The paper was published in the proceedings of the 10th European Conference on Numerical Methods in Geotechnical Engineering and was edited by Lidija Zdravkovic, Stavroula Kontoe, Aikaterini Tsiampousi and David Taborda. The conference was held from June 26th to June 28th 2023 at the Imperial College London, United Kingdom.

Proceedings 10th NUMGE 2023
10th European Conference on Numerical Methods in Geotechnical Engineering
Zdravkovic L, Kontoe S, Taborda DMG, Tsiampousi A (eds)

https://doi.org/10.53243/NUMGE2023-134

Non-linear critical speed analysis of high-speed railways

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ABSTRACT: Developing a high-speed railway network entails several geotechnical challenges, one of the major ones being related to the critical speed phenomenon when soft soils need to be crossed. In these scenarios, the ground deformations increase noticeably with the increase of train speed, far exceeding the limits of a linear elastic behaviour. Taking this fact into consideration, this research aims to study the influence of soil non-linearity on the critical speed for slab and ballasted tracks. For this purpose, a 3D FEM non-linear numerical model was used to model several geotechnical scenarios. A different behaviour was observed as a function of the railway track, the ballasted one being deeply affected by the soil non-linearity. In contrast, for the slab track a lesser influence was shown.

Keywords: Non-linear critical speed; High-speed railways; Railway Engineering; Numerical modelling

1 INTRODUCTION

A significant growth in new high-speed railway lines has been seen in the last decades, since they offer, in terms of time and sustainability, an efficient method of mass transportation. In spite of these advantages, several drawbacks can be associated with the implementation of high-speed railway lines. From the geotechnical perspective, one challenge occurs when the train speed approximates the minimum wave propagating speed of the track-ground system. When this happens, a major amplification of the displacements and stresses is induced (Bian et al., 2018), which increases both the derailment risk and the maintenance cost. This phenomenon, known as the critical speed, corresponds to the speed of a non-oscillating moving load which gives rise to the largest amplification in comparison to the static deformation field (Connolly and Costa, 2020).

From a scientific point of view, the critical speed phenomenon has been object of research for more than a century (Alves Costa et al., 2015). Moreover, the well-known Ledsgard's case supposed one of the more interesting real cases where the critical speed could be experimentally studied in the free-field (Kaynia et al., 2000, Madshus and Kaynia, 2000). Since then, several approaches to predict the critical speed have been proposed, ranging from analytical methodologies (Alves Costa et al., 2015, Dieterman and Metrikine, 1996) to complex numerical models (Alves Costa et al., 2015, Galvín et al., 2018, François et al., 2010). Despite the range of numerical models at our disposal, complex geometries and non-linear material behaviours can be modelled in the time domain numerical models with a good numerical accuracy (Fernández-Ruiz et al., 2021, Chen and Zhou, 2018).

The approach of the train speed to the critical velocity of the track-ground system is accompanied by the amplification of the strain field, exceeding the range where soil linearity behaviour is plausible. This fact has given rise to recent studies where the non-linear behaviour of the soil is modelled rather than assuming it to be a linear elastic material (Shih et al., 2017, Sayeed and Shahin, 2016, El Kacimi et al., 2013, among others). Such an assumption was found to have a major impact on the critical speed, leading to critical speed values considerably lower than the one achieved in a linear elastic scenario (Alves Costa et al., 2010, Shih et al., 2017). Taking into consideration the strong dependence between the critical speed and the non-linear soil behaviour, the present paper aims to study the influence of the soil plasticity index on the critical speed of both ballast and slab tracks. This index was selected because, as is well known, it is the most decisive parameter for simulating the non-linear behaviour of cohesive soils.

With this purpose, a reference scenario with linear elastic materials will be adopted to further assess the impact of assuming non-linear soil behaviours. A 3D time domain numerical model was used to perform the simulation of all the scenarios considered in the present paper.

2 NUMERICAL MODEL

A 3D finite element model formulated in the time domain (Plaxis software) was used to develop the study presented here. A global perspective of the model can be found in Figure 1, and a detailed view of both the slab and ballast tracks can be seen in Figure 2. The model dimensions are 80 m x 35 m x 15 m in the longitudinal, horizontal and vertical directions. The railpad is modelled as a linear spring, while the rail has been simulated as a beam, with the standardised properties of the UIC-60 type (see Table 1). The rest of the track components (sleeper, ballast and subballast) and the ground were modelled using 3D solid elements. The axis-to-axis spacing between sleepers is 0.6 m and their width is 0.2 m.

