Proceedings 10th NUMGE 2023
10th European Conference on Numerical Methods in Geotechnical Engineering
Zdravkovic L, Kontoe S, Taborda DMG, Tsiampousi A (eds)

https://doi.org/10.53243/NUMGE2023-194

Constitutive modelling of rate-dependent shearing in combination with creep of frozen granular soils

U. Schindler[1], S. Chrisopoulos[1], R. Cudmani[1]

[1] *Department of Civil, Geo and Environmental Engineering, Chair of Soil Mechanics and Foundation Engineering, Rock Mechanics and Tunnelling, Technical University of Munich, Munich, Germany*

ABSTRACT: This study tests two versions of a novel constitutive model for frozen granular soils. The original model, initially proposed by Cudmani et al. in 2022, enables the combined description of shear and creep strength for predominantly monotonic loading. In 2023, Schindler et al. enhanced the original version to take account of stepwise loading and creep by coupling creep time with stress-strain history. While the rate-dependent shear and creep responses have been separately evaluated for both models, the present study uses both model approaches to compare and validate their ability to predict rate-dependent shearing in combination with single-stage and multi-stage creep conditions. For this purpose, uniaxial compression tests, coupled uniaxial single-stage creep tests with different load application speeds and uniaxial multi-stage creep tests with stepwise loading or unloading were conducted. The experiments were recalculated with the two constitutive model versions and compared. For predominantly monotonic loading, both model predictions are similar and satisfactorily agree with the experimental data. However, and as expected, for non-monotonic loading (e.g. multi-stage loading and unloading) the model version by Schindler et al. can more precisely reproduce the observed experimental behaviour. This finding highlights the potential of the enhanced model version to increase efficiency and safety in artificial ground freezing applications under general loading conditions.

Keywords: artificial ground freezing, shear strength, creep strength, frozen soil, constitutive modelling

1 INTRODUCTION

The construction of new urban tunnelling applications under complex as well as partially unknown geological and hydrogeological boundary conditions and the resulting high damage risk often requires the use of sophisticated subsoil improvement, water tightening and dewatering techniques. Artificial ground freezing (AGF) is an advanced and environmentally friendly construction technique which temporarily increases the stiffness and strength of the subsoil as well as provides water tightness. Thus, AGF is often preferable to other soil improvement techniques as demonstrated by its impressive use for the Brenner Base Tunnel project (Casini et al., 2023). Nevertheless, for the ultimate-limit state and service-limit state analysis of complex AGF applications, existing analytical solutions are often limited and may lead to economically unbalanced designs. To tackle this limitation, advanced constitutive models have been developed during the last few decades to describe the complex rate-, stress- and temperature-dependent mechanical behaviour of frozen soils. In fact, most of these models can only capture either the shear or creep behaviour. However, from a practical point of view it is often necessary to evaluate the combination of shear and creep processes of frozen soils. For instance, a tunnel excavation supported by a frozen soil body initially leads to shearing of the frozen soil. Subsequently, the frozen soil body creeps at a nearly constant stress level until the completion of the permanent tunnel support. The different frozen soil shear and creep characteristics influence each other and thus its ultimate shear and creep strength depends on the previous and complex interaction history. Studies by e.g. Schindler et al. (2023) and Staszewska (2022) reveal the influence of the stress-strain history on the rate- and temperature-dependent behaviour of frozen granular soils based on single-stage as well as multi-stage creep tests. In the single-stage creep tests, the load was increased monotonically to the desired value and then maintained at a constant. In the multi-stage creep tests, the same final load was applied as in the single-stage creep tests, though this was achieved stepwise. The authors found that after the well-known initial strain rate decrease (primary creep), the minimum creep strain rate $\dot{\varepsilon}_{\mathrm{m}}$ (secondary creep) is mostly independent of the loading history. In contrast, the corresponding frozen soil lifetime $\mathrm{t_m}$, after which tertiary creep (strain rate increase) starts and creep failure occurs, depends on the loading history. For instance, $\mathrm{t_m}$ becomes longer in stepwise loaded creep tests in relation to their equivalent single-stage creep tests. Hence, these and other examples clearly indicate the important requirement for advanced constitutive model to describe the coupled shear-creep behaviour of frozen soils rather than capturing the shear or creep behaviour separately, as often found in the literature. In this study, two different model versions of a novel constitutive model framework for frozen granular soils was extensively tested in terms of its coupled shear-creep response based on a high-quality experimental database.

