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Coupled experimental and numerical approaches in bender element testing of geomaterials

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ABSTRACT: Bender element testing of geomaterials involves inducing a shear wave at one end of a sample (the input signal) and reading its arrival at the other end (the output signal). As compared to resonant columns, bender elements are cheaper, flexible to install, easier to use and yield similar results. However, the wave propagation induced by bender elements is complex, hindering the standardization of the testing setup and the interpretation of the output signal. The research project CEN-DynaGeo (2018-22) aimed at coupling innovative experimental and numerical modelling techniques to advance the bender element testing of geomaterials. Frequency-insensitive hybrid-Trefftz finite elements, developed for solid and poro-elastodynamics, may be used to simulate and optimize test setups and for the automatic interpretation of the output signal. Setup optimization is aimed at minimizing the pollution of the output signal with compression waves, while preserving the legibility of the shear wave whose reading is the objective of the experiment. The automatic interpretation of the output signal is based on a fixed-point model updating technique, insensitive to local extrema, implemented into the novel, user-friendly computational platform GeoHyTE.

Keywords: Bender element; hybrid-Trefftz finite element; small strain shear modulus; automatic interpretation; GeoHyTE

1 INTRODUCTION

Bender elements (BE) are shear wave piezoelectric transducers for the experimental identification of small strain shear moduli of geomaterials. Two BE are used in a typical setup: one generates a shear wave (transmitter) and the other reads its arrival at the other end of the sample (receiver). The travelling time of the wave gives the velocity needed to obtain the shear modulus. BE have been installed on various geotechnical apparatus like oedometers, triaxial apparatus, and Rowe cells. However, considerable scientific debate is still ongoing about the optimal experimental setup and interpretation of the test results.

Besides the shear wave that propagates directly towards the receiver, the lateral vibration of the transmitter also triggers laterally propagating compression waves that may reflect from the lateral envelope of the sample and pollute the output signal. This may lead to erroneous readings of the arrival time and erroneous shear modulus estimates (Yamashita, et al., 2009).

The interpretation of the output signal is notoriously uncertain (Arroyo, et al., 2003). It has been done in time and frequency domains, but all techniques are either subjective (they depend on the expertise of the analyst) or assume that input and output signals are similar. The subjectivity in the identification of the arrival time of the shear wave can lead to large variation of the shear modulus estimates, as illustrated by an international parallel test that involved 23 labs from 11 countries (Yamashita, et al., 2009). The labs were asked to measure the shear modulus of the same geomaterial (Toyoura sand), under different state and confinement conditions. For some tests under low confinement pressures, the shear modulus readings varied between roughly 3 and 140 MPa, with a standard deviation of the order of half of the average value. Automated methods (e.g. cross-correlation and cross-spectrum methods) assume that input and output signals are similar in shape or wavelength. However, that is generally not the case, as the output signal is affected by the interaction of the travelling waves with the sample-apparatus interface.

The CEN-DynaGeo Project took place between 2018 and 2022 and was aimed to develop innovative numerical methods tailored to model the propagation of high frequency waves through multi-phase media and use them to improve the setup of BE experiments and their automatic interpretation. To improve the output signal, the location of the receiver can be optimized to avoid areas where the disturbance caused by the reflected compression waves is large, while still securing good shear wave amplitude to facilitate their clear identification (Moldovan, et al., 2021). Moreover, the shape of the lateral envelope of the sample can be designed to attenuate compression waves and prevent them from reaching the receiver. For the automatic interpretation of the output signal, the GeoHyTE toolbox was developed (Moldovan, et al., 2022). GeoHyTE is both objective, in the sense that it does not depend on the experience of the analyst, and physically-consistent, in the sense that it uses finite element models that recover the physics of the experiment.

This paper summarizes the numerical outcomes of the CEN-DynaGeo Project. It is not meant to be complete, but to give pointers to its main contributions and the papers where each is presented in detail.

2 HYBRID-TREFFTZ FINITE ELEMENT MODELS

Conventional numerical modelling of BE experiments is hindered by the high frequency of the excitation and the complexity of the material behaviour. For instance, when conventional finite elements are used to model BE experiments, they need to observe the well-known restriction of using at least six (but preferably 10) finite elements per wavelength to consistently model the wave propagation. While this may not be an issue in dense, single-phase media, it typically is in multi-phase media, where the compression waves propagating through the fluid phases have very short wavelengths. For this reason, conventional finite element models may yield results that are rather inconsistent with the output signal obtained in the laboratory (e.g. (Johnson, 2008)).

