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# Response surface based probabilistic studies on static liquefaction failure of tailings dams

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**ABSTRACT:** Tailings are a mixture of sand, silt, clay, metal, chemical reagents, and processing water, employed during metal extraction, usually stored in a Tailings Storage Facility retained by dams. Commonly, tailings are deposited in a loose saturated state for which, even a small perturbance in undrained condition can cause a progressive failure, often referred to as static liquefaction. In this work, stability evaluation and reliability analysis of tailings dams were investigated through the Response Surface Method. The response surface was developed based on finite element simulations with a simple dam geometry. The finite element simulation employed NorSand constitutive model that can capture softening behaviour. The developed response surface evaluates factor of safety based on the relative density of tailings layers and ground water level. Response surface and the first/second-order reliability methods were coupled to calculate the failure probability. The main outputs of such analyses are the failure probability and sensitivity factors for the various random variables that quantify the uncertainty in input parameters. The latter provides insight on how the uncertainty in relative density affects the probability of failure.

**Keywords:** Tailings Dam; Strain Softening; Response Surface; Failure Probability; Static liquefaction

## 1 INTRODUCTION

The mining industry produces very large volumes of "waste" each year, i.e., rock dumps, tailings and other waste. Tailings from mining operations are often deposited hydraulically, as a mix of solids and water (slurry) in ponds. To retain hydraulic placed tailings material, dams and embankments combined with natural confinement are constructed. The hydraulic placement typically leads to deposits being in a loose state and prone to liquefaction. There are several recent examples of tailings dam failures where static or dynamic liquefaction of the tailings has been either the direct cause, or a consequence, of the failure. Static/flow liquefaction is associated with undrained softening behaviour. For a loose saturated material, undrained softening with a significant loss of strength occurs after the peak load is reached (contractive behaviour). There are several potential static liquefaction triggers as listed below (Davies et al. 2002):

- Change in pore pressures induced by an increase in pond levels.
- Excessive rate of loading due to rapid raising of the impoundment.
- Removal of toe support.
- Rapid foundation movements enough to create undrained loading in tailings material.

This highlights that loose saturated tailings material is in a highly unstable state and even a small trigger can lead to a progressive failure due to strain softening. Traditional limit equilibrium analysis, which is commonly used to access the stability of tailings dam, can overlook this kind of failure. Finite element (FE) framework coupled with a suitable constitutive model that can capture strain softening behaviour and hence, the initiation of static liquefaction in tailings dams and provide an estimate of factor of safety (FoS) against failure. But significant uncertainties exist in the input parameters because of existing conditions on field and in the way we calculate the factor of safety against failure. In such cases, risk and probability based approaches provides more insight than deterministic analyses. Response surface coupled with Second order reliability method (SORM) is adopted in this work. The probabilistic methods will supplement deterministic methods by providing additional information on the effects of uncertainties on safety of tailings dam. With the implementation of ISO 31000 risk management protocol to management of tailings dams, the need for a framework to enable probabilistic study of static liquefaction failure of tailings dam grows further (Cruz and Rodovalho, 2019).

Currently, most geotechnical numerical programs do not have the probabilistic analysis function. In such scenarios, the response surface method (RSM) is increasingly useful for reliability analysis because it can integrate numerical stability evaluation and reliability analysis.

This paper presents a probabilistic approach where the physical behaviour of a simple tailings dam is represented by response surface (RS), and first-order reliability method (FORM). The RS is developed based on numerical simulations carried out using the commercially available finite element software PLAXIS. The combined FE simulation and RS-FORM analysis provide a deeper insight on effect of uncertainty in input parameters on FoS and demonstrates the capacity of a computationally efficient framework to calculate failure probability of tailings dam.

## 2 NUMERICAL SIMULATIONS

As stated earlier, tailings when mobilized to a shear stress that is higher than the undrained shear strength, are in an instable state and a small trigger can cause a progressive failure by strain softening, often referred to as static liquefaction. This type of failure is not captured by traditional limit equilibrium analyses.

