Proceedings 10th NUMGE 2023
10th European Conference on Numerical Methods in Geotechnical Engineering
Zdravkovic L, Kontoe S, Taborda DMG, Tsiampousi A (eds)

https://doi.org/10.53243/NUMGE2023-133

The effect of stone columns on critical speed for high-speed railway lines

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ABSTRACT: When high-speed railway lines have to cross soils with lower propagation velocities, several geotechnical challenges arise, one of these being related to the critical speed. In this scenario, soil reinforcement is usually necessary, one of the most common materials for doing this being stone columns. Regarding the track settlements, there are several approaches to quantifying the settlement reduction factor which are frequently applied in practical engineering. Nevertheless, the effect of soil reinforcement on critical speed has received scarce attention in the scientific community. For this reason, this research focuses on the enhancement of critical speed in high-speed ballasted railway tracks produced by stone columns. To this end, different geotechnical scenarios were considered, computing the critical speed with a 3D elastodynamic numerical model for each one of them and further analysing the effectiveness of the reinforcement technique adopted. The results show that the enhancement of the critical speed is greater for a high column/soil stiffness ratio, whereas the depth of the stone columns also plays an important role although a critical length has been found, from which the critical speed remains constant.

Keywords: Critical speed; High-speed railway lines; Stone columns

1 INTRODUCTION

The critical speed phenomenon is a very important aspect of high-speed railway lines when crossing soils with lower wave propagation velocities. From a theoretical point of view, the critical speed is the speed of a non-oscillating moving load that implies the greatest amplification of the dynamic response (Dieterman and Metrikine, 1996; Dong et al., 2019; Madshus et al., 2004; Alves Costa et al., 2018; Fernández-Ruiz et al., 2021). In the last decades, the scientific community has made great advances towards understanding this concept, beginning with the well-known case of Ledsgard (Hall, 2003; Alves Costa et al., 2010, Fernández-Ruiz and Alves Costa, 2021) and developing different approaches to compute the critical speed, ranging from analytical solutions (Alves Costa et al, 2015) to complex numerical models based on the finite element method (Kaynia et al., 2000; Abu Sayeed and Shahin, 2013; Fernández-Ruiz et al., 2021, among others).

When the situation of critical speed is predictable, corrective measures are imperative, the most important being soil replacement and soil improvement. One of these measures involves the use of stone columns, which are vertical boreholes in the ground, filled with compacted gravel (Fernández-Ruiz at al., 2021). The studies on the effect of soil improvement on critical speed are scarce although it is worth noting the studies presented by Castanheira et al. (2022), where a simplified approach is presented and validated for calculating the critical speed of the railway system achieved by a soil reinforcement with jet grouting in several geotechnical scenarios and the work of Fernández-Ruiz et al. (2021), which presented a complete numerical study of the enhancement of critical speed with stone columns, focusing particularly on the critical length of the stone columns. In that research, the authors showed how the critical speed remains constant when a soil reinforcement is applied in a homogeneous soil scenario. However, in normally dispersive scenarios the effect of stone columns on critical speed is relevant, finding increases of up to 50%. It was also shown that a critical length exists, from which the critical speed remains constant.

This fact can be seen in Figure 1, where the critical speed is plotted versus column length for two different area replacement ratios (Fernández-Ruiz et al., 2021). In this study, the geotechnical scenarios correspond with very soft soils. Starting from this study and in order to expand to not so very soft soils, the current research is presented, where a new geotechnical scenario is considered. In this way, the main goal of this paper is to study and compare the effect of stone columns in three different geotechnical scenarios, all with normally dispersive profiles. Specifically, and as the main novelty, the enhancement of critical speed is also studied depending on the contrast between the stone column and soil elastic properties.

