cap is parallel to the current stress ratio. D_1 and D_2 are the dilatancy functions of cone and cap, respectively.

As a result of the foregoing, the elastoplastic stress-strain relationship of the reformulated CSSA-Model can be stated as

$$\overset{\mathrm{J}}{\boldsymbol{\sigma}}{}' = \boldsymbol{c}^{\mathrm{ep}}(\boldsymbol{\sigma}, e, \lambda_1) : \boldsymbol{d}.$$

The fourth-order elastoplastic stiffness tensor $\boldsymbol{c}^{\mathrm{ep}}$ depends on stress, void ratio e and the accumulated plastic strain λ_1 associated with the cone. In the end, $\boldsymbol{c}^{\mathrm{ep}}$ has the same shape as the stiffness tensor developed by Li (2002), but it is formed by finite deformation quantities.

The reformulation leads to a series of consequences regarding the practical application of the CSSA-Modell.

- The corotating stress-rate has to be integrated over the computational time increment by a so called incrementally objective algorithm, like for example the one developed by Hughes (1984).
- In finite-deformation theory, the relationship between displacement and strain induces an additional nonlinearity. This is supposed to influence convergency during iterative solution of boundary value problems.
- The results of laboratory tests that are used to calibrate the model have to be analyzed utilizing consistent finite deformation stress- and strain measures.

5. MODELLING OF CYCLIC COMPRESSION

This section concerns an open question regarding the modelling of cyclic stress-paths with the CSSA-Modell. As has been demonstrated in section 3, the model leads to excellent simulation results so long as cyclic shearing is concerned only. However, there is room for improvement with respect to the computation of plastic strains induced by cyclic changes of mean effective stress at constant stress ratio, e.g. cyclic compression.

The present situation is depicted in Figure 6. It shows the stress - void ratio path of a cyclic oedometer test and its back-calculation with the CSSA-Model. In the laboratory test, repeated un- and reloading induces only little compaction compared to the state after first loading. Opposed to this, the model predicts undiminished densification even after 20 cycles.

The reason for this obviously unrealistic behaviour of the model can be found in the cap-mechanism. It does not account for the influence of cyclic loading history and current material state on the computed stiffness properly.

To iron out this issue, the hardening modulus K_{p2} of the cap (which was introduced by the equation for $\boldsymbol{d}^{p}$) needs to be augmented by a cyclic mechanism h_c. Thereby, the hardening modulus takes on the following form:

$$K_{p2} = h_c G(e, p') h_4 \left(\frac{M_c g(\theta)}{R} \right) \left(\frac{\rho_2}{\bar{\rho}_2} \right) \frac{\dot{p}}{|\dot{p}|}$$

Here, h_4 and M_c are material constants. R and θ are invariants of the stress ratio tensor $\boldsymbol{r}$. ρ_2 and $\bar{\rho}_2$ quantify the stress history in pursuance of the bounding surface concept. The function $g(\theta)$ defines the width of the cone-shaped bounding surface. The new constitutive function h_c is scales K_{p2} to increase the stiffness during cyclic loading realistically.

Elgamal et al. (2003) observed that few experimental investigations have been conducted with regard to the development of a cyclic compression cap for plasticity. At least some qualitative conclusions can be drawn from the available experimental data of (Bauer, 1992; Ko & Scott, 1967; Mallwitz & Holzlöhner, 1996; Sawicki & Swidzinski, 1995; Wichtmann et al., 2004). Accordingly, a sand sample in oedometric or constant-stress-ratio conditions reacts as follows to cyclic compression with amplitude $\Delta p'$ and maximum pressure p_0.

- Compared to the p_0-state after first loading, every subsequent cycle induces further compaction.
- The larger the amplitude, the stress ratio and the void ratio, the more compaction is induced per cycle.
- The compaction per cycle decreases with increasing cycle count.
- The rate of compaction decreases with increasing volumetric plastic strain.

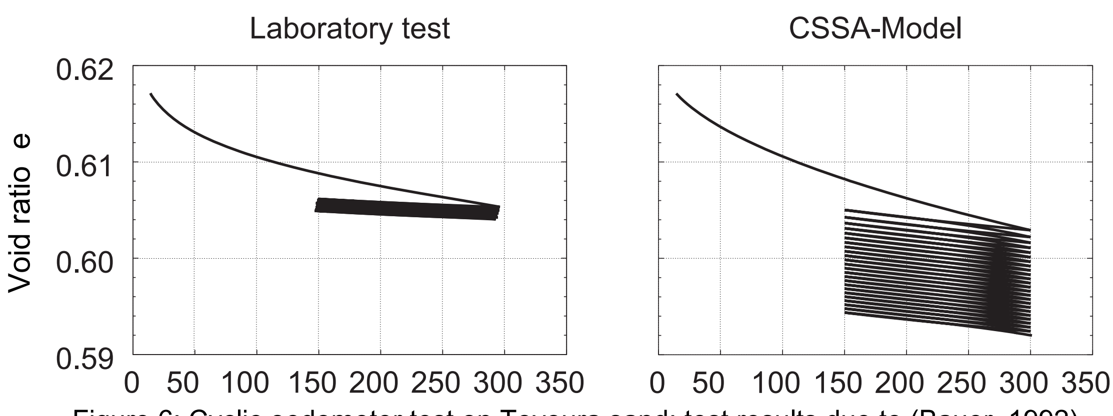

Figure 6: Cyclic oedometer test on Toyoura sand; test results due to (Bauer, 1992)

- Starting from an isotropic initial state, isotropic stress cycles induce volumetric strains only.
- Starting from an anisotropic initial state, isotropic stress cycles may induce deviatoric strains, too.
- If a sample is loaded beyond p_0 after the cyclic loading stage, the resulting stress-strain curve extends the virgin loading curve more or less seamlessly.