2 TESTING MATERIAL

This study introduces a comprehensive lab testing programme, including uniaxial compression tests, single-stage as well as multi-stage creep tests at a temperature of -10 °C. The tests were performed with frozen Karlsruhe sand. The coarse to medium sand tested is extensively described in Schindler et al. (2023) and the essential characteristics of the cylindrical specimens are summarised in Table 1. In addition, the testing procedure of the different testing types is explained in detail in the corresponding sections and will not be repeated here.

Table 1. Sample characteristics and physical properties

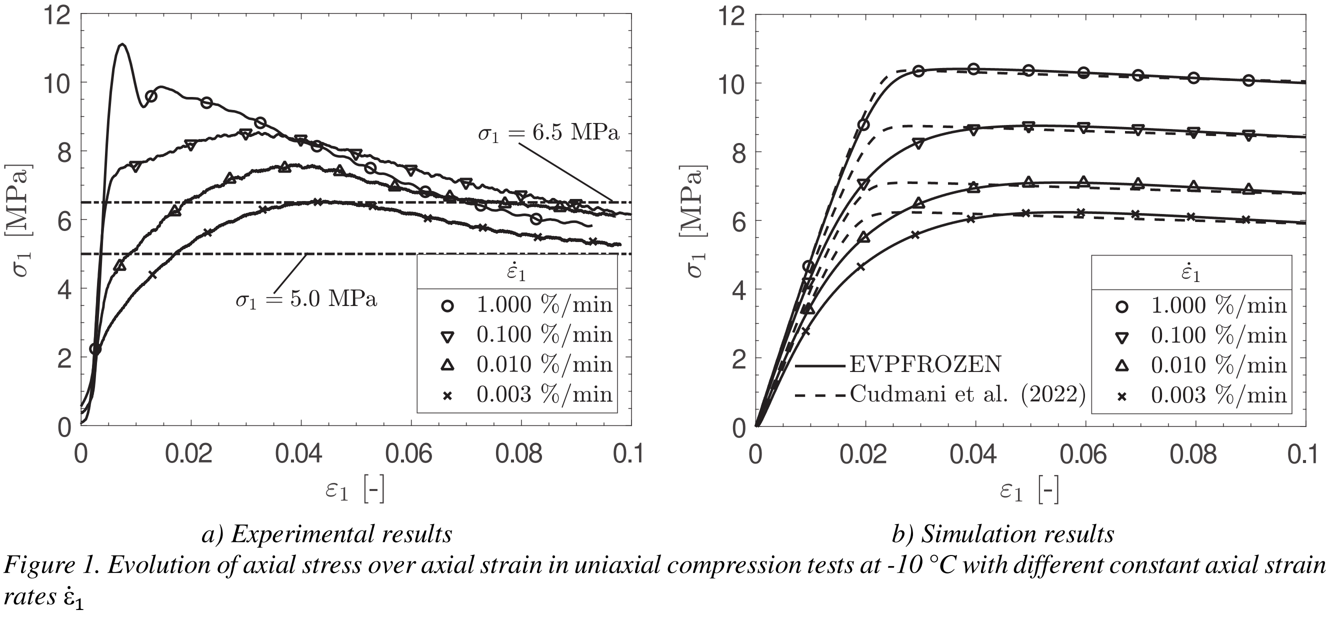
diameter [mm]	**height [mm]**	**water content [-]**	**dry unit weight [g/cm³]**	**Saturation [%]**
~ 50	~ 100	~ 0.20	~ 1.61	~ 87

3 CONSTITUTIVE MODEL BY CUDMANI ET AL. (2022) AND SCHINDLER ET AL. (2023)

Cudmani et al. (2022) proposed a constitutive model for frozen granular soils able to describe the rate-, stress- and temperature-dependent mechanical behaviour for predominantly monotonic loading. The model takes into account the influence of the confining pressure and differentiates between compressive and tensile strength and creep. In the following, the Cudmani et al. (2022) model is referred to as the original model version. We extended the original version for multi-stage creep by considering the influence of the loading history on the creep behaviour of frozen soils (Schindler et al., 2023). This enhanced model version, designated by the acronym EVPFROZEN, is able to capture the equivalent creep time after a frozen soil stress state change by coupling and transforming the creep time to the previous loading history. By contrast, in the model version developed by Cudmani et al. (2022), the creep time is taken to be equal to the global time. For more details on both constitutive model versions, e.g. their basic equations, see Cudmani et al. (2022) and Schindler et al. (2023). The calibration process and the material constants for frozen Karlsruhe sand have already been introduced in Cudmani et al. (2022) and Schindler et al. (2023) and will not be presented here. Next, we will compare both model versions in terms of their shear and creep responses and discuss important differences.

