Proceedings 10th NUMGE 2023
10th European Conference on Numerical Methods in Geotechnical Engineering
Zdravkovic L, Kontoe S, Taborda DMG, Tsiampousi A (eds)

https://doi.org/10.53243/NUMGE2023-222

Reliability analysis of jet grouting bottom plugs

J. A. Alonso[1], L. M. Muñoz[2], R. Jimenez[3]

[1]*Alpha Geotecnia S.L.P . – Escuela de Caminos, Technical University of Madrid, Spain*
[2]*TEAM Ingeniería y Consultoría S.L, Bilbao, Spain.*
[3] *Escuela de Caminos, Technical University of Madrid, Spain*

ABSTRACT: Jet-grouting bottom plugs are common to conduct deep excavations in cohesionless soils under the water level. This work studies their Limit States design using a Reliability Based Design approach. Analytical solutions for Ultimate and Serviceability Limit States are used to model the plugs behaviour, and reliability methods – as First Order Methods and Monte Carlo – are used to compute the reliability of a given design for each Limit State, and to identify the relevant design variables. Finally, for the Limit State found to be critical (i.e., the rate of water inflow, which can control short-term or long-term performance, especially when an impervious bottom raft will not be constructed), we study its sensitivity to specific design decisions, such as the thickness of treated soil within the bottom plug. This work illustrates that there are two key aspects during design and execution of the jet grouting bottom plug, to assure its reliability: (i) the stratigraphy and permeability of the ground; and (ii) the continuity and thickness of the jet grout treatment.

Keywords: reliability; jet grouting; bottom plug; permeability; flow rate.

1 INTRODUCTION

The construction of deep excavations using diaphragm walls in permeable (sandy) soils below the phreatic surface is a common challenge in civil engineering, for which constructing an "impervious" excavation with the aid of a jet grouting bottom plug is often an optimal technical or economical solution. In this way, the excavation can be conducted with reduced water inflow into the excavation, and with minimal affection to the surrounding ground. The jet grouting plug has some additional benefits, as: (i) it serves as an "underground strut", constructed prior to the excavation, and which can help optimize the cost of constructing the deep excavation; and (ii) it reduces the hydraulic problems associated to excavations under the phreatic surface in permeable soils, such as piping of the excavation bottom, since, from a practical perspective, it provides a suitable substitute to the "ideal" condition of being able to introduce the diaphragm wall toes into an impervious stratum, even when suitably impervious soils (clays or silts) are too deep to be reachable economically. That is, when impervious layers are too deep, a jet grouting plug may be needed to achieve the low permeability conditions needed for a safe excavation construction.

The design of jet grouting plugs is extensively studied in previous works (see e.g., Modoni et al., 2016), and this article builds on them to analyze the influence of uncertainties on the response of the different limit states that condition the design of jet grouting plugs, and hence on their reliability or probability of failure. To that end, we first address the problem of uncertainty characterization – i.e., characterize the uncertainty associated to random variables that affect each limit state defined – and then apply reliability techniques – First Order Second Moment or FOSM, First Order Reliability Method or FORM and Monte Carlo–to compute the reliability against failure under the identified limit states, and with the assumed uncertainty characterization. Finally, for the limit state that was found to be critical in more cases (i.e., the rate of water inflow going through the jet grouting plug, which can control short-term or long-term performance, especially when an impervious bottom structural raft will not be constructed), we study its sensibility to specific design decisions, such as the thickness of treated soil within the bottom plug.

2 DESIGN OF JET GROUTING BOTTOM PLUGS. LIMIT STATES

Following Modoni et al. (2016), the limit states that condition the design of jet grouting bottom plugs can be classified within three types of Ultimate Limit States (or ULS, of type UPL, STR, HYD) and one Serviceability Limit State (or SLS, of type TFR). In particular, the Ultimate Limit States considered represent the plug design against uplift (UPL limit states, which can be global or local), the structural integrity of the plug (STR), the piping of the ground at the excavation bottom (HYD); and the Serviceability Limit States considered study the flow rate of water ingression into the excavation, due mainly to voids or heterogeneities within the jet grouting plug (see Figure 1). Authors have preferred not to consider the inflow through the walls of the excavation pit, mainly because it is an easily controllable value and in any case it is due to execution errors.