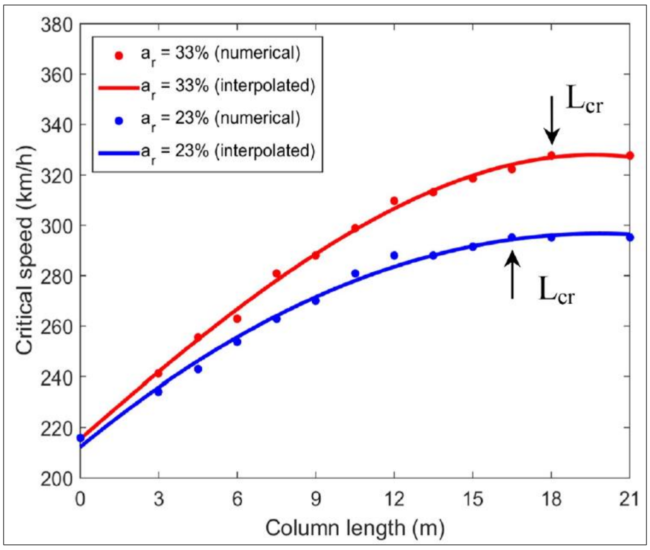

Figure 1. Critical speed vs. column length (Fernández-Ruiz et al., 2021)

2 NUMERICAL MODEL

The effect of stone columns on critical speed has been studied through a 3D elastodynamic numerical model formulated in the time domain using Plaxis 3D 2018.01 software. The dimensions of the numerical model are shown in Figure 2 and, as can be seen, this corresponds to a symmetric case, so that only half of it has been modelled. The model dimensions are 80 m x 35 m x 30 m the longitudinal, horizontal and vertical directions. The railpad is modelled as a linear spring (through a node-to-node anchor element), with a stiffness of 600 kN/mm, while the rail has been simulated as a beam, with the standardised properties of the UIC-60 type, namely EA = $1.6 \cdot 10^6$ kN and EI = $6.4 \cdot 10^3$ kN*m^2.
The rest of the track components (sleeper, ballast and subballast) and the ground and stone columns were modelled using 3D solid elements. The axis-to-axis spacing between sleepers is 0.6 m and their width is 0.2 m.

The finite element mesh used in this research is unstructured (Figure 3 and 4), with tetrahedral 10-node elements and a minimum element size in the ground of 0.4 m (in the area around the track). In this case, the model is made up of 437,071 elements and 621,670 nodes. The boundary conditions correspond to viscous dampers (Fernández-Ruiz et al., 2021). A Rayleigh damping type has been used, very suitable and widely used in numerical models formulated in the time domain.

The equivalent nodal force method has been considered to simulate the moving point load and the time step has been considered according to the criteria of Courant-Friedrichs-Lewy (Abu Sayeed and Shahin, 2013; Fernández-Ruiz et al., 2021; Galavi and Brinkgreve, 2014) and with an implicit Newmark integration scheme. The basis of the method is to consider the load as triangular pulses, as shown in Figure 5 (Fernández Ruiz et al. 2017). These triangular pulses are applied in each node of the rail as a function of speed of the load and the time step.