Recently, however, the authors proved that hybrid-Trefftz finite elements can accurately model BE experiments (Moldovan, et al., 2016), even under difficult testing conditions. Hybrid-Trefftz finite elements combine the favourable features of finite and boundary elements. Like the former, they are based on the division of the domain into finite elements and work with regular (not singular) trial functions. Like boundary elements, hybrid-Trefftz elements employ trial functions that satisfy locally the homogeneous form of the differential equations governing the problem. This means that the trial functions contain relevant information on the physics of the phenomenon they model, and the formulation is reduced to the boundaries of the elements. The physical insight built into the trial functions accounts for their insensitivity to short wavelengths (Moldovan, et al., 2012). Their dimensions can be of the order of three wavelengths with little degradation of the quality of the solutions, 30 times larger than is recommended for conventional finite elements. This feature renders hybrid-Trefftz elements ideally suited to modelling problems where high frequency waves are propagating through multi-phase media.

The trial bases of hybrid-Trefftz finite elements are built hierarchically and have no connection to the nodes of the elements. Localised basis refinements are therefore straightforward and meshes need not be conforming. As the trial functions are actual waves propagating through the medium (Figure 1), they are naturally grouped according to their type (compression or shear) and material they propagate through (solid or fluid), enabling the numerical filtration of a type of wave from the solution.

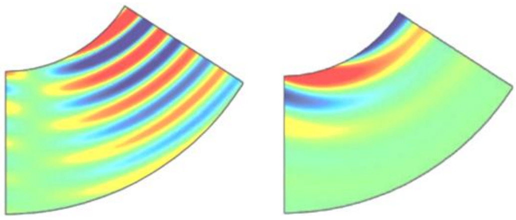

Figure 1. Trefftz trial functions for a porous medium problem: a shear wave (left), and a secondary compression wave propagating through the fluid phase (right)

A sustained effort has been invested in the last seven years to bringing the favourable features of the hybrid-Trefftz finite elements to the fingertips of the scientific community, by their integration in the FreeHyTE framework (Moldovan, et al., 2018). FreeHyTE is an open-source and user-friendly collection of hybrid-Trefftz finite elements for static and dynamic problems defined on single-phase, biphasic and triphasic materials (Moldovan, et al., 2021). It also features solvers for acoustic and heat propagation problems (Moldovan, et al., 2019), and a direct boundary method module (Borkowski, et al., 2021). The modules are bidimensional, in general, but two new three-dimensional modules have been recently developed for transient wave propagation problems defined on single phase (Climent, et al., 2022) and biphasic media (Climent, et al., 2022).

The FreeHyTE modules can be freely downloaded from an online repository (FreeHyTE, 2017), together with user's manuals to quickly get new users started. Table 1 lists the FreeHyTE modules that are currently available for download or will become available for download in 2023. A wide range of boundary conditions are implemented in FreeHyTE, well suited to modelling BE experiments. Dirichlet boundary conditions, enforcing boundary displacements, can be used to model rigid recipients; Neumann boundary conditions, enforcing boundary tractions can be used to model free surfaces or thin membranes through which confining pressures are applied to the specimen; Robin boundary conditions are used to model elastic boundaries to control the impedance between the specimen and the surrounding testing equipment; and, absorbing boundary conditions, used to model non-reflecting boundaries which absorb incoming waves (Moldovan, 2022).