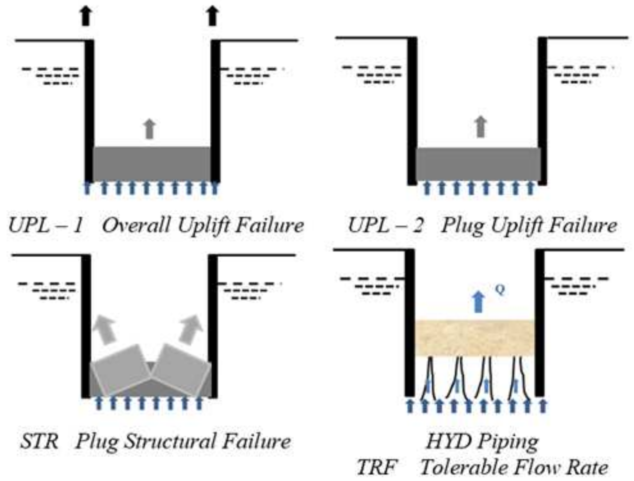

Figure 1. Limit States (adapted from Modoni et al. 2016)

To avoid the ULS Overall Uplift Failure (UPL-1) and Plug Uplift Failure (UPL-2), the destabilizing action due to uplift pressure, $V \leq G + R$, where G is the stabilizing action provided by the structure's self-weight and R is the stabilizing action due to strength of the soil-wall interface (in UPL-1) or of the jet grouted material–wall interface (in UPL-2). Under ULS-STR Failure, R is the stabilizing action due to the strength of the jet grouted material. Under ULS-HYD Piping, one checks that $hs,min \leq hs$, where hs,min is the minimum height of untreated soil to prevent piping and hs is the actual height of the untreated portion of the plug. Finally, a Tolerable Flow Rate SLS-TFR is assessed by $Q \leq Qlim$, where Q is the flow rate through the plug and $Qlim$ is the tolerable flow rate.

Equations for each Limit State are listed in Modoni et al. (2016) and they are not repeated herein. They can be expressed using "Margin of Safety" (M) and "Factor of Safety" (F) formulations: In UPL and STR Limit States, M is expressed as $M = G + R - V$, while $F = (G + R)/V$. In HYD Limit State, $M = hs,min - hs$, and $F = hs,min / hs$. If $M \leq 0$ or $F \leq 1$, failure occurs. We neglect partial safety factors (i.e., they are all equal to 1.0) to remove estimation biases that, despite their usefulness in deterministic analyses, would be inadequate in the reliability analyses conducted herein: parameters are characterized using their mean values and their associated uncertainty (expressed statistically as its standard deviation or coefficient of variation). Similarly, for the SLS considered (TFR), one could distinguish between short-term (i.e., during construction) and long-term flow rates (i.e., during the building lifetime) if a non-impervious structural bottom is chosen. However, flow rate differences between (short-term or long-term) situations are often minor when execution imperfections exist, as the permeability of the untreated ground (acting as 'flow paths' between jet grouting columns) is much higher than the permeability of the soil treated with jet grouting. In the TFR SLS, M is expressed as $M=Qlim-Q$, while F is expressed as $F= Qlim/Q$. If $M \leq 0$ or $F \leq 1$, failure is reached.

3 RELIABILITY ANALYSIS. FOSM, FORM, MONTE CARLO

Our reliability analyses aim to quantify the probability of failure, *Pf,* of a design, and for a given LSF (Limit State Function) and uncertainty characterization; equivalently, they allow us to compute its reliability index. The interest of this probabilistic approach is that quantifying the design reliability is a more consistent measure of risk than the use of a factor-of-safety approach in which uncertainties are not modelled (or their effects quantified; see e.g., Baecher and Christian, 2003). The *Pf* was computed using three reliability methods proposed in the Spanish Geotechnical Recommendations for Marine Works ROM0.5-05 (2005): i) one "Level I" Method acting as "ordered sensitivity analyses", and also known as First Order Second Moment (FOSM) method; ii) one "Level II" Method (the First Order Reliability Method, or FORM), where Pf is estimated from a first order approximation of the LSF; and iii) one Level III (or simulation) method, and in particular Monte Carlo simulation.