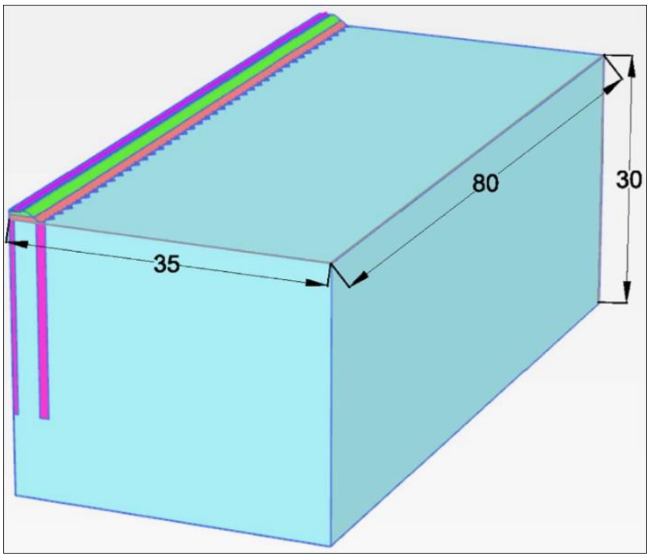

Figure 2. Numerical model dimensions

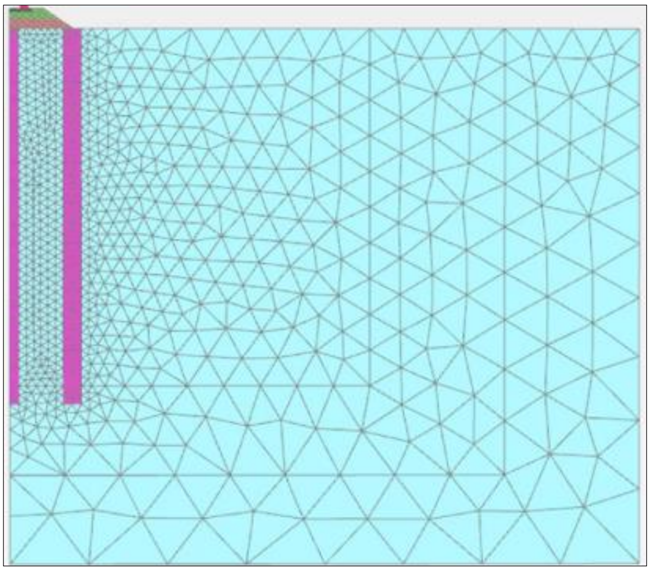

Figure 3. Finite element mesh

The relationship between time step, distance between nodes and speed of the load is controlled by the Courant-Friedichs-Lewy criteria, as shown below:

$$C_n = \frac{\Delta t \ x \ v}{L_{min}} < 1 \tag{1}$$

where: C_n is the Courant number, Δt is the dynamic time step, v is the speed of the moving load and L_{min} is the distance between two adjacent loading nodes.

It should be noted that in all cases the train is modelled as a moving 200 kN axle load, so that all results are for a single wheel passage only.

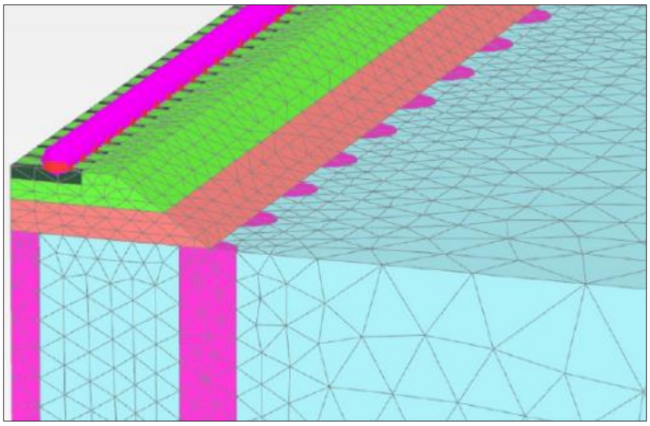

Figure 4. Detail of finite element mesh

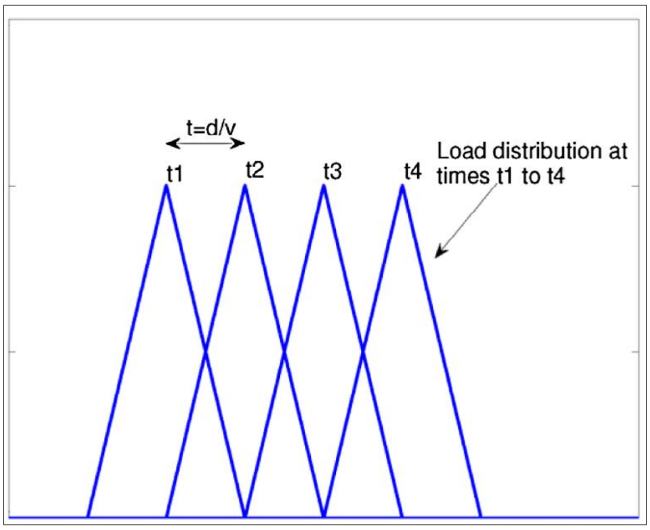

Figure 5. Schematic representation of equivalent nodal forces for simulating the moving load (Fernandez Ruiz et al. 2017)