The finite element mesh used in this research is unstructured, with tetrahedral 10-node elements. The boundary conditions correspond to viscous dampers. A Rayleigh damping type has been used, very suitable and widely used in numerical models formulated in the time domain.

The equivalent nodal force method has been considered to simulate the moving point load and the time step has been considered according to the criteria of Courant-Friedrichs-Lewy and with an implicit Newmark integration scheme. (Fernández-Ruiz et al., 2021)

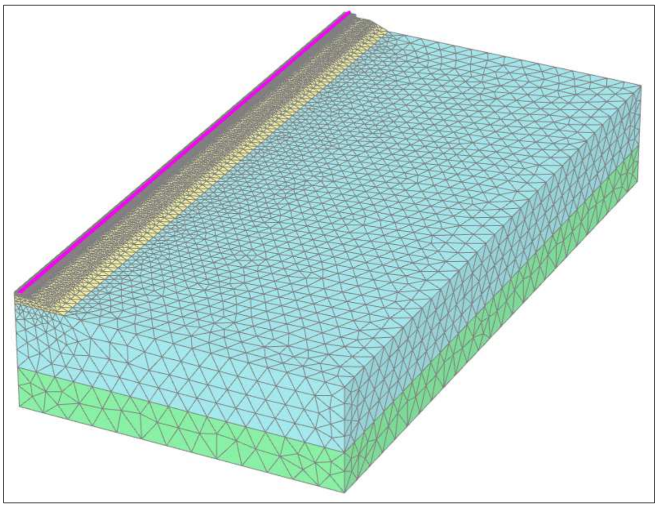
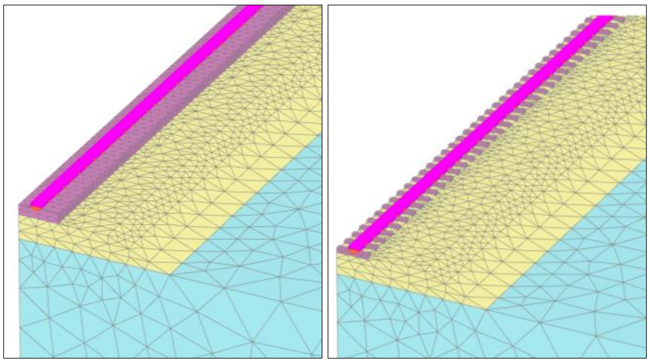


Figure 1. Global overview of the finite element model.



a) *b)*

Figure 2. Detailed view of: a) slab track; b) ballast track.

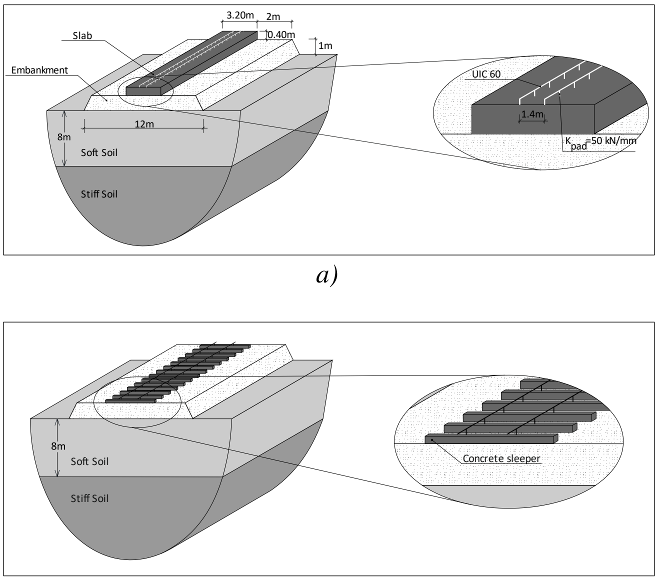
As can be observed, the geotechnical scenario consisted of a dispersive layered ground. The linear elastic mechanical properties of the track and soil can be found in Table 1. It is important to mention that all the material properties of the soil presented in Table 1 are exclusively used for the reference scenario, which will be described in the next section.