4 UNIAXIAL COMPRESSION TEST

In order to evaluate the models' shear responses, four uniaxial compression tests with different constant axial strain rates were performed at -10 °C. Figure 1 presents the experimental and numerical results of these tests.



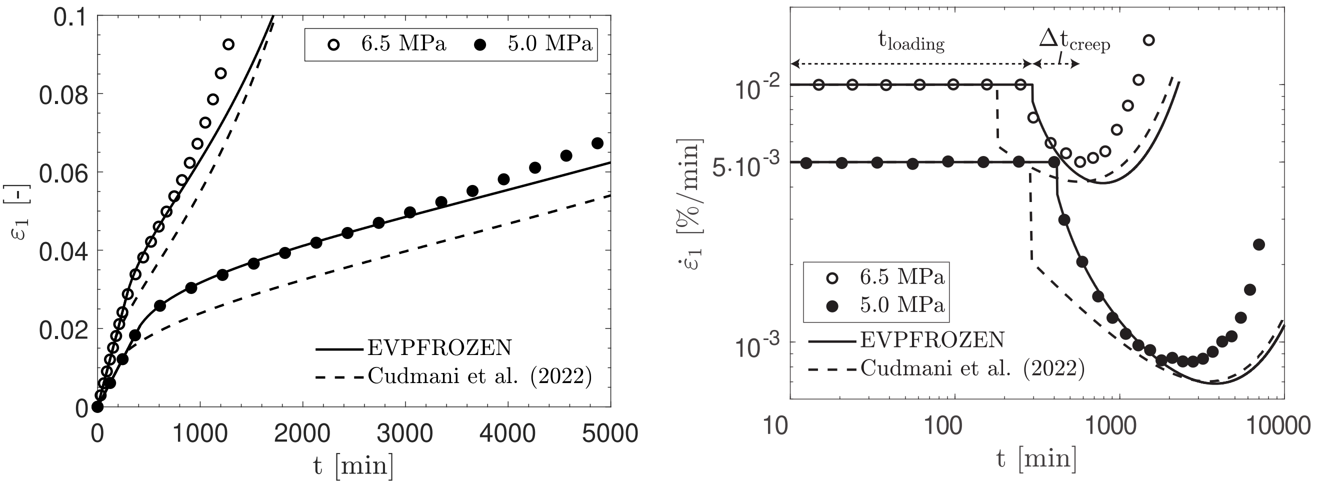
a) Experimental results *b) Simulation results*

Figure 1. Evolution of axial stress over axial strain in uniaxial compression tests at -10 °C with different constant axial strain rates $\dot{\varepsilon}_1$

The experimental results in Figure 1a illustrate the well-known rate-dependent shear behaviour of frozen soils as the ultimate shear strength increases with increasing axial strain rate. The predicted ultimate shear strength values in Figure 1b agree well with the experiments for both model versions. However, EVPFROZEN is more accurate in terms of the corresponding axial strain deformations than Cudmani's version. As already discussed by Cudmani et al. (2022), using their version reproduces a stiffer frozen soil behaviour before reaching the peak than measured in the tests. Consequently, and as can be seen in Figure 1b, the predicted axial strain (dashed lines) required to achieve the peak is mainly smaller in the simulations than in the experiments. Moreover, this axial strain is mostly independent of the axial strain rate when using the original version. In contrast, EVPFROZEN (solid lines in Figure 1b) predicts a more ductile behaviour with decreasing axial strain rate for the ultimate shear strength, which is in accordance with the measured axial strain. The axial strain corresponding to the ultimate strength is reached more slowly. The implementation of a transformed creep time in EVPFROZEN provides an improved nonlinearity stiffness response for the shear behaviour. However, note that both model versions still clearly underestimate the stiffness for a relatively fast axial strain rate of 1 %/min. In addition, the observed shear strength softening after reaching its peak value is also not well reproduced by the models. This should be taken into account for the model use in AGF applications, even though the order of strain rate magnitude in in-situ shear processes is mostly significantly smaller, for which the models' predicted deformation behaviour is in good agreement with the measured ones. To sum up, both model versions satisfactorily capture the essential characteristics of the shear behaviour of frozen soil and are thus suitable for AGF applications in which shear failure plays an important role.

