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Intelligent placement of untooled rock to form precision structures using a weighted criterion

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ABSTRACT: Innovative low carbon methods for construction are becoming increasingly necessary due to the current climate crisis. In-situ resource utilisation, where local materials are utilised with minimal externally provided resources are becoming increasingly attractive, updating traditional methods by leveraging the latest technology. This paper examines the potential for creating geotechnical structures using raw rock cobbles/boulders (or potentially demolition rubble) with optimised particle placement to minimise the need for concrete or reinforcement. While such work requires the input of computer vision and robotics, this paper focuses on the mechanics aspect of the problem - how to place the rock to give the optimal performance of the final structure. The presented work adopts an optimisation methodology that scores particle placement using a weighted criterion in a two-dimensional scenario. Results in the form of final volume achieved are presented and the use of more sophisticated criteria are discussed. The key methodological concepts and challenges are introduced using shapes from the classic video game Tetris, before addressing more realistic particle shapes.

Keywords: Particle packing; untooled rock; precision structures; fourier-voronoi; Tetris

1 BACKGROUND

1.1 Introduction

As the climate crisis becomes more prominent the need for a reduction in material use and low carbon techniques becomes more pressing. The construction industry accounts for nearly 30% of raw material use and 33% of related greenhouse gas emissions globally (Chau et al. 2015). Additive manufacturing methods are already being utilised to try and provide solutions by reducing material usage (Paolini et al., 2019; Ghaffar et al. 2018). Techniques involving 3D printing using earthen material have been explored (Perrot, et al. 2018; Gomaa et al., 2021). Currently, previous research in this area has focused on material suitable for extrusion through a nozzle.

Precision structures are those which are created using only the optimal quantities of material required for the required function. The concept of creating a precision structure from geomaterials includes minimal use of the materials through smart placement and the strategic use of reinforcement if necessary.

The aim of this paper is to report initial work into developing an additive method that can create precision structures from untooled rock, cobbles, boulders, and which would therefore allow the exploitation of locally available materials to minimise environmental impact. The investigation is carried out using two-dimensional soil particle shapes to simplify the geometry of the problem. The initial aim of the structure is to be stable enough to be self standing with global strength being prioritised.

1.2 Particle system geometry

To determine rock "particle" placement, factors towards soil strength need to be analysed. Oda (1977) states that the strength of a system can be determined by coordination number where a soil with higher mean coordination number possesses a larger internal angle of friction. Coordination number is quanitified by the number of neightbouring particles that touch a particle and is important in context to the force transferred by force chain development when loaded (Muir Wood, 2008) as forces are transmitted through interparticle contacts. Particles with higher coordination numbers are more likely to be involved in force chains (Fonseca et al., 2016) meaning the force can be distributed within the soil through the strong network as well as a subset of weak networks. Additionally, a lower coordination number can lead to a particle being unstable in the system resulting to a lower internal angle of friction (Oda, 1977).

There are other factors which are shown to affect strength in a system. Larger areas of a particle in contact with other particles reduces the pressure on that particle and helps distribute the force (Thornton, 2000). Particle size does not have an effect on shear resistance. Rather it is due to the change of other surface characteristics and particle shape (Vallerga et al., 2009; Winterkorn, 1967). Leslie (1969) showed that the higher the coefficient of uniformity, C_u, the higher the value of peak strength of a particle system. However Wang et al. (2013) concluded that angle of friction increased as C_u decreased. No detailed description of the soil is given in Wang et al. (2013). Therefore, it could be considered that a well graded sample does not necessarily mean a higher angle of friction as stated by Winterkorn (1967). Rather, it is the soil particle shape.

It can be understood that a high strength will generally correlate with a low void ratio. A low void ratio tends to lead to higher mean coordination number using rounded shapes (Graton and Fraser, 1935; Field, 1963). However, note that this would not be the case with squares, as a system using a regular grid of squares would have minimal resistance to shearing. In terms of grading and particle shape, systems with a minimum void ratio imply that the particles are packed together to form a stronger structure. In addition, more areas of particles are in contact leading to a higher interparticle friction.