Table 1. Material properties of the tracks and soils

Layer	E (MPa)	ρ (Kg/m3)	ν (-)	ξ (-)	Cs (m/s)
Slab/Sleeper	25e3	2500	0.20	0.01	2236
Embankment (ballast and subballast)	200	2000	0.30	0.03	196
Soft soil	30.5	1600	0.35	0.03	80
Stiff soil	208	2000	0.30	0.03	200
Rail (UIC 60)	210e3	7850	0.30	0.01	5170
Rail pads	K_{pad}= 50 kN/mm and 0.6m of longitudinal spacing.				

3 REFERENCE SCENARIO

Before studying the non-linear critical speed, a reference scenario is defined (corresponding to the one shown in fig. 1 and 2), both slab track and ballasted track, where an elastic linear behaviour has been considered. This is relevant in order to compare the non-linear critical speed with the elastic approach. This geometry of the scenario is shown in Figure 3, where a soft soil is considered with a thickness of 8 meters. Note that the dimensions in both scenarios are the same, except the sleeper, which has 2.6 m and a height of 0.2 m for ballast track.



a)

b)

Figure 3. Reference scenario for: a) slab track; b) ballast tack

The dynamic amplification factor (DAF) for the ballast track is shown in Figure 4 while the DAF for the slab track is shown in Figure 5. As can be perceived, the critical speed for the ballast track is smaller than for the slab track because its bending stiffness is much smaller. In fact, the critical speed for the ballast track is 84 m/s, whereas for the slab track it is 97 m/s. The difference is relevant since the slab track develops a critical speed 15% higher than the ballast track.

This is due to the fact that the deeper soil has little influence on the critical speed for the ballast track, whereas for the slab track the relevance is higher. Therefore, the ballast track is more sensitive to shallow soil properties, whereas for slab tracks the deeper soil also plays an important role. This different pattern behaviour of slab and ballast tracks can be explained from a dispersive analysis, as showed by the authors (Fernández-Ruiz et al., 2022).

Moreover, it is worth noting that the DAF is higher in the ballast track than in the slab track, therefore developing greater displacements.