The FOSM method (see e.g., ROM 0.5-05, J. Michael Duncan 2000) starts computing a "centred" safety factor, F^*, for average values, and it then repeats computations changing one input variable at a time (out of the set of input variables whose variability is considered) and keeping the others constant. Usually, the change is equal to the standard deviation of the random variable, and changes are commonly conducted in both favourable and unfavourable directions. A sensitivity coefficient, νi, can be obtained as $\nu_i = \frac{F^{+} - F^{-}}{2 \cdot F^*}$, where $F+$ is the safety factor associated to the change in the favorable direction (and $F-$ to the unfavorable one). Then, the global sensitivity is computed as $\nu_F = \left[\sum_i \nu_i^2\right]^{1/2}$. The reliability index β can be computed as how many standard deviations one needs to move away from F^* so that failure (theoretically) occurs; i.e., so that $F = 1$. Assuming a lognormal distribution of F as stated by J. Michael Duncan (2000), the solution is:

$$\beta = \frac{lnF^*}{\xi} - \frac{1}{2} \cdot \xi \quad (1)$$

where $\xi = \sqrt{[ln(1 + \nu_F^2)]}$.

Then, following the standard definition of "generalized reliability index", the probability of failure is uniquely associated to the reliability index by $p_f = \Phi(-\beta)$, where $\Phi(\cdot)$ is the CDF (Cumulative Density Function) of the standard normal distribution.

FORM computations have been carried out with the ellipsoid method proposed by Low (1997), using an easy spreadsheet (Excel) implementation, as:

$$Minimizing\ \beta = \sqrt{\sum_i \frac{(x_i^* - \mu_i)^2}{\sigma_i^2}} \qquad (2)$$

subjected to $M(x_i) \leq 0$, where $\sum i$ indicates summation over all random variables considered; x_i^* are the values of such random variables at the minimum (i.e., "design point"); μ_i, σ_i^2 are the means and variances of each random variable; and $M(x_i)$ is the Limit State Function. For simple cases with uncorrelated normal variables (as considered herein) FORM results directly provide sensitivity coefficients (α_i), useful to identify random variables with a stronger influence on the reliability results (Jimenez Rodriguez et al. 2006):

$$\alpha_i = \frac{-(x_i^* - \mu_i)}{\beta \cdot \sigma_i} \qquad (3)$$

Monte Carlo (MC) method approximates *Pf* as $p_f \approx \frac{1}{N} \cdot \sum_{i=1}^{N} I(x_i)$, where N = number of simulations to be conducted, and $I(x_i)$ is a Boolean function indicating whether the randomly generated x_i produces failure ($I = 1$ is failure).

4 APPLICATION EXAMPLE

4.1 Introduction

The reliability methods described are applied to compute the *Pf* associated to each LSF considered, for an application example based on Modoni et al. (2016) –see Figure 2 and Table 2– inspired from a case history in which a double-fluid jet grouting bottom plug in non-cohesive soils was employed at the excavation bottom of a train station near Barcelona (Spain).

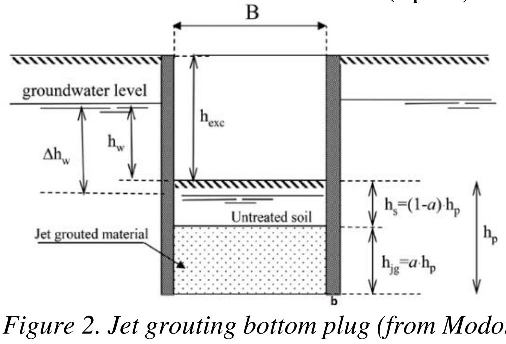

Figure 2. Jet grouting bottom plug (from Modoni et al. 2016)

Table 2. Geometrical Characteristics of the Bottom Plug, with additional parameters of interest

Parameter	Value	Comments
L (m)	107	*L >> B*, calculations per lineal meter Tolerable Flow Rate over an area *A* = 1000 m²
B (m)	25	--
h_{exc} *(m)*	15	--
h_w *(m)*	12	--
Δh_w *(m)*	12.5	Assumed value
b (m)	1	--
h_{jg} *(m)*	3.47	Modoni et al. 2016
h_p *(m)*	10.24	Modoni et al. 2016
h_s *(m)*	6.77	Modoni et al. 2016
$\alpha = \frac{h_{jg}}{h_p} (m)$	0.339	--
γ_w *(kN/m³)*	10	--
Soil type	Cohesio n-less	--
Qlim (m³/s)	2E-03	Over *A* = 1000 m² Compilation made by Modoni et al. 2016

4.2 Uncertainty characterization

For simplicity, the reliability analysis assumed uncorrelated random variables, whose statistics – mean value, coefficient of variation (*COV*), and standard deviation (*SD*) – are listed in Table 3; Table 3 also indicates the sources to define uncertainties (as measured by *COV* or *SD* values) for each random variable. While uncertainties associated to some of these parameters are commonly discussed in other projects, the uncertainty assigned to Ω i.e., to the untreated area ratio of the plugs (Figure 3) deserves some discussion, since it is specific to jet grouting plugs and can affect the key flow rate SLS.