Table 1. FreeHyTE modules available for download by the end of 2023

Designation	Description	Available
Direct Boundary Methods	Three solvers for 2D Laplace and Helmholtz problems: direct Boundary Element Method, direct Method of Fundamental Solutions and direct Trefftz-Herrera Method	Yes
Solid Transient	Hybrid-Trefftz finite element solver for 2D transient problems defined on continuous media	Yes
Biphasic Transient	Hybrid-Trefftz finite element solver for 2D transient problems defined on biphasic (porous, saturated) media. Uses the Biot's theory of porous media	Yes
Triphasic Transient	Hybrid-Trefftz finite element solver for 2D transient problems defined on triphasic (porous, unsaturated) media. Uses the theory of mixtures with interfaces	Yes
Structural (displacement)	Hybrid-Trefftz finite element solver for plane stress/strain problems. Uses the displacement-based formulation	Yes
Structural (stress)	Hybrid-Trefftz finite element solver for plane stress/strain problems. Uses the stress-based formulation	Yes
Heat conduction (steady-state)	Hybrid-Trefftz finite element solver for 2D steady-state heat conduction problems	Yes
Heat conduction (transient)	Hybrid-Trefftz finite element solver for 2D transient heat conduction problems	Yes
Hyperbolic non-homogeneous	Hybrid-Trefftz finite element solver for 2D non-homogeneous hyperbolic (e.g. transient acoustic) problems	Yes
3D Solid Transient	Hybrid-Trefftz finite element solver for 3D transient problems defined on solid (single-phase) media	In 2023

3 OPTIMISATION OF BENDER ELEMENT SETUP

The hybrid-Trefftz finite elements described in the previous section were adopted in the CEN-DynaGeo Project to optimise the experimental setup of BE tests. The objective was to minimise the pollution of the output signal with compression waves reflected from the lateral envelope of the sample. This objective was pursued at two levels: by optimising the location of the receiver and by minimising the reflections from the lateral envelope.

3.1 Optimisation of the receiver location

The single most important reason of uncertainty in the interpretation of the BE signal is its pollution with compression waves. These waves are generated by the motion of the transmitter, travel laterally, hit the lateral envelope of the specimen, and are then reflected towards its axis, where receivers are typically located. Numerical experiments have shown that the intensities of the compression and shear waves are not uniform in the area of the specimen where the receiver is inserted. This means that the optimisation of the location of the receiver may help secure an output signal of higher quality. By *higher quality* we mean a signal less polluted with compression waves, but strong enough to allow a clear reading of the shear wave.

To quantify the amount of compression wave noise, the wave filtration capability of the hybrid-Trefftz elements is used. The BE experiment is first simulated in full, using an adequate FreeHyTE module. Then, compression and shear waves are sequentially filtered out from the solution and the relevant displacement amplitudes caused by only shear and compression waves (respectively) are plotted to identify the area where the receiver is best located. Relevant displacement amplitudes are displacement maxima in the direction that triggers the receiver, recorded from the onset of the experiment until shortly after the shear wave arrives at the receiver.

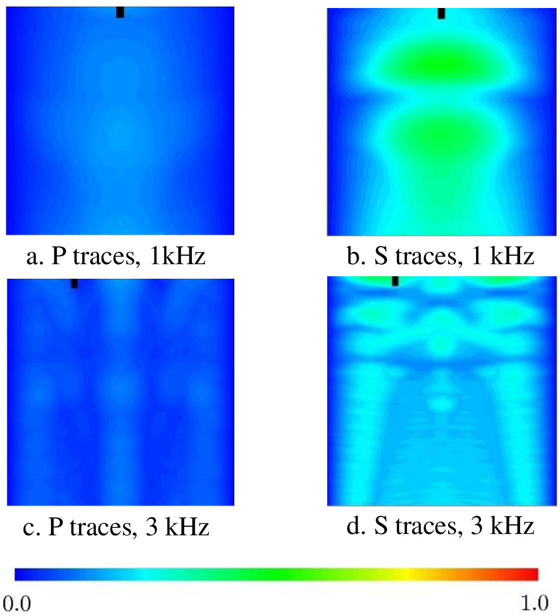

Figure 2. Normalised compression (P) and shear (S) relevant displacement amplitudes map for two input frequencies and the optimal positions of the BE (black rectangles)

The application of this procedure enables the identification of the regions of the specimen where the different types of waves are best expressed. Typical maps for compression and shear waves are presented in Figure 2, for two frequencies of the (pulse) input signal. The compression wave has larger amplitudes in the central axis of the mould, meaning that the conventional localisation of the receiver may not be ideal. However, these maps are problem-dependent, so a ‘thumb rule’ regarding the optimal location of the receiver cannot be established. For instance, at lower frequencies, the compression and shear amplitudes occur in the same (central) region of the end platen, so moving the receiver away would not only mitigate the compression wave pollution, but also diminish the shear wave signal. Conversely, for higher frequencies, the amplitudes of the shear traces are more spread out over the top end platen, so moving the receiver away does not hamper the strength of the output signal. Conversely, the compression wave traces are concentrated in the centre, so their pollution is reduced away from it. Thus, the optimal location of the sensor is shown in Figure 2.c and d. These findings are confirmed experimentally, as detailed in (Moldovan, et al., 2021). A procedure generally applicable for the optimisation of the receiver location is given in the same paper.