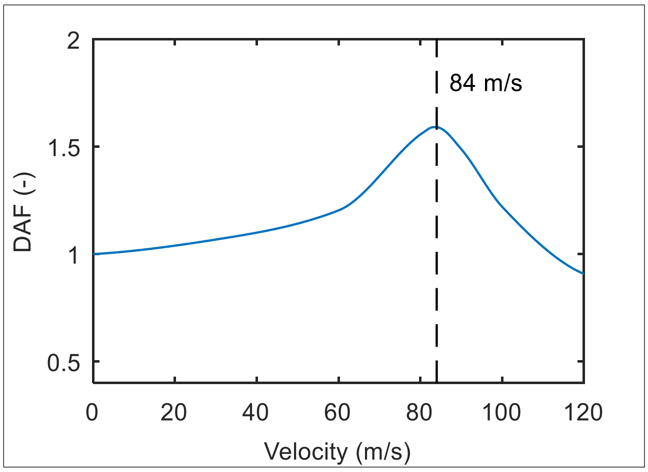


Figure 4. DAF curve for ballast tack

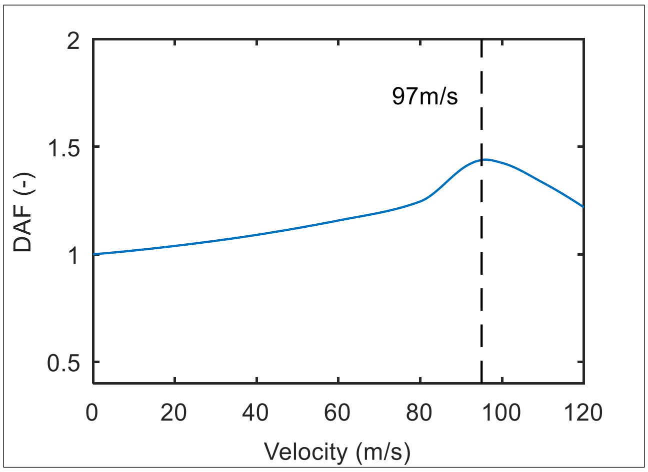


Figure 5. DAF curve for slab track

4 PARAMETRIC STUDY

4.1 Generalities

As previously mentioned, the present research aims to study the influence of the soil plasticity index on the critical speed of both the slab and ballast tracks. To do this, the soft soil, which was assumed as linear elastic in the previous section, is now modelled with a non-linear constitutive model, the Hardening Soil Model with Small Strains Stiffness, previously validated by the authors (Fernández-Ruiz and Alves Costa, 2021). Although the reference scenario has a soft soil with a shear wave velocity of 80 m/s, in the present study two more cases were considered with shear wave velocities of 100 and 120 m/s. Each one the soft soil scenarios were simulated assuming several plasticity indexes (15, 30, 50 and 70), its material properties being presented in Table 2. As in the reference scenario, the thickness of the soft soil was assumed to be 8 meters in all cases.

Table 2. Material properties of the embankment and soils.

Element	E_{50} (kN/m^2)	E_{oed} (kN/m^2)	E_{ur} (kN/m^2)	Φ (°)	c (kN/m^2)	Ψ (°)	$\gamma_{0.7}$
Embankment (ballast and subballast) (PI 0)	35x10^3	35x10^3	70x10^3	45	5	10	7.5x10^{-5}
Soft soil (PI 70)	1.3x10^3	1.3x10^3	4x10^3	0	50	0	1.4x10^{-3}
Soft soil (PI 50)	1.3x10^3	1.3x10^3	4x10^3	0	50	0	9.7x10^{-4}
Soft soil (PI 30)	1.3x10^3	1.3·10^3	4x10^3	0	50	0	6.7x10^{-4}
Soft soil (PI 15)	1.3x10^3	1.3·10^3	4x10^3	0	50	0	3.6x10^{-4}
Stiff soil (PI 0)	40x10^3	40·x10^3	80x10^3	35	5	10	2.4x10^{-4}

4.2 Numerical results

As previously cited, the main objective of this paper is to assess the influence of the non-linear soil behaviour on the critical speed in railway lines. For that, the non-linear critical speed achieved in the present section is compared to the one obtained assuming linear elastic behaviours (reference scenarios). Bearing that in mind,

Figure 6 shows a comparison of the non-linear critical speed with the linear elastic one for the ballast track scenario with soft soils with shear wave velocities of 80, 100 and 120 m/s.