Multiple algorithms have been created that look at packing with the aim to provide minimum voids in scenarios such as bin packing (Baker et al., 1980; Martello and Vigo, 1998; Wang and Hauser, 2019). While this research is considered relevant, it should be noted that the purpose of bin packing is to optimise space in a container, not to optimise for the strength of a system.

A simple scenario for the particle placing problem where minimum void ratio is desired is in the videogame Tetris. In Tetris, the player attempts to place shapes made of 4 squares (tetroids) consecutively in a rectangular domain of 10 squares by 20 squares. There are 7 possible tetroids which are represented in Figure 1. These are randomly selected using a "pull from bag" technique which creates a list of the 7 tetroids and selects one at random. The player uses rotations and horizontal translations to orientate the shape as they fall. Once that piece is positioned in the domain it is removed from the list and another is selected. The list is renewed and the process is repeated when each set of 7 tetroids are placed. The aim of Tetris is to build up layers of squares. Once one layer is created across the width of the domain, this line is removed and all other squares shift downwards. If a void pocket is accidently created this will mean that a line is potentially unfillable, hence the need to create minimum void space. The game ends when a tetroid is placed outside of the top of the domain. There is a variety of research trying to optimise methods in which Tetris is played (Böhm et al., 2005; Breukelaar et al., 2004; Kostreva and Hartman, 2003) These tend to focus on systems only using one, two or three tetroids.

This paper takes the idea of creating minimum void space within a simple system like Tetris in order to develop and illustrate basic concepts and algorithms and then applies this to a system represented by untooled rock. It is clear that lines of soil will not delete themselves, so this aspect of the game is removed. The technique of placing the particles in a top-down approach is kept. A heuristic method is developed using the classic tetroid shapes before moving to use two-dimensional granular soil shapes. The criteria used to quantify placement of particles will consider void ratio as well as the aspects that contribute to soil strength discussed earlier: coordination number and area of particle in contact with other particles.

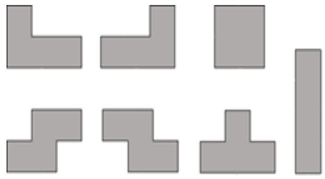

Figure 1.Tetroids that are utilised in the videogame Tetris

2 METHOD: TETROIDS

2.1 Particle Placement

The work presented was conducted in the C++ like vector graphics language Asymptote (Hammerlindl et al., 2014). The 7 basic tetroid shapes and a rectangular domain of 10 squares by 10 squares are adopted for this work and constructed from coordinates at the positions of the corners that make up the outer shape. A domain of this size was adopted to save on run time.

Iterations of particles are selected using the pull from bag method. Particles were set to be contained inside this domain and were able to touch the edges without any overlap. If overlapping occured, this possible placement was marked as outside of the boundaries and denied as a potential solution.

An example of two particles being placed is shown in Figure 2 with the "bag" of particles available displayed. A particle is taken from the bag. It is positioned above and to the left of the domain and the lengths from each coordinate to the base of the domain are calculated as shown in Figure 2a. Simultaneously, lengths from coordinates that make up the base of the domain that are directly below the particle to the bottom of said particle are found. This is to avoid any smaller parts that might be missed by a gap in coordinates. This is unlikely in a system using Tetroids however may occur in systems using other shapes. The minimum length is taken as the distance that the particle can be shifted downwards in the system. This is repeated at different locations along the domain using the different rotations of the particle and each position is scored using a weighting system. The position that outputs the highest score is chosen for the placement of that particle. If the highest score is shared, following Baker et al., (1980), the position closest to the bottom left of the domain is prioritised. The top surface outline of the particle is connected to the line that creates the base of the domain and becomes the new target shape for particles to be lowered to. The next particle is then taken out of the bag and this is repeated until no space for the next particle is available in the domain. It should be noted that the placement displayed in Figure 2 is not optimised for minimum void ratio to call attention to the upper surface of the particles placed and the bottom of the domain and to highlight voids can be present below this line. Underneath the upper surface is the area in which the final void ratio of the system is calculated once all particles are placed and the line also represents the exterior of the domain onto which a particle can be lowered to as demonstrated in Figure 2b.