5 UNIAXIAL SINGLE-STAGE CREEP TESTS

As discussed in Section 4, the ultimate shear strength of frozen soils is rate-dependent. Hence, the time- and stress-dependent creep strength of frozen soils also depends on its previous shear history (load application speed) since the creep strength is obviously limited to its ultimate shear strength. For instance, and as shown in Figure 1a, in the experiments, uniaxial creep deformations for a creep stress of $\sigma_1 = 6.5$ MPa are observable, only if $\dot{\varepsilon}_{1,\mathrm{loading}}$ is higher than 0.003 %/min. Otherwise, shear failure occurs first and no creep strength exists for this intended stress level. In addition, relatively slow load application speeds reduce the maximum creep time period $\Delta t_{\mathrm{creep}} = t_{\mathrm{m}} - t_{\mathrm{loading}}$ until the lifetime t_m of the frozen soil is reached (see Figure 2b). From a practical point of view, it is important that sophisticated constitutive models for frozen soils capture these coupled shear-creep characteristics. In order to validate this, we simulate uniaxial single-stage creep tests under different load applications speeds. Figure 2 compares both model responses with each other and single-state creep tests at 5.0 and 6.5 MPa with two different load application speeds $\dot{\varepsilon}_{1,\mathrm{loading}}$. Note that the plotted experimental results include the total strain evolution (Figure 2a) and the total strain rate evolution (Figure 2b).



a) *b)*

Figure 2. Evolution of axial strain (a) and axial strain rate (b) over time in uniaxial single-stage creep tests at -10 °C with $\dot{\varepsilon}_{1,loading} = 0.01\ \%/min$ (open symbols) and $\dot{\varepsilon}_{1,loading} = 0.005\ \%/min$ (filled symbols). Experiments: symbols. Simulations: lines.

In general, the time-dependent axial strain evolution in Figure 2a is in good accordance with the two experiments for both model versions, even though EVPFROZEN is more accurate than Cudmani's version. The same applies to the strain rates in Figure 2b, which first decrease (primary creep) and then increase (tertiary creep) with time for both the experimental and numerical results. In particular, the predicted minimum axial strain rates $\dot{\varepsilon}_{\mathrm{m}}$ in Figure 2b agree with those measured. Moreover, studies like (Schindler et al., 2023) and (Staszewska, 2022) provide evidence that $\dot{\varepsilon}_{\mathrm{m}}$ is mostly independent of the previous stress-strain history and thus both versions predict the same $\dot{\varepsilon}_{\mathrm{m}}$. Nevertheless, the corresponding lifetime t_m (testing time at which $\dot{\varepsilon}_{\mathrm{m}}$ is reached) differs in the two simulations. As already discussed in Section 4, Cudmani's model predicts a stiffer shear behaviour than EVPFROZEN. Hence, in Figure 2b, the Cudmani version's response results in a shorter load application time until the intended creep stress level is reached. This is indicated by the sudden drop of $\dot{\varepsilon}_1$ after its previous constant period in Figure 2b. Consequently, the creep stage in Cudmani's model simulation (dashed lines) begins earlier and the total time until reaching t_m is also shorter since it is shifted by nearly the same time difference. On the one hand, the use of extended model version EVPFROZEN may result in longer lifetimes of the frozen soil and thus improves on AGF designs economically. On the other hand, Cudmani's model version response is stiffer for the load applications and therefore predicts an earlier lifetime reach, which from a practical point of view is on the safe side. Nevertheless, both model version are able to capture the coupled shear-creep behaviour for predominantly monotonic loading, which is an essential feature for advanced AGF designs.