3.2 *Minimisation of side reflections*

An alternative approach to the minimisation of output noise is to attenuate compression waves through scattering near the lateral envelope, to avoid them reaching the receiver as a coherent wave pack. This can be done by profiling the lateral walls of a sample mould with spikes. The spikes must be as unintrusive as possible to avoid tampering with the geomaterial, but large enough to have a scattering effect. Their optimal dimension was established using the hybrid-Trefftz numerical models available in FreeHyTE. The propagation of the waves triggered by the lateral motion of the transmitter was simulated in a mould with flat walls and in one with spikes. The spikes have a height of 3 mm, a base of 8 mm, and stand 5 mm apart.

Figure 3 shows the finite element models of the tested moulds and the horizontal displacement field throughout the specimen when the reflected compression wave and the shear wave arrive at the receiver (respectively). The mitigation of the intensity of the compression wave that arrives at the receiver is clearly noticeable in the centre figures. On the other hand, the spikes also seem to slightly dampen the shear wave, as visible in the right plots, but not to the point where its clear reading is jeopardized. Lab tests were conducted to validate the numerical findings. Three moulds were 3D-printed based on the spikes design (Figure 4) and used to perform BE tests. Figure 5 presents the input signal used for the experiment (a sine pulse with 3.5 kHz frequency) and the time-histories of the output signals for the flat-walled setup and the three damping moulds.

As expected, the output signal obtained in the mould with acrylic (flat) walls is considerably polluted by the early arrival of the compression waves, as seen in the detail presented in the lower part of the figure. Conversely, the compression waves are strongly dampened for all tested spike shapes (although the vertical strips are slightly less efficient), rendering the arrival of the shear wave more clearly visible. The intensity of the shear signal is also slightly reduced, as predicted by the numerical model, but not to a point where it can be missed by the analyst.

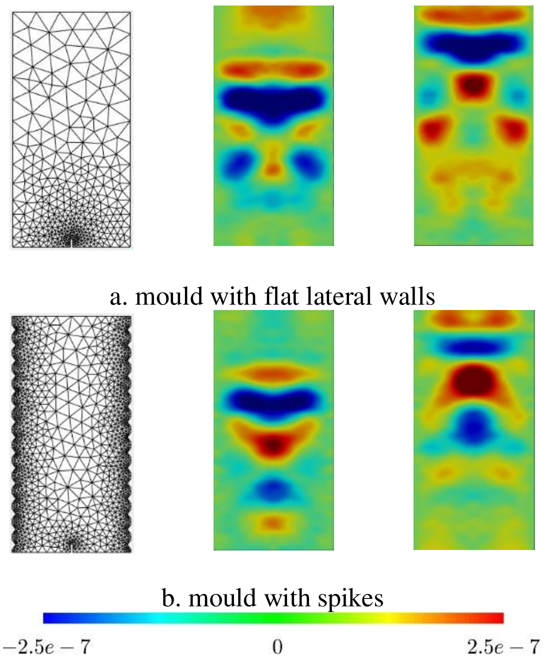

Figure 3. Flat and rugged moulds. Finite element meshes (left); arrival of the compression wave at the receiver (centre); arrival of the shear wave at the receiver (right). Units:m