The foregoing list indicates which material state quantities probably have to be accounted for by the cyclic mechanism function h_c. These are the stress ratio, the mean effective stress, the void ratio and the accumulated plastic volume-strain.

The available qualitative experimental findings, on the other hand, are not sufficient to derive a specific form for h_c. To this end, a comprehensive series of cyclic triaxial tests with constant stress ratio is currently being conducted at the soil mechanics laboratory of TU Berlin. The results and their evaluation with regard to the constitutive functions will be published in following papers.

6. CONCLUSIONS

The paper highlighted that elaborate constitutive models for sand are needed to solve contemporary boundary value problems which feature cyclic loading and large soil deformations.

After selecting the bounding surface model of Li (2002) as one of the most comprehensive approaches and outlining strategies for its further development, the following conclusions can be drawn.

Complex elastoplastic models like the CSSA-Model can be extended into the finite deformation theory without having to re-compile the underlying model mechanisms. Nevertheless, some consequences regarding the practical application of the extended model have to be considered.

In elastoplasticity, material behavior in compression is accounted for by cap-like surfaces in stress space. They work quite well for monotonic loading, but fail to capture cyclic material behavior properly. To resolve this, additional constitutive functions should be developed in accordance with findings from experimental investigations. A short review of the state of the art showed that existing experimental data is sparse and additional tests have to be performed.

Such tests are being conducted currently. On the basis of their results, the new cyclic function h_c for the CSSA-Model's cap will be designed. Subsequently the reformulated and extended CSSA-Model will be validated by applying it to benchmark boundary value problems.

REFERENCES

Aubram, D.: Development and experimental validation of an arbitrary Lagrangian-Eulerian (ALE) method for soil mechanics. In: geotechnik Vol. 38 (2015), No. 3, p. 193–204

Aubram, D., Rackwitz, F., Savidis, p. A.: Vibro-Injection Pile Installation in Sand: Part I - Interpretation as Multi-material Flow. In: Triantafyllidis, T. (Hrsg.): Holistic Simulation of Geotechnical Installation Processes, Lecture Notes in Applied and Computational Mechanics. Vol. 77 : Springer, 2015 — ISBN 978-3-319-18169-1, p. 73–102

Bauer, E.: Zum mechanischen Verhalten granularer Stoffe unter vorwiegend ödometrischer Beanspruchung, Universität Karlsruhe, Dissertation, 1992. — Veröffentlicht in Institutsreihe

Been, K., Jefferies, M. G.: A state parameter for sands. In: Géotechnique Vol. 35 (1985), No. 2, p. 99–112

Belytschko, T., Liu, W. K., Moran, B.: Nonlinear Finite Elements for Continua and Structures : John Wiley & Sons Ltd, 2000

Dafalias, Y. F.: Bounding Surface Plasticity. I: Mathematical Foundation and Hypoplasticity. In: Journal of Engineering Mechanics Vol. 112 (1986), No. 9, p. 966–987

Dafalias, Y. F., Manzari, M. T.: Simple Plasticity Sand Model Accounting for Fabric Change Effects. In: Journal of Engineering Mechanics Vol. 130 (2004), No. 6, p. 622–634

Elgamal, A., Yang, Z., Parra, E., Ragheb, A.: Modeling of cyclic mobility in saturated cohesionless soils. In: International Journal of Plasticity Vol. 19 (2003), No. 6, p. 883–905

Guo, Z.: Time Derivatives of Tensor Fields in Non-Linear Continuum Mechanics. In: Archivum Mechaniki Stosowanej Vol. 15 (1963), No. 1, p. 131–161

Hughes, T. J. R.: Numerical Implementation of Constitutive Models: Rate-Independent Deviatoric Plasticity. In: Nemat-Nasser, S., Asaro, R. J., Hegemier, G. A. (Hrsg.): Theoretical foundation for large-scale computations for nonlinear material behavior, Mechanics of elastic and inelastic solids 6 : Martinus Nijhoff Publishers, 1984 — ISBN 978-94-009-6215-6, p. 29–63

Jaumann, G.: Geschlossenes System physikalischer und chemischer Differentialgesetze (I. Mitteilung). In: Sitzungsberichte / Akademie der Wissenschaften in Wien, Mathematisch-Naturwissenschaftliche Klasse, Abteilung 2a Vol. 120 (1911), p. 385–530

