The paper was published in the proceedings of the 10th European Conference on Numerical Methods in Geotechnical Engineering and was edited by Lidija Zdravkovic, Stavroula Kontoe, Aikaterini Tsiampousi and David Taborda. The conference was held from June 26th to June 28th 2023 at the Imperial College London, United Kingdom.

Proceedings 10th NUMGE 2023
10th European Conference on Numerical Methods in Geotechnical Engineering
Zdravkovic L, Kontoe S, Taborda DMG, Tsiampousi A (eds)

https://doi.org/10.53243/NUMGE2023-275

Effects of spatial variability on Bayesian model updating using measured excavation responses

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ABSTRACT: In view of the challenges in the characterization of soil properties, particularly the spatial variability, through conventional site investigation activities, the measured responses of geosystems have been increasingly used to back-calculate soil properties. However, the presence of spatial variability, which is commonly considered as one of the most critical uncertainties, may add to the challenges in the back-analysis of geosystems using field-response measurements. In this paper, we used a synthetic excavation example to examine the effects of spatial variability on the Bayesian model updating of soil properties using measured excavation responses. Specifically, we adopted the random-field finite-element model to generate synthetic wall deflection measurements. In this way, the spatial variability is manifest in the generated measurements. Bayesian model updating was then carried out in conjunction with the deterministic analysis following the conventional back-analysis procedures adopted by engineering practitioners. We showed that the omission of spatial variability leads to (i) a bias in the posterior distributions of soil properties, and (ii) underestimations in the variations of the predicted wall deflections. The results highlight the critical need of an advanced model updating framework that considers spatially variable soil properties.

Keywords: Back analysis; Bayesian model updating; Spatial variability; Geotechnical excavation; Finite-element method

1 INTRODUCTION

Underground construction activities are usually monitored, and back analysis is often the technique adopted by engineering practitioners to interpret the monitoring data and extract the information of system behaviour (Finno and Calvello, 2005; Wang et al., 2019). Among the many available techniques, Bayesian model updating is a popular back analysis technique that has been successfully applied to geotechnical engineering applications (Kelly and Huang, 2015; Qi and Zhou 2017; Wang et al., 2020). In this regard, this technique is chosen for the present study.

In geotechnical engineering, it is well known that soil properties can vary spatially in both horizontal and vertical directions (Phoon and Kulhawy, 1996), which can significantly influence the performance of geotechnical systems (Griffiths et al., 2009). However, studies on Bayesian model updating of geotechnical systems in spatially variable soils using field-response measurements are still limited. The work of Lo and Leung (2019) is one of the studies on Bayesian model updating of geotechnical excavations considering the spatial variability. However, they do not explicitly update the spatial variability of soil properties.

In this paper, we investigated the effects of spatial variability on the Bayesian model updating of a synthetic geotechnical excavation example using field-response measurements. Specifically, we adopted the random-field finite-element model to generate synthetic wall deflection measurements to manifest the effects of spatial variability in the field-response measurements. Bayesian model updating is then carried out in conjunction with the deterministic analysis following the conventional back-analysis procedures adopted by engineering practitioners. In this way, the effects of spatial variability on conventional back-analyses were investigated.

2 BAYESIAN MODEL UPDATING

In the Bayesian model updating framework, the posterior distribution of the material property values can be calculated as follows:

$$p(\Omega|\hat{y}_i) = k_1 \cdot L(r; \mu_U, C) \cdot p(\Omega) \qquad (1)$$

where Ω represents material parameter values; $\hat{y}_i$ represents field-responses measurements at location $i \in \{1, \ldots, n\}$ with n represents the total number of field-response measurements. k_1 is a normalization constant that makes the PDF valid. $p(\Omega)$ is the prior PDF of the material parameter values to be identified. $L(r; \mu_U, C)$ is the likelihood function, which is defined as the joint probability that the residuals, r, between model predictions and field-response measurements are equal to the values with a given set of material parameter value and uncertainties, with μ_{U} and C being the mean and covariance matrix of the uncertainties. Mathematically, the likelihood function can be expressed as follows:

$$L(r;\mu_U,C)=\frac{1}{2\pi^{\frac{n}{2}}|C|^{\frac{1}{2}}}e^{\left\{-\frac{1}{2}(r-\mu_U)^T C^{-1}(r-\mu_U)\right\}} \qquad (2)$$

Essentially, the likelihood is formulated assuming the uncertainties are normally distributed, and Equation (2) describes a multivariate normal distribution. For simplicity, the uncertainties at the different measurement locations are assumed to be independent.