6 UNIAXIAL MULTI-STAGE CREEP TESTS

6.1 Loading paths

The influence of the loading history on the mechanical behaviour of frozen granular soils was experimentally investigated both for uniaxial stepwise loaded as well as stepwise unloaded creep. Figure 3 describes the testing procedure of the different uniaxial multi-stage creep tests. The multi-stage creep tests were conducted at -10 °C and consist of an initial loading and then one or two loading/unloading stages. After the initial loading stage, the second or third load change occurred after an axial strain $\varepsilon_1 = 2.5$ % and $\varepsilon_1 = 3.5$ %, respectively. In each step, the load was maintained at a constant, and the creep deformations of the frozen soil samples were monitored.

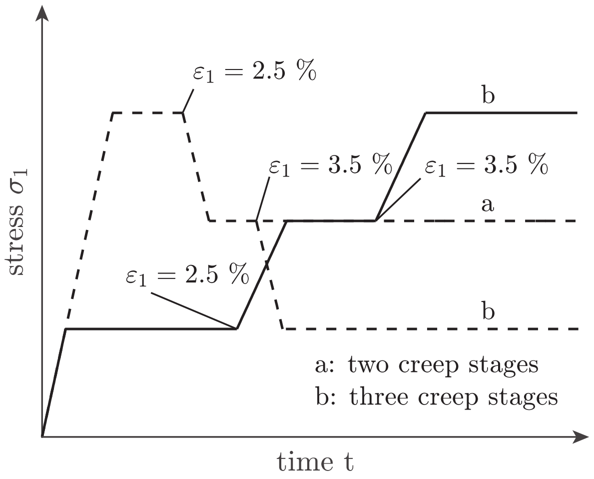
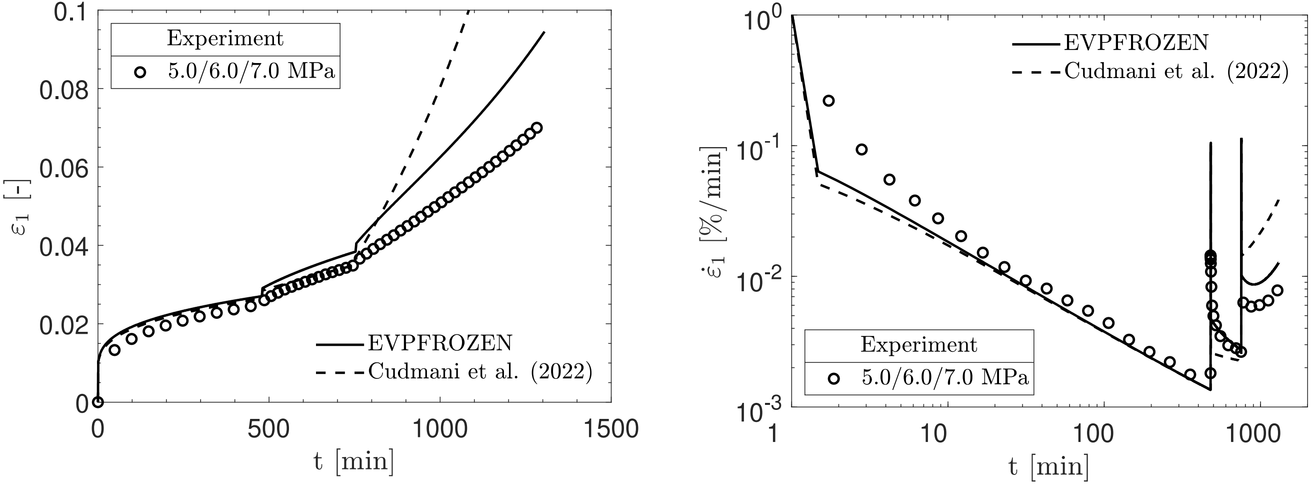


Figure 3. Experimental procedure for uniaxial multi-stage creep tests with stepwise loading (solid lines) and stepwise unloading (dashed lines)

6.2 Stepwise loading

Figure 4 compares the experimental and numerical results of a multi-stage creep test with three loading stages, viz. $\sigma_1 = 5.0$ MPa, $\sigma_1 = 6.0$ MPa and $\sigma_1 = 7.0$ MPa.



a)

b)

Figure 4. Evolution of axial strain (a) and axial strain rate (b) over time in uniaxial multi-stage creep tests with three loading stages at -10 °C. Experiments: symbols. Simulations: lines