### *2.1 Tailings dam geometry*

An idealised geometry rather than a real slope of the tailings dam was numerically simulated as presented in Figure 1 (Reid and Fourie, 2023). The slope was 50 m high with 1V:3H side slope. A 10m wide bench was included at a height of 25m, making the overall average slope 1V:3.2H. An initial phreatic line commencing from 30 m upstream of the toe till 26 m height as shown in Figure 1, was considered. The initial conditions mimic a tailings dam at the end of deposition. However, subsequent events such as recommissioning of the facility, or a degradation of surface water management structures can lead to a rise in the phreatic surface.

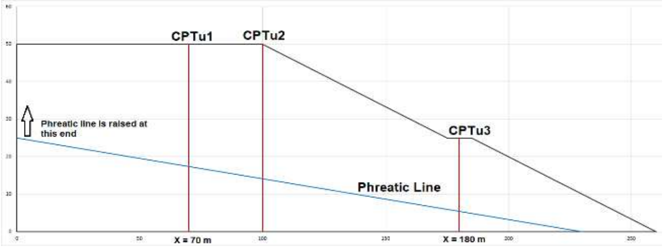

*Figure 1. Geometry of the tailings dam with initial phreatic condition*

### *2.2 Tailings dam soil profile*

Publicly available data from Cadia and Fundão tailings dam failure was utilised to define the material parameters for the tailings material. Three CPT data profiles were considered, and they were scaled to represent saturated and unsaturated conditions below and above phreatic line respectively. Further details can be found in Reid and Fourie (2023). Synthetic CPTu data was not used to ensure that realistic tailings material behaviour such as, contractive undrained shearing and brittle strain softening are captured. The CPT data were used to interpret the in-situ density in terms of the state parameter ($\psi = e - e_c$, where e is the in-situ void ratio and $e_c$ is the critical void ratio at same mean pressure p). CPTU correlations after Been and Jefferies (1992), Plewes et al. (1992) and Robertson (2016) were employed to determine $\psi$ and are as depicted in Figure 2. It is to be noted that a bulk unit weight of $\gamma_{unsat} = 20$ kN/m3 above phreatic surface and $\gamma_{sat} = 22$ kN/m3 below phreatic line was considered alongside $K_0 = 0.7$. The interpreted state parameter from various correlations as function of depth is presented in Figure 2. It is evident that there is a huge scatter in the results, and this served as the strongest motivation to carry forward the work with a probabilistic approach. State parameter evaluated based out of the correlation after Jefferies and Been (1992) was chosen and the dam was discretized into 4 layers based on the state parameter as depicted in Figure 3. The mean and standard deviation for each layer was evaluated as in Table 1. The scatter in the data of layer 1 is high, making it difficult to determine whether is contractive or dilative. This reemphasises the need to analyse tailings dam in a probabilistic framework.

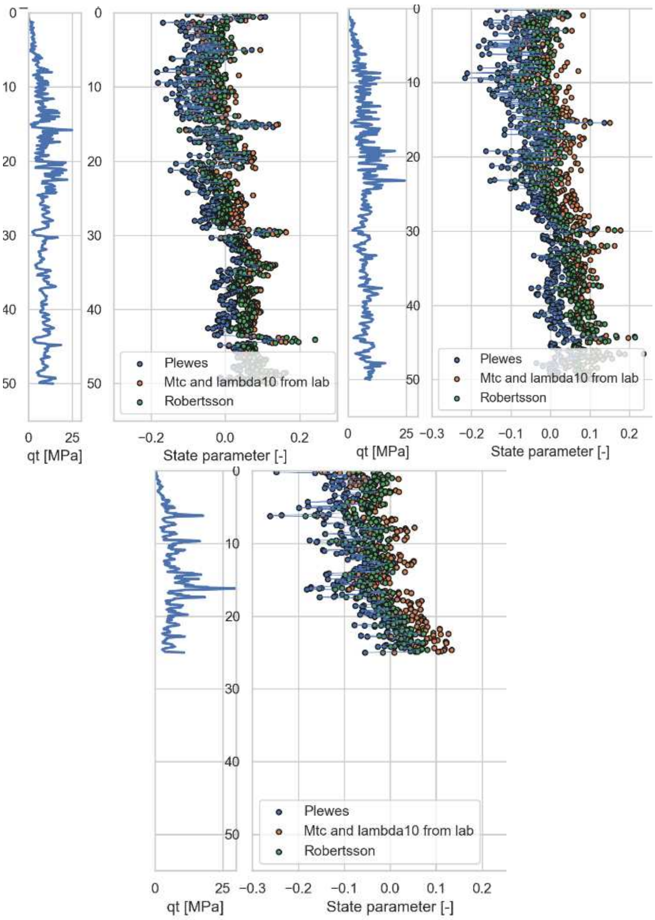

*Figure 2. State parameter interpreted from CPTu1 (left) CPTu2 (middle) and CPTu3 (right) using different methods.*