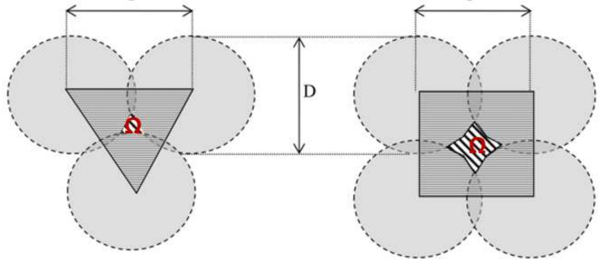

Figure 3. Untreated portion of the soil mass (adapted from Modoni et al. 2016)

To estimate it, we considered a triangular grid of jet grouting columns with a surface separation between centers *so* = 0.85 m and with average column diameter *Dm* = 1.0 m; then, using the *so/Dm* ratio (=0,85), and the average depth (below the surface from where the jet grout columns are constructed) of the jet-treated plug, we obtained the average and *SD* of Ω following the guidelines by Modoni et al. (2016), who computed them using Monte Carlo simulation of representative column arrays in which both column diameter (*D*) and angle of column inclination (β) were assumed variable, with: *COV(D)* = 0.1 & 0.2; and *DS(β)* = 0.1°, 0.2° & 0.3°. Modoni´s diagram provides an average Ω equal to 0.03, with a *SD* of 0.01 applying the well-known Two-Sigma Rule. To investigate the influence of different 'levels' of construction control (or "quality"), we considered cases with lower and larger column diameter variability (*COV(D)* = 0.1 and =0.2), which produce average Ω values of 0.02 and 0.04, respectively. The standard deviation of Ω is still considered equal to 0.01.

Table 3. Uncertainty Characterization of random variables considered

Random variables: mean values, *COVs*, standard deviations					
Variable	**Mean value**	***COV***	***S.D.***	**ut.**	***COV* & *S.D.* Estimation**
γ'_s = submerged unit weight of the untreated soil	10	10%	1.0	kN/m³	EC7 (2019)
γ_{jg} = unit weight of the jet grouted material	15	10%	1.5	kN/m³	EC7 (2019)
γ_c = unit weight of the retaining structure	25	10%	2.5	kN/m³	EC7 (2019)
$ks \cdot tan\ \phi'$	0.25	20%	0.05	-	Modoni et al. (2016) Variability in skin friction calculations according to different Codes
$q_{u,jg}$ = uniaxial compressive strength of the j-g material	6	15%	0.9	MPa	Modoni et al. (2016)
c_{jg} = cohesion of the j-g material $cjg = 0.2 \cdot q_{u,jg}$	1.2	15%	0.18	MPa	Modoni et al. (2016)
n = compressed block depth (percentage of *hjg*)	25%	15%	3.75%	-	Modoni et al. (2016) Sofianos (1996) Two-Sigma Rule
Ω = untreated area ratio	0.03	33%	0.01	-	Modoni et al. (2016) Two-Sigma Rule
dh = representative cross-sectional dimension of the hole	0.11	40%	0.04	m	Obtained from Ω and Two-Sigma Rule
k = coefficient of soil permeability	10-5	150%	1.5·10-5	m/s	EC7 (2019)

4.3 *Results of the reliability analysis*

The *Pf* results listed in Table 4 were obtained after applying the reliability methods discussed in Section 3 to the Jet Grouting Limit States presented in Section 2, and considering the deterministic parameters of Table 2 and the random variables listed in Table 3. Results are expressed as (i) *Pf* values and as the β index corresponding to each *Pf* value, and (ii) as the safety factor (F^*) corresponding to average values for each limit state. Sensitivity coefficients from the FORM analyses are listed in Table 5.

Results indicate a good agreement between *Pf* results obtained with FORM and MC methods; note, however, that FOSM sometimes departs from these results – hence illustrating its lower precision – although the overall tendency is also similar, with probabilities of failure that are very low for the Ultimate Limit States (UPL-1, UPL-2, STR and HYD) and which are higher for the Serviceability Limit State of TFR.

4.4 *Sensitivity analyses*

The sensitivity analyses presented below were conducted for the SLS TFR, since it has a much higher probability of failure that the others, hence controlling the reliability of the whole design. It analyzes how the TFR *Pf* changes as the thickness of the jet-grouted zone increases, keeping everything else equal. Analyses were conducted for three average values of Ω (0.03-0.02-0.04), and with the same value of *SD* for Ω, of 0.01.