3 CASES STUDIED

To study the effect of stone columns on critical speed, three geotechnical scenarios have been considered, although two scenarios were already studied in Fernández-Ruiz et al. (2021). Thus, a new geotechnical scenario is analysed in this research since the authors consider it relevant to show the three scenarios in order to improve the comparison between them. The elastic properties of the three scenarios are shown in Table 1. Ideal cases of only one soil layer have been considered to facilitate the analysis and interpretation of the results.

Regarding the ballasted track, the geometry is shown in Figure 6, where the ballast and subballast layer are founded on natural ground. Their elastic properties are listed in Table 2.

It is worth noting that the Young's modulus considered in the tables 1 and 2 corresponds to very small strains. This is reasonable because is expectable a very low degradation stiffness in presence of the stone columns.

3.1 Results without soil reinforcement

Figure 7 shows the maximum rail displacement versus velocity of point load for the threee scenarios without reinforcement. As can be seen, the critical speeds are 216 km/h, 324 km/h and 432 km/h for soil 1, soil 2 and soil 3 respectively. It is worth noting that the shapes of the three curves are different, showing smoother shapes for the stiffest soils. In this sense, a higher soil stiffness causes a lower dynamic amplification.

Table 1. *Soil and reinforcement properties*

	Layer depth (m)	**Specific weight (kN/m^3)**	**Young's modulus (MPa)**	**Poisson's ratio**	**Rayleigh Damping α (s^{-1})**	**β (s^{-1})**
Soil 1	0-2	14.75	11.90	0.49	1.58	$0.67 \cdot 10^{-3}$
	2-∞	14.75	$11.90 + 2 \cdot (z-2)$	0.49	1.58	$0.67 \cdot 10^{-3}$
Soil 2	0-2	14.75	32.00	0.49	1.58	$0.67 \cdot 10^{-3}$
	2-∞	14.75	$32.00 + 5.40 \cdot (z-2)$	0.49	1.58	$0.67 \cdot 10^{-3}$
Soil 3	0-2	14.75	64.00	0.49	1.58	$0.67 \cdot 10^{-3}$
	2-∞	14.75	$64.00 + 10.80 \cdot (z-2)$	0.49	1.58	$0.67 \cdot 10^{-3}$
Stone Columns	Variable	18.00	432.00	0.20	2.38	$1.00 \cdot 10^{-3}$

Table 2. *Track properties*

	Layer depth (m)	Specific weight (kN/m^3)	Young's modulus (MPa)	Poisson's ratio	Rayleigh Damping	
					α (s^{-1})	β (s^{-1})
Sleeper	0.22	25.00	30.00*10^3	0.20	0.39	0.16·10^{-3}
Ballast	0.35	16.00	97.00	0.12	2.38	1.00·10^{-3}
Subballast	0.55	19.00	212.00	0.20	1.58	0.67·10^{-3}

3.2 Effect of stone columns

To study the effect of stone columns on critical speed, a typical configuration has been adopted (Figure 8). In this case, a unique area replacement ratio (a_r) has been considered, whose value is 23%. This value may be high for static cases (with usual values between 5-15%) but in dynamic cases such values are necessary in order to be an effective measure. The diameter of the stone columns is 1 m and their centre-to-centre distance is 2 meters.