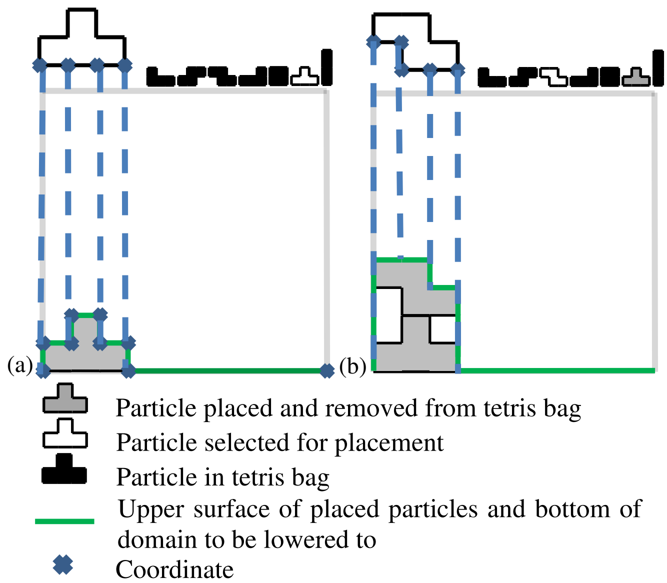

Figure 2. Placement of two particles using the drop down method with Tetris bag of available particles presented to the top right of the domain.

2.2 Scoring placement

Weighting for placement of particles was determined using four scoring criteria which include void ratio, contact area between the placed particle and particles already in-situ, and coordination number. It was seen with initial runs of the program that towers of tetroids were created as the program tended to favour the left-most position if the highest weight was found in multiple positions. To counter this, a weight was given for how close the particle was to the base of the domain, or the depth of the particle. This was considered logical as if a structure was being created it would be carried out in a layer by layer process rather than in towers to ensure stability. This was achieved by normalising distance lowered from the top of the domain to the centre of gravity of the placed particle by the length of the domain.

Void ratio was initially quantified using the total void ratio of the system (Method 1). However, it became obvious that as more particles were placed an increase in void space would have diminishing effect on the weighting than initial placements. Instead, two other methods were trialled; one calculating void ratio using a localised area around the placed particle (Method 2) and another calculated as a ratio of the volume of void created beneath the particle to the volume of the particle placed (Method 3). Figure 3 presents the relative performance of these three methods with the inclusion of the depth weighting, W_D. Each violin plot represents the results of 100 tests using tetroids. As can be determined from scenario f, Method 3 is optimal as this outputted much lower final void ratios of the total system.

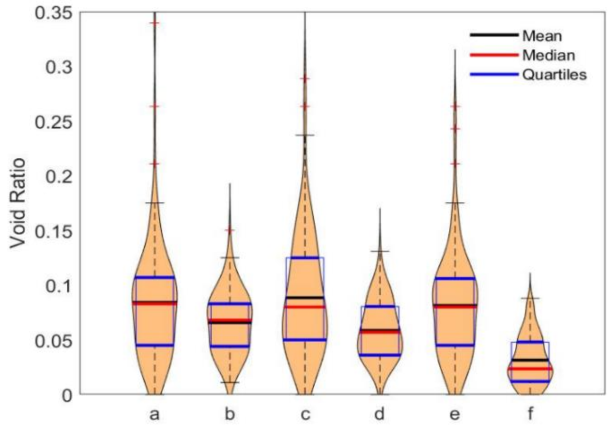

Figure 3. Violin plots, each representing 100 results using the different methods of quantifying the void ratio score. a, b, c, d, e and f represents the scenarios using Method 1, Method 1 with W_D included, Method 2, Method 2 with W_D included, Method 3 and Method with W_D included

Contact area of the particle was calculated by taking the length of the particle that was in contact with the domain or other particles and normalising this by the total length of the bottom of the particle. As the particle is lowered until it makes contact with the upper surface, it can be envisaged that this can lead to only one point of contact for other particle shapes that are not tetroids. For this initial work, a contact distance error between each particle coordinate and the upper surface line was allowed for. If this distance error was less than 0.1 then the coordinate at that point was deemed as touching.

Coordination number was calculated by taking the particle placed and expanding it by 10% of its original size in the placed position. If the particle intersects with surrounding particle, these are counted as touching and are taken to contribute towards the coordination number.