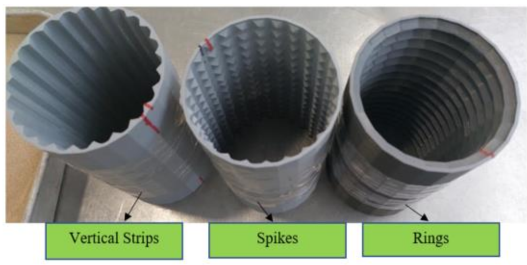

Figure 4. 3D printed moulds

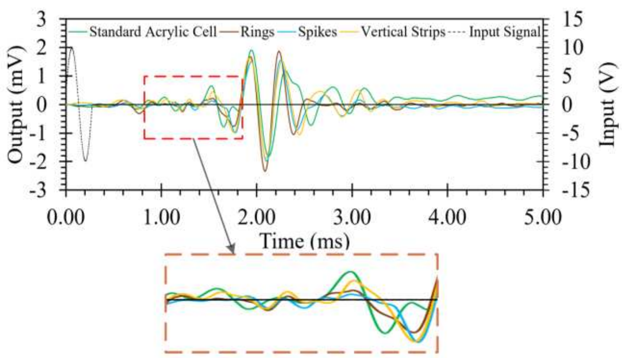

Figure 5. Input and output signals

4 AUTOMATIC INTERPRETATION OF THE OUTPUT SIGNAL

GeoHyTE is a new software for the automatic interpretation of output signals in BE experiments. Unlike conventional interpretation techniques, GeoHyTE is objective, as minimal user intervention is required, and physically-meaningful, as signals of the same nature (output) are correlated. GeoHyTE features Graphical User Interfaces (GUI), automatic mesh generators, and is insensitive to coarse finite element models and erroneous initial input. The beta version of GeoHyTE is available for download by users that register as testers (GeoHyTE, 2022). Here, the model updating procedure and computational architecture of GeoHyTE are presented, followed by an illustration of its validation using a benchmark geomaterial. More details on GeoHyTE can be found in reference (Moldovan, et al., 2022).

4.1 Model updating procedure

GeoHyTE finds an estimate of the shear modulus of a geomaterial by maximizing the correlation between the experimental output signal and those simulated by a numerical model based on hybrid-Trefftz finite elements. The correlation maximisation procedure starts with an analysis of the output signal, where the time over which the correlation should be measured is defined, and a coarse estimate of the arrival time of the shear wave is made. Then, the numerical model is constructed using a shear modulus based on the arrival time estimate. Next, the model is run, and the simulated output signal (time-history of the displacements at the tip of the receiver) is extracted. The cross-correlation between the experimental and numerical output signals is computed and the time lag that corresponds to its maximum is automatically identified. The time lag enables the definition of a new arrival time, based on which a new shear modulus estimate is computed. The process is repeated until a fixed point of the shear modulus is found within a user-defined tolerance. This shear modulus is delivered to the user as the likely shear modulus of the material.

GeoHyTE's computational architecture follows the model updating procedure presented above. It consists of six main GUI, complemented by a GUI for the definition of non-regular domains and meshes, and an optional visualization interface to assist with the definition of boundary conditions. Free sequential navigation is supported between interfaces, in both directions.

4.2 Validation

BE measurements were performed in a triaxial apparatus on a natural sand extracted from a borehole. A total of six isotropic consolidated drained triaxial tests were conducted on this sand. Each specimen, with a dimension of 70 mm diameter and height of 140 mm, was saturated (Skempton's B-value of at least 0.9) and subsequently consolidated. For the saturation phase, an isotropic effective stress around σ'= 20 kPa was applied to the geomaterial. For the consolidation phase, the effective stress was σ'= 100 kPa. The BE tests were performed at the end of the saturation and consolidation phases. For illustration, the input and output signals of the BE test after saturation are presented in Figure 6 for the case where the input signal is a 5.8 kHz sine pulse.

The output signal was loaded into the first GeoHyTE GUI and the tentative arrival time (magenta circle) and correlation window (blue rectangle) were defined as shown in Figure 7. GeoHyTE converged in two iterations to a shear modulus estimate of G_0 = 19.9 MPa (Figure 8), which is similar to the readings using two conventional techniques, namely the peak to peak and zero crossing methods (20.8 and 20.9 MPa, respectively). Similar analyses were performed for all six samples. All experiments were interpreted using the same conventional techniques and GeoHyTE. The shear moduli obtained with these techniques are presented graphically in Figure 9. The results obtained with all techniques are consistent for all tested situations.

5 CONCLUSIONS

CEN-DynaGeo was a research project (2018-2022), conducted by the Portuguese universities of Lisbon and Minho. Its objective was to use numerical models of BE experiments to improve their testing setup and automatize their interpretation. The models exploit favourable features of hybrid-Trefftz finite elements, mainly their wavelength insensitivity and the capacity to filter entire types of waves out from the solution.