Results are presented in Figure 4. Figure 4 also includes the admissible β according to its consequences, as defined by ISA (Social and Environmental Impact Index) in ROM 0.5-05 (2005).

The sensitivity coefficients (squared), which illustrate the relative importance of random variables in the *Pf*, are included in Figure 5.

Table 4. Deterministic safety factor for average values (F) and probability of failure and reliability indices for each limit state function*

LS	F* (mean values)	FOSM		FORM		MONTE CARLO	
		β	***Pf***	***β***	***Pf***	***β***	***Pf***
UPL – 1	1.315	3.969	3.61E-05	4.041	2.66E-05	3.832	6.35E-05
UPL – 2	2.341	8.705	<1.00E-14	5.900	1.82E-09	5.768	4.00E-09
STR	1.622	5.355	4.28E-08	5.022	2.56E-07	4.823	7.06E-07
HYD	23.650	6.658	1.39E-11	4.802	7.85E-07	4.894	4.94E-07
TFR	1.959	0,568	2,85E-01	0,597	2.75E-01	0,690	2.45E-01

Table 5. FORM. Sensitivity coefficients

LS	FORM		SENSITIVITY COEFFICIENTS	
	β	***P_f***	**RANDOM VARIABLES**	***$alpha_i$^2***
UPL-1	4.041	2.66E-05	*γ's*	0.44
			γjg	0.03
			γc	0.00
			ks·tan φ'	0.55
UPL-2	5.900	1.82E-09	*γ's*	0.03
			γjg	0.03
			cjg	0.94
STR	5.022	2.56E-07	*γ's*	0.04
			γjg	0.04
			qu,jg	0.64
			n	0.28
HYD	4.802	7.85E-07	*γ's*	0.39
			ks·tan φ'	0.39
			dh	0.22
TFR	0.597	2.76E-01	*Ω*	0.12
			k	0.88

Results show that the minimum jet grouting thickness considered for design *(hjg* = 3.47 m) provides relatively high probabilities of failure, even for a SLS. However, as expected, the reliability increases significantly as *hjg* increases, so that treated thickness in the order of 5-9 m (depending on the average value of *Ω*) already provide a very low *Pf* in this case. Following Alonso and Jimenez (2011), this also illustrates the importance of construction quality and its control: note that, as illustrated by Modoni et al (2016), the average of *Ω* depends greatly on the variability of jet grout column diameters; and on their deviations from theoretical alignment. These aspects are probably a proxy of 'construction quality', in which more carefully executed treatments, or treatments executed with better equipment, will produce more reliable designs.

Sensitivity coefficients show that the permeability *k* is often the most influential variable: for low *hjg* values the relative importance of k is in the range 0.74 - 0.95, hence being key for reliability. As *hjg* increases, *Ω* becomes more relevant

5 CONCLUSION

We conducted reliability analyses of a jet grouting design, inspired by a real case history. ULS and SLS are considered, following their formulation by Modoni et al. (2016). Results show that the ULS (UPL-1, UPL-2, STR, HYD) provide very low *Pf*; however, the *Pf* for the SLS associated to groundwater flow through the bottom plug (TFR) is much higher, clearly above its threshold in ROM 0.5-05 (2005), of p_{fmax} about 5.0E-02 to 7.0E-02.