For the first two creep stages, both models' predictions are similar and agree well with the experimental data, especially for the time-dependent strain evolution in Figure 4a. However, a first underestimation of the axial strain rate evolution (dashed line in Figure 4b) already occurs using Cudmani's model version for the second creep stage at $\sigma_1 = 6.0$ MPa when compared to EVPFROZEN and the experiment. Despite the actual continuous steep decrease of $\dot{\varepsilon}_1$, Cudmani's model reproduces a nearly constant strain rate, indicating the imminent arrival at lifetime t_m. This trend and the resulting essential difference between the two model versions becomes even more evident for the third and final loading stage. Despite the observed initial decrease of $\dot{\varepsilon}_1$ in the experiments, the original model directly predicts a strong axial strain rate increase corresponding to tertiary creep. In contrast, EVPFROZEN appropriately reproduces the initial ongoing primary creep phase directly after the third load increase. Subsequently, both the arrival at lifetime and the axial strain rate increase can be well reproduced by EVPFROZEN. In order to further evaluate the different model responses for multi-stage loaded creep, we performed additional creep tests at -10°C with two loading stages including different total stresses (σ_1 from 6 up to 9 MPa) and different incremental stress increases ($\Delta\sigma_1 = 1.0$ or 2.0 MPa). Figure 5 illustrates these experimental results and their back-calculations.

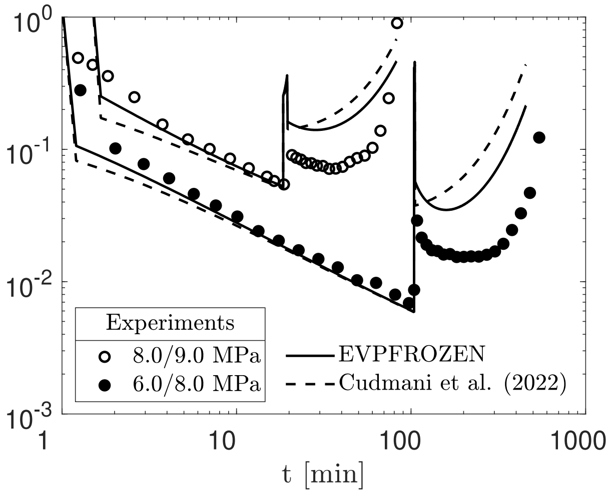


Figure 5. Evolution of axial strain rate over time in uniaxial multi-stage creep tests with two loading stages at -10 °C.

As can be seen, the multi-stage creep simulations with two load steps confirm the already observed essential differences in Figure 4 in predicting the lifetime (t_m) between both model versions. Cudmani's model directly predicts tertiary creep after the last load increase, while the EVPFROZEN model response at first results in decreasing axial strain rates. This is followed by reaching the minimum axial strain rate $\dot{\varepsilon}_m$ and the frozen soil lifetime t_m. Later on, $\dot{\varepsilon}_1$ increases since tertiary creep occurs. Here, the EVPFROZEN model response is in good accordance with the experimental measurements. In summary, the multi-stage creep comparison for stepwise loading revealed the influence of the loading history on the creep behaviour. In the original model, the creep time is coupled with the global time. This assumption may lead to an unexpected increase in the strain rate after an increasing stress state at a constant temperature. For instance, and as schematically shown in Figure 7, t_m can suddenly become smaller than the creep time t (total time) after a stress change using the original model. Hence, Cudmani's model may immediately predict increasing axial strain rates, which clearly differs from the essential mechanical behaviour frozen soils observed in the lab tests and studies from the literature. In contrast, the EVPFROZEN model with its coupled transformed creep, which is independent of the global time, precisely reproduces the rate-dependent evolution of the axial strain rate both for different numbers of load steps as well as for varying incremental stress increases and total stress levels.

6.3 Stepwise unloading

This section deals with the multi-stage creep responses of both models upon unloading (see dashed curves in Figure 3). For this purpose, a uniaxial multi-stage creep test with two stepwise unloading stages after an initial loading phase was performed at -10 °C. Figure 6 shows the comparison of the experimental and numerical results. Note that the non-continuous curves in Figure 6 are related to the short-term negative value of $\dot{\varepsilon}_1$ occurring at the beginning of each unloading stage, which is not shown in the scaling.