Each weighting was assigned a coefficient and the total score for each placement was calculated by

$$W_{ij} = C_V(1 - e) + C_D D + C_T T + C_{CN} CN \qquad (1)$$

where W_{ij} is total weight (or score) for the horizontal position and particle rotation, e is void ratio calculated using Method 3, D is the depth of the particle in the system normalised by the height of the domain, T is the area of the particle in contact with other particles normalised by the total area of the bottom of the particle, and CN is coordination number. C_V, C_D, C_T, C_{CN} are the coefficients applied to e, D, T and CN respectively. The highest score of W_{ij} for each placement was selected to determine the positioning of the particle. The coefficients that were selected for the tetroid system described are presented in Section 3. Results are presented using the final void ratio of the system as this is usually a good indication of the strength of the system as discussed in Section 1.2. A more complex technique including, for example, mean coordination number will be deployed in future work.

3 RESULTS: TETROIDS

An investigation into the influence of the coefficients C_V, C_D, C_T and C_{CN} on final void ratio was conducted and is presented in Figure 4. These violin plots represent the void ratio results of 100 runs of each combination of coefficients investigated. Selected combinations have been chosen to highlight the effect of each coefficient. The values of coefficients can be found in Table 1. Of the combinations of coefficients, combination O was selected as this gave the highest frequency of results closest to zero void ratio. This can also be represented by the overlying box plot that shows the mean, median and lower quartile values being closer to zero void ratio than combinations A-H. The mean is lower than values for the similar plots of combinations K-P. Figure 5 is an example of the packing produced with the order of particle placement displayed on top of the particle, 0 indicating the first particle placed.

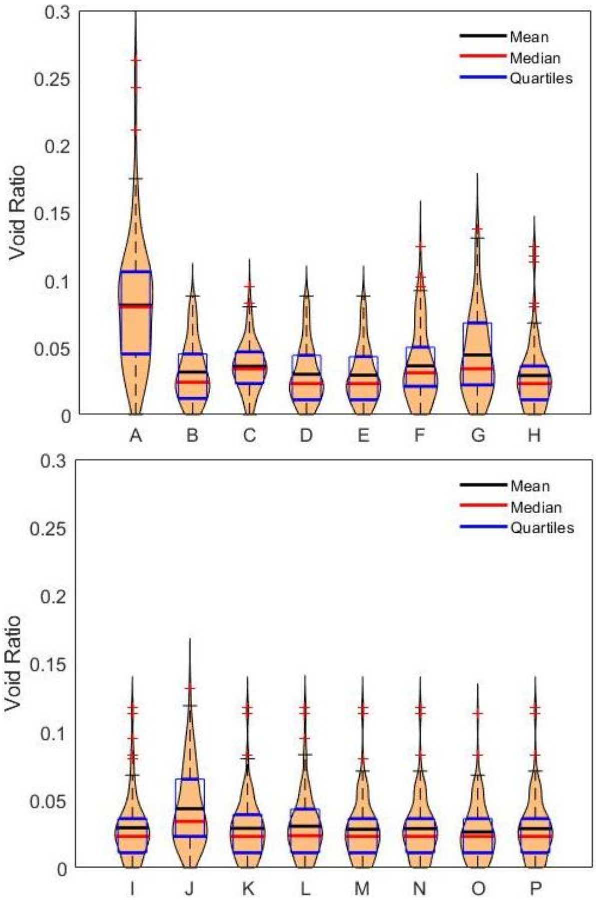

Figure 4. Violin plots, each of 100 results using coefficient values represented in Table 1 overlaid by the corresponding box plot.