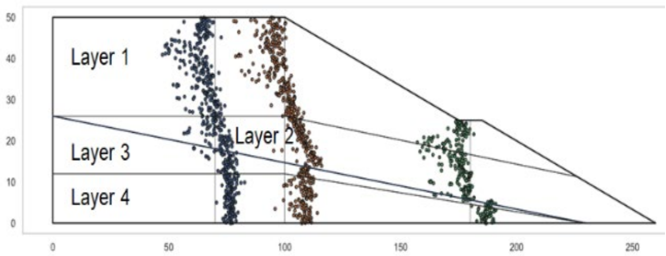

*Figure 3. Layering of tailings dam based on state parameter evaluated as per Jefferies and Been (1992)*

*Table 1. Mean and standard deviation of state parameter for each layer*

| Layer | Mean ψ (M) | Std. Dev. Ψ (D) |
|---|---|---|
| Layer 1 | -0.01756 | 0.04513 |
| Layer 2 | 0.05206 | 0.03794 |
| Layer 3 | 0.07968 | 0.02977 |
| Layer 4 | 0.07968 | 0.02977 |

### 2.3 *Constitutive model for tailings material*

The constitutive behaviour of the tailings material was modelled using the NorSand model (Jefferies, 1993), which is an elastoplastic model based on the critical state theory and is well suited to capture tailings material behaviour (Been, 2016). NorSand model requires initial state parameter as model input to define the initial state of tailings material. The NorSand model parameters for the tailings material are a mixed data set of critical state parameters based out of the Fundão and Cadia Panels report and are as tabulated in Table 2. It is to be noted that barring the state parameter for the 4 layers, the remaining other NorSand model parameters are held constant across all the simulations.

*Table 2. NorSand model parameters for tailings*

| Parameter | Value |
|---|---|
| $\Gamma$ | 0.92 |
| $\lambda_e$ | 0.021 |
| $M_{tc}$ | 1.45 |
| $\varphi'_{cs}$ | 36 |
| $N$ | 0.3 |
| $\chi$ | 6.3 |
| $H$ | 125-690 ψ |
| $G_0$ | 50 MPa |

### 2.4 *Numerical modelling in PLAXIS*

The stability analyses were carried out in Plaxis 2D, using the NorSand model. The model was divided into four layers to account for the variability in state parameter and variation of phreatic line as depicted in Figure 4. The bottom boundary of the model is completely fixed, and the vertical boundaries are fixed in the lateral direction. The model is discretised with 15 noded triangular elements. The phreatic line for the initial case is defined as 26 m on the left boundary of the model as shown in Figure 4. As described before the NorSand model was used to define tailings material behaviour.

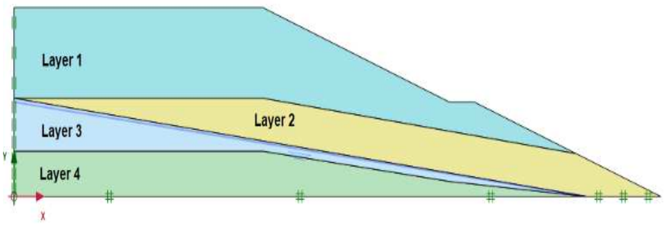

*Figure 4. PLAXIS 2D model of tailings dam with 4 layers and phreatic line*

The simulation was segmentized to the following phases:

- Initial Phase: The initial stresses are initiated under $K_0$ conditions. The procedure calculates the initial vertical stresses based on depth below surface and unit weight of soil. It then calculates the horizontal stresses based on $K_0$ parameter. Equilibrium is not calculated.
- Equilibrium Phase: As the ground surface is not horizontal $K_0$ procedure will lead to the existence of unbalanced forces or non-equilibrium of initial forces within the soil body, which is obviously not correct. In such cases, to maintain equilibrium, shear stresses develop within the soil body. Therefore, the $K_0$ step is followed by a gravity loading procedure in PLAXIS, which calculates shear stresses and ensures equilibrium.
- Drained Phase: All the soil layers are drained.
- Undrained Phase: The soil layers below the phreatic line are classified to be undrained and the system is checked for equilibrium.
- Perturbation Phase: The system is subjected to an increase in loading to mimic any sudden small perturbation which can render the system to behave under undrained conditions. In this paper, this is simulated by gradually increasing the gravity load on the system.

It is important to note that tailings can exhibit a sudden switch from drained to undrained behaviour following that even a small perturbation can lead to failure as discussed earlier. Hence, safety calculations under completely drained conditions would never be able to capture this static liquefaction failure condition. In our work we apply additional gravity to mimic this perturbation and the additional gravity (above 1) that is required to lead the dam to failure is termed as FoS henceforth in this paper.

### 2.5 *Numerical model results: static liquefaction*

Simulation results could capture static liquefaction dam failure by usage of a suitable constitutive model. Dam failure was triggered by a perturbation consisting of an increase in the dam gravity by a small portion (less than 5%) under undrained condition as described earlier. Simulation was able to capture the softening behaviour (shear strain increases with decreasing shear stress due to the shear-induced pore water pressure) of tailings material, which is key to simulate static liquefaction (Figure 5). Simulation results help us identify the initial failure triggering point located near the toe depicted as A in Figure 5. The corresponding stress-strain behaviour of triggering point is described in Figure 6, and it depicts that strain softening is observed immediately after the material state is switched from drained to undrained. The reduction of mean effective stress p' for point A due to pore water pressure increase leads it to the failure surface.

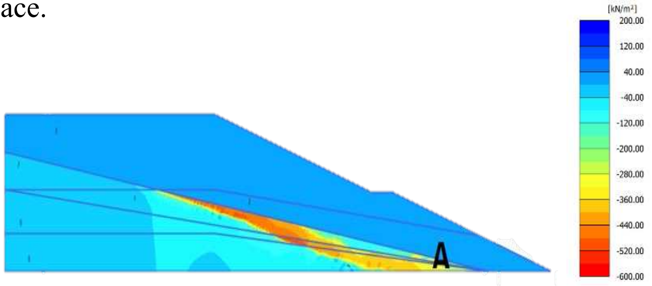

*Figure 5. Excess pore pressure contours depicting liquefaction and softening at toe of the tailings.*

*Table 3. FoS from simulations for various values of state parameters of 4 layers and location of phreatic line*

| **Layer/Phreatic Line** | **Distribution** | **FoS** |
|---|---|---|
| 1 (ψ)/2 (ψ)/3 (ψ)/4 (ψ)/ Y (m) | (M)/M/M/M/Y | 1.051 |
| " | (M + 0.75 D) /M/M/M/Y | 1.048 |
| " | (M + 1.5 D)/M/M/M/Y | 1.044 |
| " | M/(M + 0.75 D)/M/M/Y | 1.055 |
| " | M/(M + 1.5 D)/M/M/Y | 1.051 |
| " | M/M/(M + 0.75 D)/M/Y | 1.047 |
| " | M/M/(M + 1.5 D)/M/Y | 1.042 |
| " | M/M/M/(M + 0.75 D)/Y | 1.027 |
| " | M/M/M/(M + 1.5 D)/Y | 1.009 |
| " | M/M/M/M/ (Y-12 m) | 1.078 |
| " | M/M/M/M/ (Y+12 m) | 1.031 |

M = Mean; D = Standard Deviation from Table 1;
Y = 26 m

Scenarios were simulated varying the state parameter of tailings layers and the corresponding FoS for each case is as tabulated in Table 3. It is to be noted that the state parameters for each scenario were chosen based on mean and standard deviation for each layer as tabulated in Table 1. The elevation of the phreatic surface at the left end of the model was varied as tabulated in Table 3. To quote an example (M + 0.75D)/M/M/M/Y means state parameter in layer 1 is at mean + 0.75 times standard deviation, state parameters of layer 2-4 is at mean and phreatic line at left end of model is at 26 m. It is evident that the initiation of softening heavily relies on state parameter. This sets motivation to develop a response surface using results of FE simulations to determine probability of failure of tailings dam by static liquefaction.