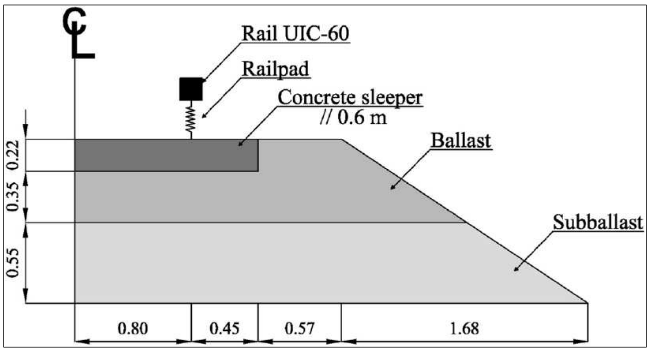

Figure 6. Railway track (in meters)

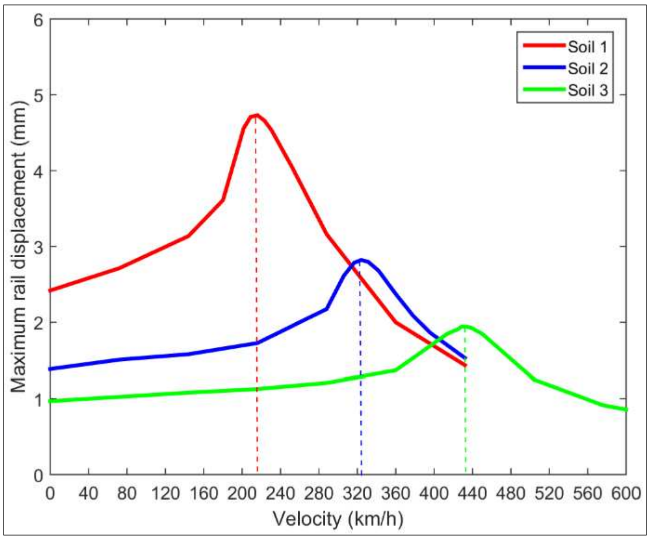

Figure 7. Maximum rail displacement vs. train velocity for 3 soils

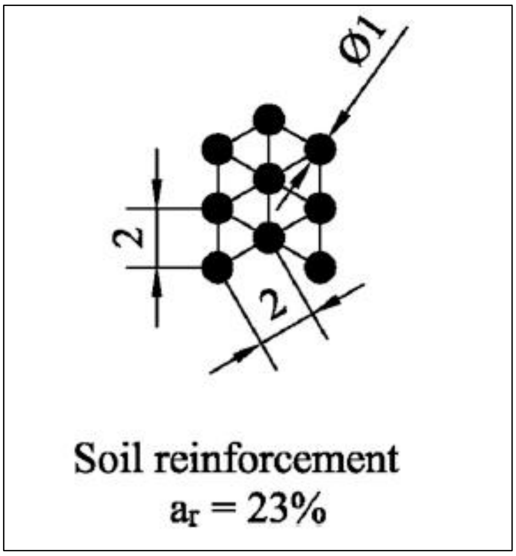

Figure 8. Stone columns configuration (in meters)

To perform the parametric analyses and to study the critical speed, the length of the columns has been varied in increments of 1.5 m, starting at 0 m and going up to 21 m. In this way, the critical speed for the three reinforced scenarios is shown in Figure 9 as a function of column length. As can be seen, the effect of the stone columns is positive and the critical speed increases considerably. For example, for soil 1, the critical speed increases from 216 km/h to 298 km/h, whereas for soil 3, the critical speed varies from 432 km/h to 510 km/h. In all scenarios the critical speed is improved and therefore the stone columns can be considered as an interesting technique for enhancing the critical speed. Moreover, the critical speed increases with the length of the stone column, although a critical length is found. This fact will be further discussed in detail later.