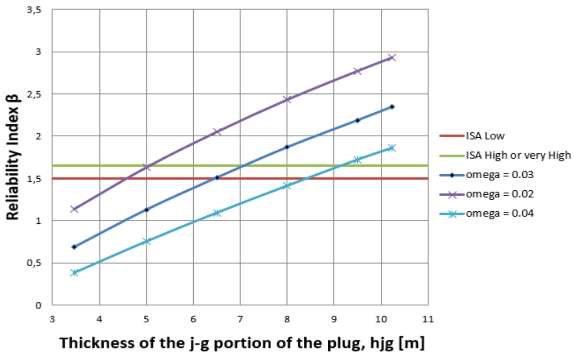

Figure 4. Reliability Index β vs. Thickness of jet grouting, hjg

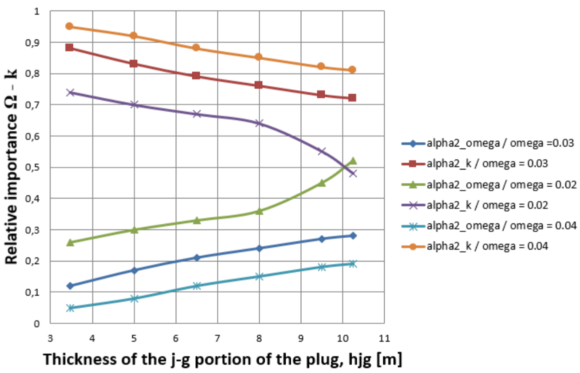

Figure 5. Relative importance Ω – k vs. Thickness of jet grouting, hjg

We argue that this is probably common in the design of jet grouted bottom plugs, whose design thickness should often be controlled by maximum flow rates. The importance of such maximum flow rate considerations is mainly associated to: i) the flow rate must be low enough so that water flowing into the excavations can be pumped out, to conduct the excavation in a "dry state", until completion of the impervious bottom raft; and ii) if a solution with a light drained slab-on-grade is admissible, then the long term water inflow rates (very similar to those during construction) must be small, so that their pumping costs (considering costs to discharge them into the sewage system) is not excessive.

This work illustrates that there are two key aspects that should be defined and assured during design and execution of the jet grouting bottom plug, to be able to assure its reliability: (i) the stratigraphy and permeability of the natural ground; and (ii) the continuity and thickness of the jet grout treatment (i.e., to avoid excessively large flow paths, through 'gaps' in the treated zone, as defined by *Ω*). They are both very relevant variables for the design reliability, but with the most relevant aspect being the characterization of the natural ground permeability. That is, if one wants to increase the reliability of a given bottom plug design, probably the most efficient path is to characterize the ground permeability as good as possible, to reduce its associated uncertainty. To that end, laboratory tests can be conducted (e.g., granulometry tests), although "in-situ" tests (e.g., pumping tests) are probably the best way ahead.

In any case, the flow rate can be observed as the excavation (and the lowering of the phreatic surface associated to it) is conducted, so that this provides information on the expected long-term flow rates so that, at that moment, one can decide whether an impervious base structural raft foundation (or a light slab-on-grade solution) should be preferred.

Finally, from a practical viewpoint, we would like to highlight the comment made by J. Michael Duncan & Sleep (2015) regarding the accuracy in calculating the probability of failure: *"It is important to remember that estimates of the probability of failure are just that – estimates. Their value lies in the order of magnitude – is it 0.1%, 1% or 10%? These orders of magnitude, 0.1%, 1%, and 10% can be viewed as low, medium (or normal), and high. Additional digits, such as 1.76%, do not add more value to the estimate".*

6 ACKNOWLEDGMENTS

This research was conducted with the partial support of the Spanish Ministry of Science and Innovation, under Grant PID2019-108060RB- I00.

7 REFERENCES

Alonso, J.A., Jimenez, R. 2011. *Reliability analysis of stone columns for ground improvement*. In JuangC.H. et al (eds) In: Proceedings of Georisk 2011: Geotechnical Risk Assessment and Management (GSP-**224**). ASCE, Reston (VA). 493 500.

Baecher G. B., Christian, J.T. 2003. *Introduction – uncertainty and risk in geotechnical engineering*. Reliability and Statistics in Geotechnical Engineering, Wiley, West Sussex, England.

Michael Duncan, J. 2000. Factors of safety and reliability in Geotechnical engineering. *Journal of Geotechnical and Geoenvironmental Engineering*. **126(4)**, 307 316

EC7, 2019. Eurocode 7: Geotechnical design

Jimenez Rodriguez, R., Sitar, N., Chacón, J. 2006. System reliability approach to rock slope stability. *International Journal of Rock Mechanics and Mining Sciences*, **43(6)**, 847 859.

Low, B. K. 1997. Reliability analysis of rock wedges. *Journal of Geotechnical and Geoenvironmental Engineering*, **123(6)**, 498 505.

Michael Duncan, J., Sleep, M.D. 2015. *Evaluating reliability in geotechnical engineering*. Risk and reliability in geotechnical engineering. Taylor and Francis, Boca Raton.

Modoni G., Flora A., Lirer S.; Ochmanski M., Croce P. 2016. Design of Jet Grouted Excavation Bottom Plugs. *Journal of Geotechnical and Geoenvironmental Engineering*. 04016018. **142(7)** 1 15.

ROM0.5-05.2005, Geotechnical Recommendations for Marine Works. Puertos del Estado, Madrid, Spain.

Sofianos A.I. 1996. Analysis and Design of an Underground Hard Rock Voussoir Beam Roof. *International Journal of Rock Mechanics and Mining Sciences*, **33(2)**, 153 166.