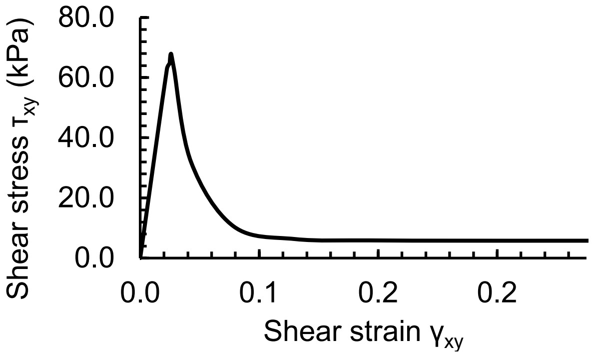

*Figure 6. Stress -strain curve of point A depicting softening of the tailings material due to liquefaction*

## 3 RESPONSE SURFACE METHOD (RSM)

Uncertainties in geotechnical engineering can be both due to natural randomness of a property such as variation in soil and uncertainty that can be reduced by collecting more information, improving the measurement methods, or improving the calculation methods (Lacasse and Nadim 1996). Probabilistic methods require definition of unacceptable (failure) and acceptable (non-failure) state of a physical system. The performance function, $G$, links the system response ($y$ and in this case the FoS of tailings dam) to the system state, considering the model uncertainty, $m$, which is defined as a random variable with an appropriate probability distribution function as described in Equation 1. If performance function is less than or equal to zero it indicates failure else represents non-failure.

$$G = my - 1 \tag{1}$$

RSM comprises an approximation of the behaviour of a physical system by a function with which a probabilistic analysis can be carried out.

The relationship between response, $y$, and model parameters, $X_i$, is obtained by a computer code. The code is run a limited number of times and the output is used to fit an approximate function (Box and Wilson 1951; Liu et al. 2018; Wong 1985). In this case the results from the numerical simulations as tabulated in Table 3 were used as input to define the response surface function which can predict the FoS based on the input parameters which are the state parameters of each layer and location of phreatic surface.

### *3.1 Development of response surface*

The non-linear response surface, y is defined by the following exponential relation.

$$FoS = FoS_0 \prod_{i=1}^{n}(1 + a_i\check{x} + b_i\widecheck{x^2}) \qquad (2)$$

where, $a_i$ and $b_i$ are fitting parameters. The letter i is the random variable index and n is the total number of random variables which are 5 in this case. $\check{x}_i$ are normalised values of random variables as follows:

$$\check{x}_i = \frac{x_i - x_{0i}}{\Delta x_i} \qquad (3)$$

where, $x_i$ is random variable, $x_{0i}$ is mean and $\Delta x_i$ is standard deviation as tabulated in Table 1. The random variables are normalized to have a well-conditioned and dimensionless set of variables with similar order of magnitude. The values serve as data points to develop the response surface regression as per Equation 2.

In principle, any set of input parameters can be used, which can often be expressed as $x_{0i} \pm k\Delta x_i$, where $k$ is an integer. In this case we have used k values of 0, 0.75 and 1.5 as evident from Table 3. Equation 2 was used to develop the following response surface equation to predict FoS of the tailings dam for a static liquefaction failure.

$$FoS = 1.5066\ .\left(1 - 0.00214\check{x}_1 - 0.00151\widecheck{x_1^2}\right).\left(1 + 0.000114\check{x}_3 - 0.00426\widecheck{x_3^2}\right).(1 - 0.034254\check{x}_4 + 0.005076\widecheck{x_4^2}).(1 - 0.02222\check{x}_5 + 0.003517\widecheck{x_5^2}) \qquad (4)$$

where, $\check{x}_1$ *to* $\check{x}_4$ are the normalised state parameters and $\check{x}_5$ is the normalised phreatic line elevation.