Nevertheless, the improvement of the critical speed is not the same for the three scenarios, showing a higher efficiency for the softer soils. To clarify this aspect, Figure 10 displays the critical speed improvement factor (VIF) as a function of column length for the three scenarios. This factor was proposed by the authors (Fernández-Ruiz et al., 2021) and is defined as follows:

$$\mathrm{VIF} = \frac{v_{cr,reinf}}{v_{cr,noreinf}} \qquad (1)$$

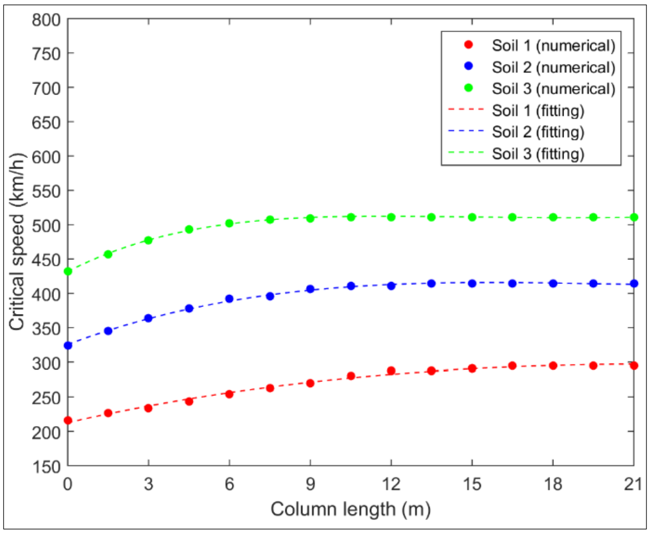

Figure 9. Critical speed vs column length

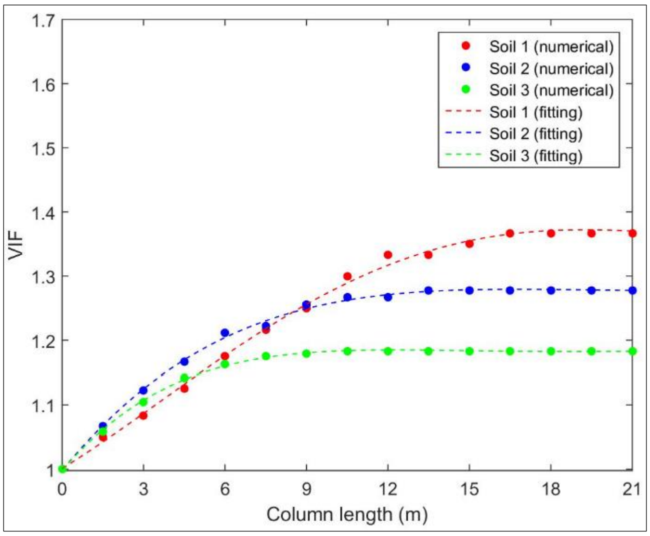

Figure 10. Comparison of critical speed improvement factor vs column length

Figure 10 clearly shows that the stone columns are more effective for the softer soils. Specifically, in soil 1 the critical speed increases by 37%; for soil 2, the rise of the critical speed is 27% and, for the last case, the increment is 18%. Therefore, the efficiency of the stone columns is greater the bigger the contrast in the stiffness between them and the ground. This fact can be seen in Figure 11, where the VIF factor is shown for the three scenarios as a function of critical speed of non-reinforced scenario. It should be pointed out that in this figure the value of VIF corresponds with the highest value of the critical speed. As can be observed, the stiffer the soil, the lower the efficiency of the stone columns.

Moreover, the relation between the critical speed of the non-reinforced scenario and the VIF factor is linear. Note that this linearity is found for the three cases analysed in this research and should not be extended to other cases without a more extensive and in-depth analysis.

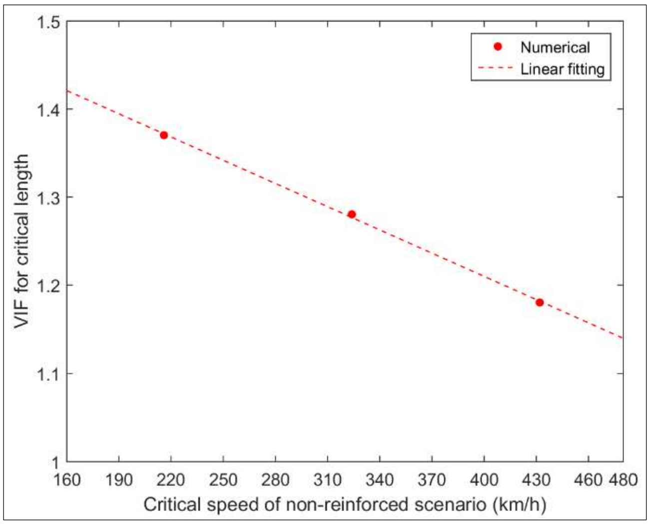

Figure 11. Critical speed improvement factor vs critical speed of non-reinforced scenario