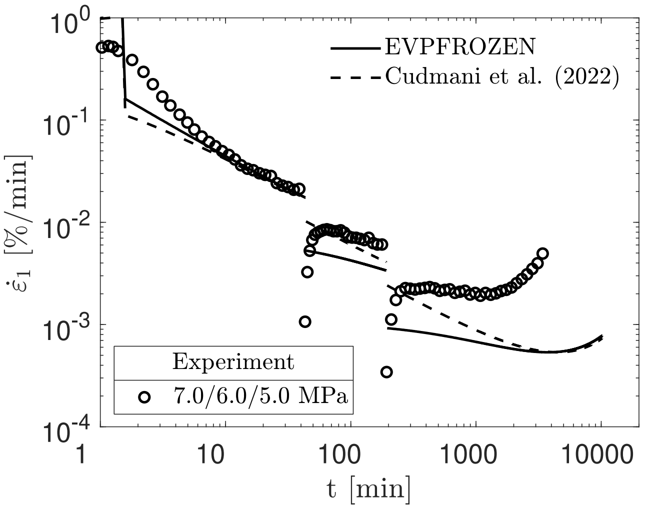


Figure 6. Evolution of axial strain rate over time in uniaxial multi-stage creep tests with two unloading stages at -10 °C.

As can be seen in Figure 6, there are only slight differences in the model response for both models. The axial strain rate evolution is more accurate for EVPFROZEN when compared to the experimental one. Here, $\dot{\varepsilon}_1$ only decreases slightly after the first and second load decrease while Cudmani's model predicts a more pronounced axial strain rate reduction. However, both simulations underestimate the measured axial strain rate evolution. In addition, they overestimate the frozen soil lifetime t_m compared to the experiment, which is not on the safe side. From a practical and safety point of view, both observed model deviations from the experimental measurements upon unloading should be taken into account since the model responses may not be on the safe side. Nevertheless, an essential model difference as observed in stepwise loaded creep (see Section 6.2) cannot be found here. This is in any case expected since the predicted lifetime t_m due to unloading always increases at steady-state temperature conditions. Consequently, both model versions keep predicting decreasing axial strain rates during primary creep independent of the incremental stress reduction. A direct jump to tertiary creep as observed during multi-stage loaded creep in Section 6.2 can be theoretically ruled out for both model versions. However, theoretically, we expect a switch from tertiary to primary creep after unloading in Cudmani's model based on its constitutive formulation. Figure 7 further explains this phenomena. Moreover, it summarizes the model response differences between the original version and EVPFROZEN for multi-stage loading and unloading in accordance with their constitutive formulation in terms of the creep time.

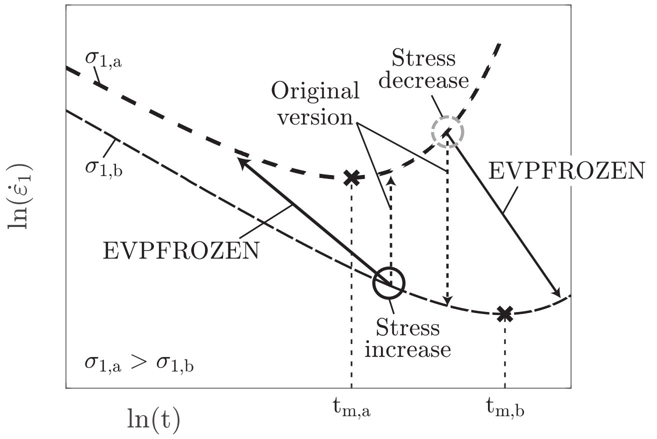


Figure 7. Model response differences for multi-stage loading or unloading at a constant temperature. Original model version after Cudmani et al. (2022); EVPFROZEN after Schindler et al. (2023).

To experimentally and numerically validate the theoretical model differences upon unloading shown in Figure 7, a second multi-stage creep test was performed but with only one instead of two unloading steps and at high stress levels so that the lifetime of the frozen soil is relatively small. The test result, together with its back-calculation, is illustrated in Figure 8.