## 4 PROBABILITY OF FAILURE

First- and second-order reliability methods (FORM and SORM) provide accurate estimate of failure probability with modest computational effort. They transform physical random variables to normal variables with zero mean and unit variance (Hasofer and Lind 1974; Madsen et al., 2006; Nataf 1962; Rosenblatt 1952;). The objective is to find the shortest distance between the limit state surface and the origin in the transformed variable space. This distance is the reliability index, β. The failure probability of the system can then be approximated by:

$$P_f = \emptyset(-\beta) \qquad (5)$$

where, $\emptyset()$ is the cumulative distribution of the standard normal variate (Madsen et al., 2006). The FORM is better suited when limit state surface is linear and SORM is well suited for non-linear limit state surfaces.

COMREL V9.0 commercially available reliability analysis software was used to carry out SORM analysis. State parameters for the 4 layers were considered to be normally distributed with mean and standard deviation defined as per Table 1. The phreatic surface elevantion was considered to increase in predefined steps of 1 m.

Figure 7 describes the variation of probability of failure with increasing phreatic surface as per SORM. It can be observed that the failure probability increases by 22 % when the phreatic surface is at 50 m compared to at 26 m. Figure 8 demonstrates the influence coefficients and it can clearly seen that state parameter of layer 4 is of demand type and has 94 % influence on the reliability of the system. Figure 9 depicts that the effect of layer 4 slightly reduces as the phreatic surfaces rises but the influence of layer 1 and 3 on FoS drastically rises beyond 36 m of phreatic line as more of areas under these layers get saturated. Layer 2 has no impact at all.

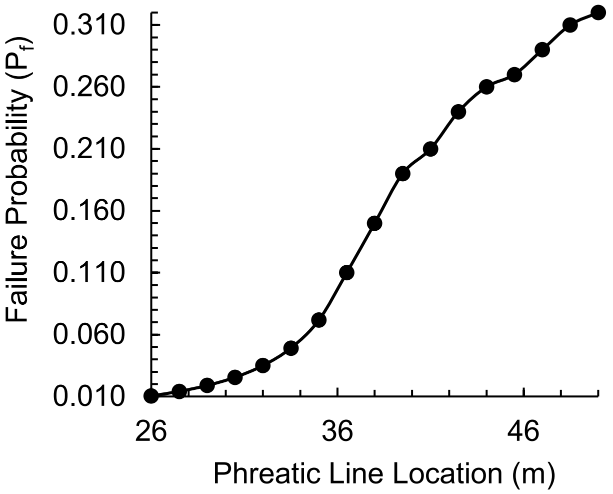

*Figure 7. Variation of $P_f$ with normally distributed state parameters for all layers at each phreatic line location*

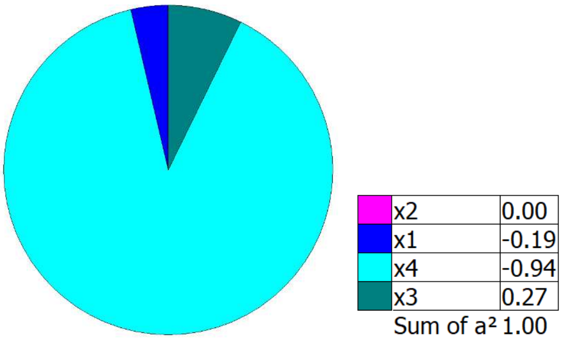

*Figure 8. Representative alpha variables for state parameter of each tailings layer as per SORM*

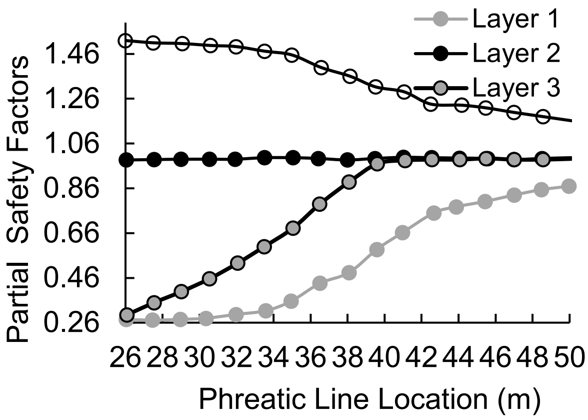

*Figure 9. Influence of state parameter of each layer on partial safety of the dam with increasing phreatic line location as per SORM*