Ko, H.-Y., Scott, F.: Deformation of sand in hydrostatic compression. In: Journal of the Soil Mechanics and Foundations Division Vol. 93 (1967), No. SM3, p. 137–156

Kolymbas, D.: A rate-dependent constitutive equation for soils. In: Mechanics Research Communications Vol. 4 (1977), No. 6, p. 367–372

Li, X. S.: A sand model with state-dependent dilatancy. In: Géotechnique Vol. 52 (2002), No. 3, p. 173–186

Li, X. S., Dafalias, Y. F.: Dilatancy for cohesionless soils. In: Géotechnique Vol. 50 (2000), No. 4, p. 449–460

Mallwitz, K., Holzlöhner, U.: Verfahren zur Ermitlung der Setzung von Fundamenten infolge zyklischer Beanspruchung. In: Bautechnik Vol. 73 (1996), No. 3, p. 175–186

Ming, H. Y., Li, X.-S.: Fully coupled analysis of failure and remediation of lower San Fernando Dam. In: Journal of Geotechnical and Geoenvironmental Engineering Vol. 129 (2003), No. 4, p. 336–349

Niemunis, A., Herle, I.: Hypoplastic model for cohesionless soils with elastic strain range. In: Mechanics of Cohesive-frictional Materials Vol. 2 (1997), p. 279–299

Nübel, K., Cudmani, R.: Examples of finite element calculations with the hypoplastic law. In: Kolymbas, D. (Hrsg.): Constitutive Modelling of Granular Materials. Berlin, Heidelberg : Springer, 2000 — ISBN 978-3-642-63115-3, p. 523–538

Prada Sarmiento, L. F.: Implementation of the elastoplastic Model Sanisand 2004 into Incremental Driver (Presentation given at the Karlsruhe Institute of Technology, slides available at http://geotecnia.uniandes.edu.co/Descargas.aspx), 2009

Savidis, p. A., Aubram, D., Rackwitz, F.: Vibro-Injection Pile Installation in Sand: Part II - Numerical and Experimental Investigation. In: Triantafyllidis, T. (Hrsg.): Holistic Simulation of Geotechnical Installation Processes, Lecture Notes in Applied and Computational Mechanics. Vol. 77 : Springer, 2015 — ISBN 978-3-319-18169-1, p. 103–131

Sawicki, A., Swidzinski, W.: Cyclic compaction of soils, grains and powders. In: Powder Technology Vol. 85 (1995), No. 2, p. 97–104

Seed, H. B., Lee, K. L., Idriss, I. M., Makdisi, F. I.: The Slides in the San Fernando Dams During the Earthquake of Febuary 9, 1971. In: Journal of the Geotechnical Engineering Division Vol. 101 (1975), No. 7, p. 651–688

Taiebat, M., Dafalias, Y. F.: SANISAND: Simple anisotropic sand plasticity model. In: International Journal for

Numerical and Analytical Methods in Geomechanics Vol. 32 (2008), No. 8, p. 915–948

Taiebat, M., Jeremić, B., Dafalias, Y. F., Kaynia, A. M., Cheng, Z.: Propagation of seismic waves through liquefied soils. In: Soil Dynamics and Earthquake Engineering Vol. 30 (2010), No. 4, p. 236–257

Tsegaye, A. B., Molenkamp, F., Brinkgreve, R., Bonnier, P. G., de Jager, R., Galavi, V.: Modeling liquefaction behavior of sands by means of hypoplastic model. In: Benz, T., Nordal, p. (Hrsg.): Numerical Methods in Geotechnical Engineering. London : CRC Press, 2000 — ISBN 978-0-415-59239-0, p. 81–87

Wang, D., Bienen, B., Nazem, M., Tian, Y., Zheng, J., Pucker, T., Randolph, M. F.: Large deformation finite element analyses in geotechnical engineering. In: Computers and Geotechnics Vol. 65 (2015), p. 104–114

Wang, Z., Dafalias, Y. F., Shen, C.: Bounding Surface Hypoplasticity Model for Sand. In: Journal of Engineering Mechanics Vol. 116 (1990), No. 5, p. 983–1001

Wichtmann, T., Niemunis, A., Triantafyllidis, Th.: The effect of volumetric and out-of-phase cyclic loading on strain accumulation. In: Cyclic Behaviour of Soils and Liquefaction Phenomena : Balkema, 2004. — Proceedings of CBS04, Bochum, ISBN 90-5809-620-3, p. 247–256

von Wolffersdorff, P.-A.: A hypoplastic relation for granular materials with a predefined limit state surface. In: Mechanics of Cohesive-frictional Materials Vol. 1 (1996), No. 3, p. 251–271

von Wolffersdorff, P.-A., Schwab, R.: The Uelzen I Lock - Hypoplastic finite-element analysis of cyclic loading. In: Bautechnik Vol. 86 (2009), No. S1, p. 64–73

Zienkiewicz, O. C., Shiomi, T.: Dynamic behaviour of saturated porous media; The generalized Biot formulation and its numerical solution. In: International Journal for Numerical and Analytical Methods in Geomechanics Vol. 8 (1984), No. 1, p. 71–96