Another interesting aspect is that the critical speed does not increase indefinitely with the column length, but rather a critical length can be found, as previously mentioned. This fact can be clearly seen in Figure 10, where the VIF factor has a well-defined limit. This depends on several factors (Fernandez-Ruiz et al., 2021) but is clearly affected by the soil properties, showing how an increase in the stiffness of the soil reduces the critical length. This is because the critical speed phenomenon in stiffer soils occurs at a higher frequency than in softer soils, thus reducing the wavelength and therefore the depth of influence. This fact is represented in Figure 12, where the critical length of the stone columns is related to the critical speed of the non-reinforced scenario.

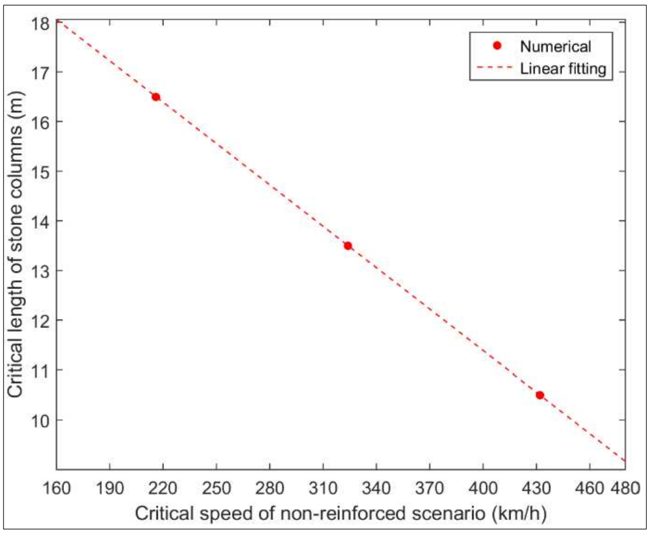

Figure 12. Critical length of stone column vs critical speed of non-reinforced scenario

As can be perceived, a higher soil stiffness causes a lower critical length. Also in this case, the relation between the critical speed of the non-reinforced scenario and the critical length of the stone column is linear.

It is worth noting the linearity relationships shown in Figures 11 and 12 are valid for the range of critical speed considered in this research. Out of this range, the relationship will probably tend to be non-linear.

4 CONCLUSIONS

In this research, the effect of stone columns on the critical speed in railway lines has been studied. Through a parametric tridimensional numerical study, the improvement of critical speed has been analysed in ballasted tracks. With the asumptions of this research, where the area replacement ratio has been considered of 23%, with a triangular pattern, Young's modulus in very small strians and with "idealised" soft soils, the main conclusions are listed below:

- Stone columns are an efficient reinforcement technique for improving the critical speed in presence of soft soils. Increases in the critical speed from 18% to 37% have been obtained, which are substantial amounts.
- The efficiency of the stone columns is greater the softer the soil. In this sense, the contrast of stiffness between stone column and soil is a very important parameter.
- The enhancement of the critical speed and the critical speed of the non-reinforced scenario follows a linear relationship in the analysed cases, the improvement being greater the lower the critical speed of the non-reinforced scenario.
- In the cases studied, a critical length of stone columns has been found. This critical length is lower the stiffer the soil . Furthermore, the relation between the critical length and the critical speed of the non-reinforced scenario is also linear.

The authors consider that these conclusions are limited to the cases studied in this research and should not be extrapolated to different geometries and geotechnical properties.

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