Table 1. Weighting coefficients guide for Figure 4

Letter	Coefficients Values (C_V,C_D,C_T, C_{CN})	Letter	Coefficient Values (C_V,C_D,C_T, C_{CN})
A	1,0,0,0	I	5,2,0.4,0
B	1,1,0,0	J	5,2,0.4,1
C	1,2,0,0	K	5,2,0.4,0.01
D	2,1,0,0	L	5,2,0.4,0.005
E	5,2,0,0	M	5,2,0.4,0.02
F	5,2,1,0	N	5,1,0.4,0.01
G	5,2,2,0	O	5,1.25,0.4,0.01
H	5,2,0.5,0	P	5,1.75,0.4,0.01

4 METHOD: SOIL PARTICLES

The exact same approach for particle placement and scoring using Equation (1) applied to tetroid shapes is next applied to shapes that represent two-dimensional soil particles. As Tetris does not conform to normal gravitational laws, a stability check was not needed. However, when using soil particle shapes, a stability check was introduced using simple toppling and sliding calculations. Contact points of the particle to the surface were identified and if the centre of gravity of the particle was outside of these points it was considered unstable. For sliding, if the angle created by the line of the contact points to the horizontal was greater than the assumed friction angle then it was considered unstable. In this research, the friction angle was taken as 31° as a conservative estimate from the measurements conducted by Buffington et al. (1992). Examples of these checks are visualised in Figures 6 and 7. Unstable positions were excluded from the possible placements when determining the best position for that particle.

Particle shapes were created using the Fourier-Voronoi method described in Mollon and Zhao (2012). Software for Fourier-Voronoi generated particles is

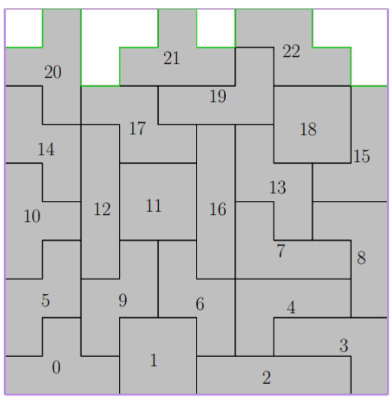

Figure 5. Single result of Tetroids using combination O of coefficient values C_V = 5, C_D = 1.25, C_T = 0.4, C_{CN} = 0.01) with the order of placement represented on top of each particle, the first placed particle indicated by 0. The green line represents the area void ratio is calculated and void ratio = 0

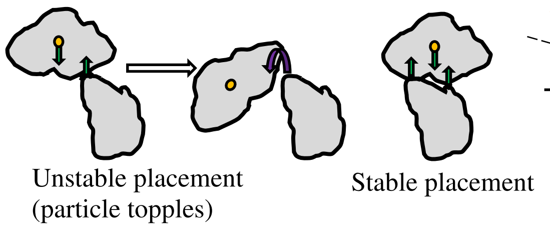

Figure 6. Demonstration of a toppling stability check with example of unstable and stable particles

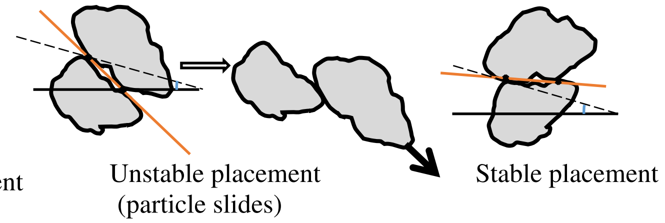

Figure 7. Demonstration of a sliding stability check with examples of unstable and stable particles

discussed and linked to in Mollon and Zhao (2013). Fourier Descriptors D_2, D_3 and D_8 were set to 0.2, 0.1, and 0.02 respectively and the target solid fraction was set to be 0.7. An arbitrary radius limit between 3 and 7.5 units was chosen to exclude any oversized or undersized particles. An example of the range of particles that were produced is represented in Figure 8.

A square domain of 50 by 50 units was selected. This was to try to simulate a similar size to the Tetris domain used previously as this came to roughly 10 particles across the base, representing the width of 10 squares in the Tetris domain. A maximum limit of 50 particles was set as this filled the domain to a reasonable height. A spacing of 1 unit was chosen for the different locations of placement to try and fit particles closely together. 16 rotations of each particle were trialled and a contact distance error of 0.1 units.