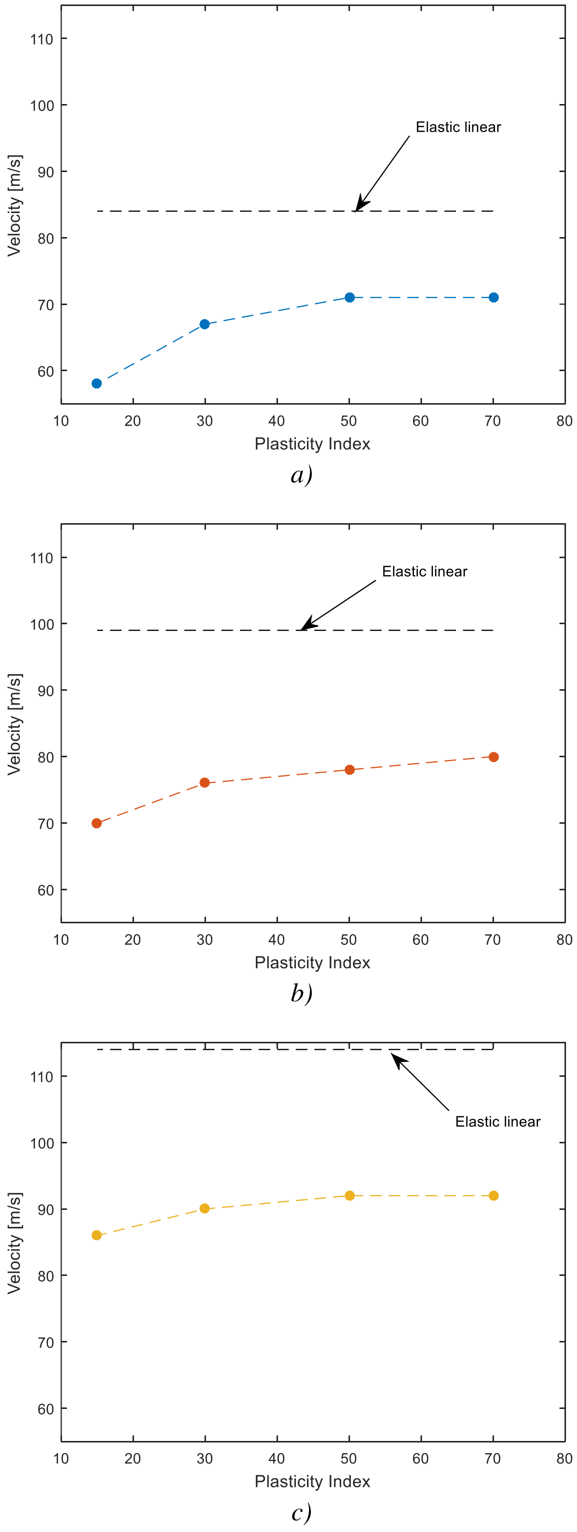
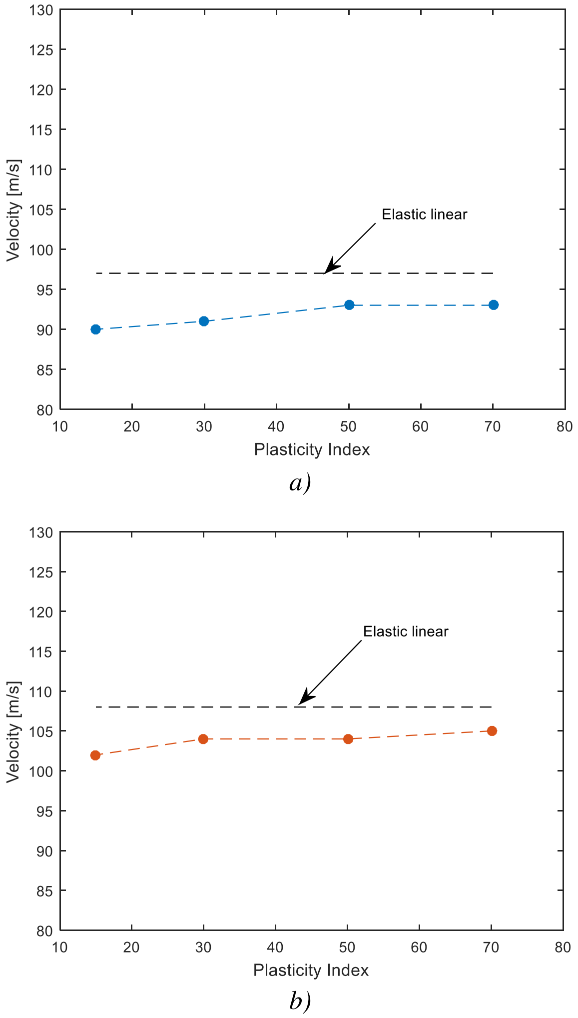
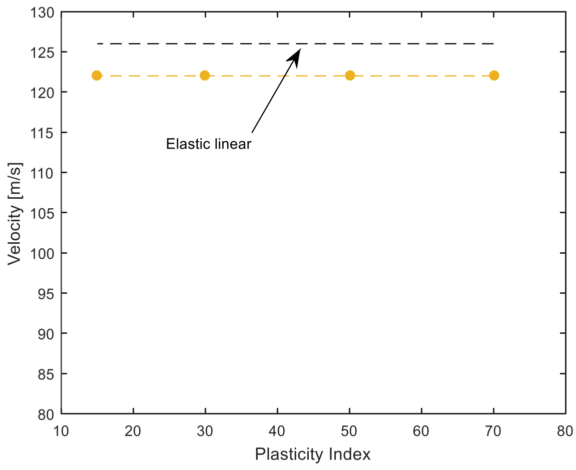


Figure 6. Critical speed for ballast track considering soft soils with a shear wave velocity of: a) 80 m/s; b) 100 m/s; c) 120 m/s.