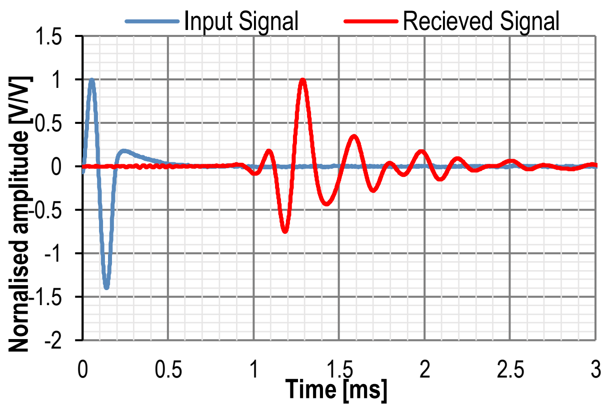

Figure 6. Normalised amplitudes of input and output signals (saturation phase)

A considerable stock of new hybrid-Trefftz models has been developed, implemented on the FreeHyTE platform, and published in open-source. They were used to fuel a new technique for the optimisation of the location of the receiver BE and to design moulds that scatter the (polluting) compression waves before they reach the receiver, thus yielding clearer output signals that are easier to interpret. Moreover, GeoHyTE, a new toolbox for the automatic, objective, and physically sound interpretation of the output signal was developed and its beta version is available for testing. Above all, the CEN-DynaGeo Project illustrated the advantages that can be gained by coupling experimental and numerical methods in BE testing of geomaterials.

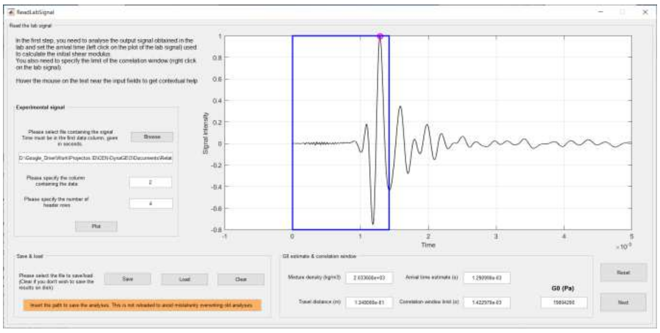

Figure 7. Initial GUI in GeoHyTE: preliminary analysis of the output signal

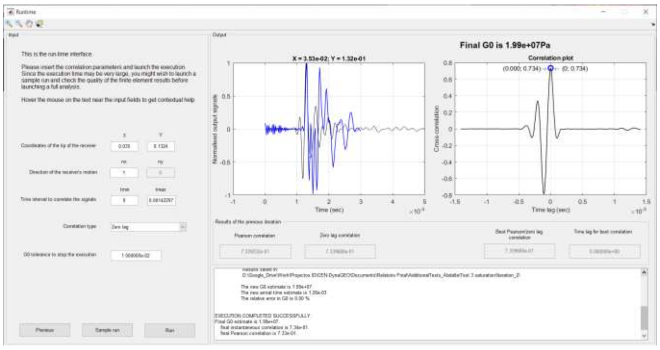

Figure 8. Final GUI in GeoHyTE: results after two iterations

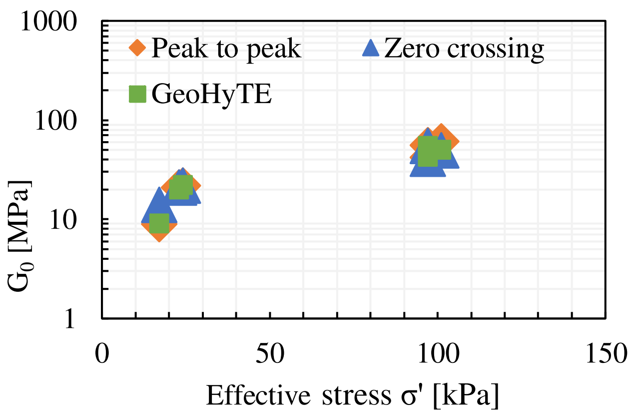

Figure 9. Shear modulus readings obtained with two conventional interpretation techniques and GeoHyTE

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