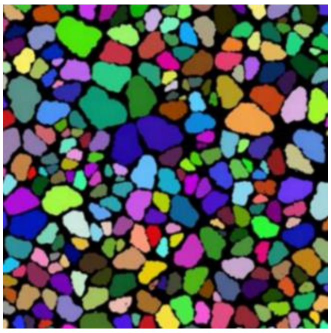

Figure 8. Output of the Fourier-Voronoi software reported in Mollon and Zhao (2013) using values of D_2, D_3 and D_8 as 0.2, 0.1 and 0.02 respectively and a target solid fraction of 0.7

5 RESULTS: SOIL PARTICLES

Coefficient values from combination O were taken and used in a simulation using particle shapes. Results of the packing arrangement produced with the stability check deactivated and activated are represented in Figure 9 and Figure 10 respectively.

While the results show that the method can produce packings of particles placed in a one-by-one approach, it is clear that the optimal method used with Tetroids needs reconfiguration given the large void spaces present in Figures 9 and 10.

Figure 11 presents a system using 100 particles in a domain of 75 units by 75 units. The number of rotations was set to 32 and the spacing between placements was

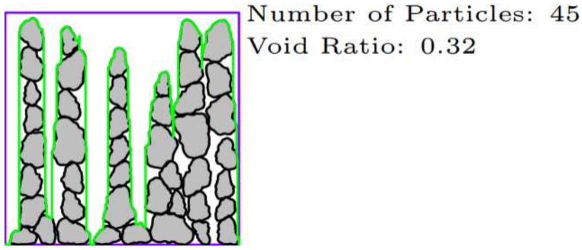

Figure 9. Packing produced using coefficient values from combination O (Table 1) with stability check deactivated.

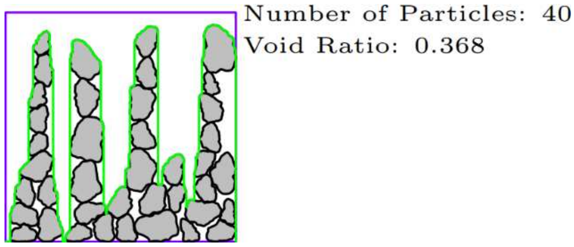

Figure 10. Packing produced using coefficient values from combination O (Table 1) with stability check activated.

1. The coefficient values C_V, C_D, C_T, and C_{CN} were set to 1, 2, 0, and 0.01 respectively. This does not take into account the effect of T, however gives an example of the type of packing that can be achieved. A more in depth investigation into the coefficients is to be completed in future work.

From Figures 9 and 10, it can be determined that combination O for weighting coefficients leaves much to be desired in terms of returning a minimum void ratio. Therefore it can be deduced that different coefficients are needed for different shaped particles.

Figure 10 shows that with the stability check, the void ratio found is higher. This is most likely due to some positions being deemed as unstable, so there are fewer options for particle placement. In addition, the configurations generated in Figure 10 would be unstable if constructed.

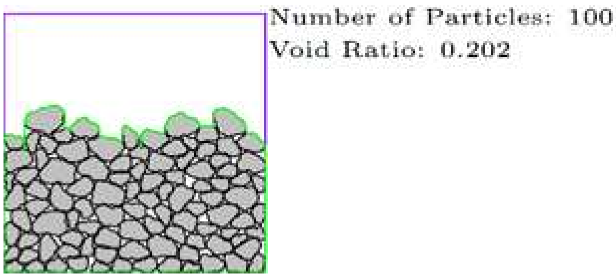

Figure 11. Packing produced using coefficient values C_V=1, C_D=2, C_T=0, C_{CN}=0.01 with stability check activated.

6 SUMMARY AND FUTURE WORK

A method has been developed for additive placement of particles that represent untooled rock in a two-dimensional system using a top-down approach. This was first investigated through the use of tetroids from the video-game Tetris before taking this technique and applying it to more realistic shaped granular particles. Particle placement was scored using a weighting criteria described in Section 2.2. While it is certain that more research is to be done, it is clear to see that construction of an efficient two-dimensional system is possible.

The problem with stability seen in Figure 10 could be fixed by a more rigourous stability check that considers connecting particles or the system as a whole at the expense of additional runtime, or by adjusting the weighting of the other criteria. Other enhancements include allowing choice of particle from an initial set and refining the number of displacement/rotation increments that are evaluated for each particle placement. A two stage approach is under evaluation, with an initial coarse set of position/rotation increments evaluated and the identified optimal position/increment subject to a refinement using a finer set of increments about that position.

7 ACKNOWLEDGEMENTS

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