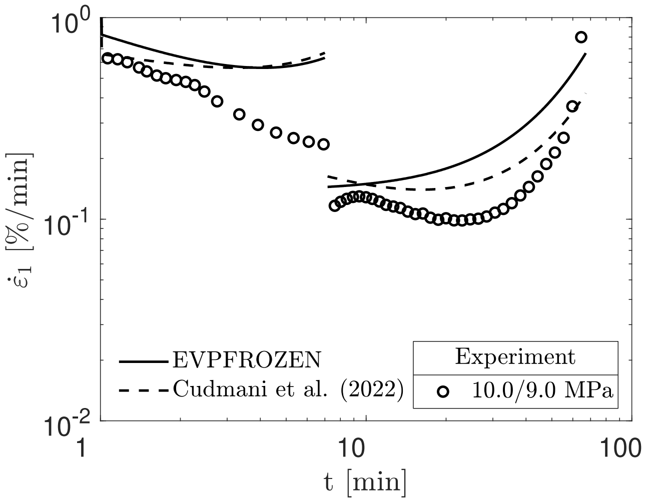


Figure 8. The same as in Figure 6 but for one unloading stage

In this case, both model versions overestimate the axial strain rate evolution in comparison to that measured. Moreover, they already predict increasing strain rates and thus underestimate the frozen soil lifetime t_m at the first loading stage. This is not in accordance with the experiment, even though the prediction is on the safe side. However, the essential and important finding here is the unrealistic switch of an increasing $\dot{\varepsilon}_1$ at the end of the first loading step to an initial decrease at the beginning of the second loading step in Cudmani's model simulation (dashed lines). In contrast, the EVPFROZEN model response for the second loading stage only includes a continuous strain rate increase. From a theoretical point of view, this behaviour is expected since creep failure indicated by the lifetime t_m cannot be reversed. Moreover, in Cudmani's model the creep time (equal to the global time) may appear falsely smaller than the lifetime after a stress decrease. This is not on the safe side as the model prediction may lead to an unsafe AGF design for unloading construction stages.

7 CONCLUSIONS

In this study, two different versions of a novel constitutive model for frozen granular soils were extensively tested for combined rate-controlled loading and creep. Both models are able to capture the rate-, stress- and temperature-dependent frozen soil behaviour for predominantly monotonic loading, as shown by the back-calculation of uniaxial compression and single-stage creep tests at constant temperatures. Nevertheless, essential differences between the models emerged for multi-stage creep after loading and unloading. Here, the original model proposed by Cudmani et al. (2022) results in an erroneous prediction of the frozen soil lifetime reach. In cases of stepwise loaded creep, Cudmani's model predicts creep failure too early, resulting in less efficient AGF designs. In addition, for stepwise unloaded creep, it may even predict decreasing strain rates (primary creep) after unloading, even though the frozen soil lifetime has already been reached earlier. In contrast, the enhanced EVPFROZEN model by Schindler et al. (2023) precisely captures the influence of the loading history on the frozen soil lifetime both for stepwise loaded and unloaded creep. Moreover, it did not falsely predict early arrival at lifetime nor a sudden switch between primary and tertiary creep after a stress change was observed in the simulations. Consequently, the EVPFROZEN model version is preferable for AGF designs in which varying stress states of the frozen soil body are expected.

8 REFERENCES

Casini, F., Guida, G., Restaini, A., & Celot, A. (2023). Water Retention Curve–Based Design Method for the Artificial Ground Freezing: The Isarco River Underpass Tunnels within the Brenner Base Tunnel Project. *Journal of Geotechnical and Geoenvironmental Engineering*, 149(3), 04023007. doi: 10.1061/JGGEFK.GTENG-10723.

Cudmani, R., Yan, W., and Schindler, U. (2022). A constitutive model for the simulation of temperature-, stress- and rate-dependent behaviour of frozen granular soils. *Géotechnique*, doi:10.1680/jgeot.21.00012.

Schindler, U., Cudmani, R., Chrisopoulos, S., and Schünemann, A. (2023). Multi-stage creep behaviour of frozen granular soils: Experimental evidence and constitutive modeling. *Canadian Geotechnical Journal*. doi: 10.1139/cgj-2022-0637.

Staszewska, K. (2022). Towards a constitutive description of creep in frozen soils. *PhD thesis*, Gdańsk University of Technology. doi: 10.13140/RG.2.2.35977.11364.