## 5 CONCLUSIONS

The FoS of tailings dam against static liquefaction is difficult and depends on the accuracy and efficiency of state parameter determination using CPT test results. The uncertainty associated with the estimation of state parameter from CPT test is largely evident questioning the credibility of using a deterministic approach for the estimation of FoS for a tailings dam. The work outlined a computationally efficient framework to estimate the probability of failure of tailings dam based on a response surface which was developed against FE simulation results. It was demonstrated that one needs to rely on FE simulation coupled with sophisticated strain softening constitutive models to capture the phenomenon of static liquefaction. The FE results showed that saturated loose tailings material under undrained conditions could liquefy even with a small perturbation and does not depend on the location of the phreatic line. Though, the probability of failure was found to drastically increase with increasing phreatic surface. Existence of saturated loose materials in some layers could exponentially increase failure risk and contribution for various layers of a tailings dam could be highly non-linear. The recent catastrophic tailings dam failures necessitate use of reliability-based methods to adhere to new regulations and make better risk informed decisions. The response surface FE coupled framework can be a promising framework for quick and accurate reliability analysis of tailings dams against static liquefaction failures.

## 6 REFERENCES

Been, K. 2016. Characterizing mine tailings for geotechnical design. *Geotechnical and Geophysical Site Characterisation* (Eds: Acosta-Martínez & Kelly), Australian Geomechanics Society, Australia.

Been, K., Jefferies, M.G. 1992. Towards systematic CPT interpretation. *Predictive soil mechanics: Proceedings of the Wroth Memorial Symposium*, 121–134.

Box, E.P.G., Wilson, K.B. 1951. On the experimental attainment of optimum conditions, *Journal of the royal statistical society*, Series B (Methodological), **13**, 1-38.

Cruz, C.O., Rodovalho, E.C. 2019. Application of ISO 31000 standard on tailings dam safety, *International Engineering Journal*, **72**(1), 47-54.

Davies, M.P., McRoberts, E.C., Martin, T.E. 2002. Static liquefaction of tailings – fundamentals and case histories. *Proceedings of Tailings Dam*, ASDSO/USCOLD, USA.

Hasofer, A.M., Lind, N.C. 1974. Exact and invariant second-moment code format, *Journal of the Engineering Mechanics division*, **100**(1), 111-121.

Jefferies, M. G. 1993. Nor-Sand: a simple critical state model for sand, *Géotechnique*, **43**(1), 91-103.

Lacasse, S., Nadim, F. 1996. Uncertainties in characterising soil properties. Uncertainty in the Geologic Environment: From Theory to Practice. *Proceedings of Uncertainty,* American Society of Civil Engineers. Geotechnical Special Publication, 58, 49-75.

Liu, Z., Choi J.C., Nadim, F., Lacasse, S. 2018. Reliability Analysis of Sensitive Clay Slope with the Response Surface Method. *Proceedings of GeoShanghai International Conference*, *Geoenvironment and Geohazards*, 63-72. Springer Singapore.

Madsen, H.O., Steen, K., Lind, N.C. 2006. *Methods of structural safety*, Dover Publications, Mineola, New York, USA.

Nataf, A. 1962. Détermination des distributions de probabilités dont les marges sont données, *Comptes Rendus de l'Académie des Sciences*, **225**, 42–43.

Plewes, H.D., Davies, M.P., Jefferies, M.G. 1992. CPT based screening procedure for evaluation liquefaction susceptibility. *Proceedings of 45th Canadian Geotechnical Conference*, 41–49.

Reid, D., Fourie, A.B. 2023. Development and outcomes of a tailings slope stability comparative design exercise. *Canadian Geotechnical Journa*l. In press.

Robertson, P.K. 2016. Cone Penetration Test (CPT)-Based Soil Behaviour Type (SBT) Classification System—An Update, *Canadian Geotechnical Journal*, **53**, 1910-1927.

Rosenblatt, M. 1952. Remarks on a multivariate transformation, *The annals of mathematical statistics*, **23**, 470-472.

Wong, F.S., 1985. Slope reliability and response surface method, *Journal of geotechnical Engineering*, **111**, 32-53.