Firstly, as can be seen, the assumption of non-linear behaviours gives rise to a reduction in the critical speed achieved, which is due to the stiffness degradation of the soil. In addition, it can be observed that for lower plasticity indexes, the higher the stiffness degradations will be and, therefore, a lower value of the critical speed is reached. Another interesting aspect that must be highlighted is related to the reduction of the critical speed for every soft soil scenario considered. A similar reduction magnitude of the critical speed was found for all the geotechnical scenarios, i.e., for a given plasticity index, the critical speed reduction in comparison to the elastic linear one was independent of the soft soil stiffness. Also, a strong dependence was found between the non-linear behaviour and the plasticity index. Soils with a lower plasticity index tend to experience more degradation and consequently achieved a lower critical speed. It is worth noting that the difference between the non-linear critical speed and the elastic critical speed was very relevant, finding reductions of up to 30%.

The same analysis was conducted for the slab track in order to assess the influence of non-linear soil behaviour on the critical speed. As before, three geotechnical scenarios were modelled, varying the plasticity index of the soft soil, the results being shown in Figure 7.





c)

Figure 7. Critical speed achieved for slab track considering soft soils with a shear wave velocity of: a) 80 m/s; b) 100 m/s; c) 120 m/s.

Contrary to what was found for the ballast track, the non linear critical speed for a slab track is much closer to the linear elastic one. In this sense, it can be concluded that these track structures do not induce the degradation levels in the soil beneath as the ballast tracks do. This is due to the high stiffness of the slab in comparison to the surrounding soil which gives rise to a stress concentration in the slab and a relief in the soil.

Another interesting aspect that must be highlighted is related to the relationship between the non linear critical speed and the plasticy index of the soil. As can be seen, the impact of analysing soils with different values of plasticity index does not induce a significant change in the non linear critical speed. This can be justified by using the same principle mentioned above: the stress and strain concentration in the slab reduces the strain levels of the surrounding soils. Since the non-linear soil behaviour is governed by the strain levels to which it is sujected , for lower strain levels a linear elastic behaviour is achieved and, therefore, the importance of the plasticity index is reduced. This fact explains why the reduction of the critical speed achieved, between the non linear critical speed and the linear elastic one, is e the same regardless of the plasticity index considered for the soil.

5 CONCLUSIONS

This study was presented to analyse the influence of the non linear soil behaviour on the critical speed of high-speed railway lines. Two different structures were modelled, one with a ballast track and another with a slab track. Both scenarios were studied for three soils with different shear wave velocites (80, 100 and 120 m/s), four plasticity indexes being considered for each soil (15, 30, 50 and 70). The non linear critical speed was computed for every scenario and compared to the linear elastic one to assess the reduction obtained.

For the ballast track a considerable difference was identified between the non linear critical speed and the linear elastic one (with differences of up to 30%), motivated by the high degradation of the stiffness of the embakment and the foundation soil. Also, an inverse relationship was observed between the degradation achieved and the plasticity index of the soil, i.e., soils with a lower plasticity index will experience more stiffness degradation and, consequently, develop a lower critical speed.

In contrast, a different behaviour was identified for the slab track case. In these cases, a lesser impact was observed when assuming the non linear soil behaviour in the critical speed, the results being close to the ones obtained considering linear elastic materials. Furthermore, and contrary to what was identified for the ballast track, the dependance between the plasticity index and the non linear critical speed is not so strong. The higher slab stiffness induces a strain and stress concentration which leads to lower strains levels in the surrounding soil and therefore a lower stiffness degradation. This fact justifies the closeness between the non linear critical speed and the linear elastic one for the slab track scenarios.

6 ACKNOWLEDGEMENTS

This work was financially supported by: Base Funding - UIDB/04708/2020 and Programmatic Funding - UIDP/04708/2020 of the CONSTRUCT - Instituto de I&D em Estruturas e Construções – funded by national funds through the FCT/MCTES (PIDDAC); Base Funding - UIDB/04029/2020 – of ISISE (Institute for Sustainability and Innovation in Structural Engineering) funded by national funds through the FCT/MCTES (PIDDAC); Project PTDC/ECI-EGC/29577/2017 - POCI-01-0145-FEDER-029577 - and Project PTDC/ECI-EGC/3352/2021, both funded by FEDER funds through COMPETE2020 - Programa Operacional Competitividade e Internacionalização (POCI); National funds (PIDDAC) through FCT/MCTES; Individual Grant: PD/BD/143004/2018 and from the European Union's Horizon 2020 Programme Research and Innovation action under Grant Agreement No 101012456

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