The calculation of the posterior distribution is facilitated by (i) the response surface method and (ii) the Markov Chain Monte Carlo (MCMC) Simulation using the Metropolis-Hastings Algorithm. Details of the two techniques can be found in Juang et al. (2013) and Qi and Zhou (2017), and will not be elaborated.

3 SYNTHETIC EXCAVATION EXAMPLE

3.1 Model setup

As shown in Figure 1, this excavation example, which is constructed using Optum G2 (Krabbenhøft et al., 2016), consists of a 15 m deep excavation supported by concrete diaphragm walls embedded to a depth of 25 m in two layers of soils with bedrock underlying a layer of residual soils. The retaining wall, which is modelled using elastic plate elements, is socketed into the rock layer supported by three layers of struts, which are modelled using node-to-node anchors. The residual soil is modelled as an elastic-perfectly plastic Mohr-Coulomb material following the effective stress approach (Whittle and Davis, 2006).

This excavation involves 4 stages in total, with stage 1 corresponding to the initial cantilever stage and stage 4 corresponding to the excavation to the formation level. During the analysis, each strutting phase and excavation phase are combined in a single stage. For example, stage 2 combines the installation of the first strut with the excavation to the second strut level.

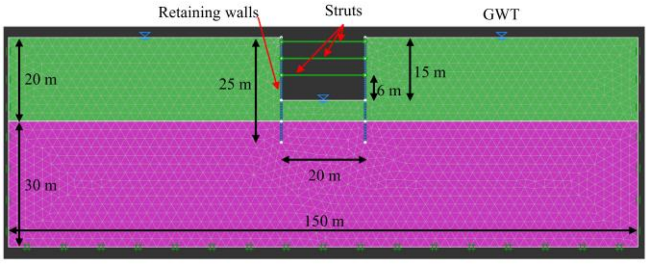

Figure 1. Detailed configurations of the synthetic excavation example.

Table 1. Analysis cases considered in this study.

Case	Synthetic Measurements	Measurement uncertainty	Modelling uncertainty
A	Generated without spatial variability	√	-
B	Generated with spatial variability	√	-
C	Generated with spatial variability	√	Spatial variability

In this study, we presented three back-analysis cases, which are summarized in Table 1. Following the procedures shown in Figure 2, the Bayesian model updating is executed in a progressive manner. Phase 1 of the model updating involves the use of measurements at stages 1 and 2. With the posterior distributions, predictions to wall deflections at stages 3 and 4 can be made. Once the measurements at stage 3 become available, Phase 2 of the model updating will consider the additional measurements and refine the posterior distributions. Wall deflection predictions at stage 4 can then be made.

Table 2. Soil parameter for the example.

Property	Case A	Cases B and C			
		Mean	CoV*	$\mathrm{SoF_v}$*	$\mathrm{SoF_h}$*
Young's modulus, E (MPa)	12	15	0.4	20	1
Friction angle, ϕ (°)	32	28	0.4	20	1
Cohesion, c (kPa)	10	10	-	-	-

*CoV: Coefficient of variation; $\mathrm{SoF_v}$: Vertical scale of fluctuation; $\mathrm{SoF_h}$: Horizontal scale of fluctuation

Table 2 summarizes the soil parameters used in the three cases. Essentially, the Bayesian model updating considers only the Young's modulus and friction angle of the residual soil since cohesion is often not a sensitive parameter to retaining wall deflections. In addition, the CoV of friction angle is intentionally set as 40% to demonstrate how Bayesian model updating can reduce

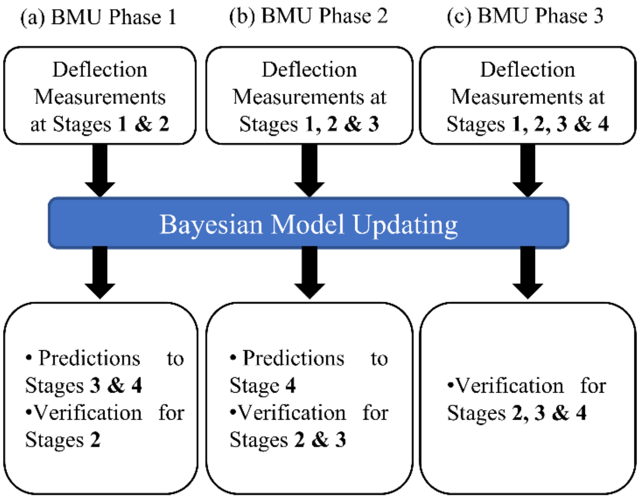

Figure 2. Procedures of the back-analysis and predictions.

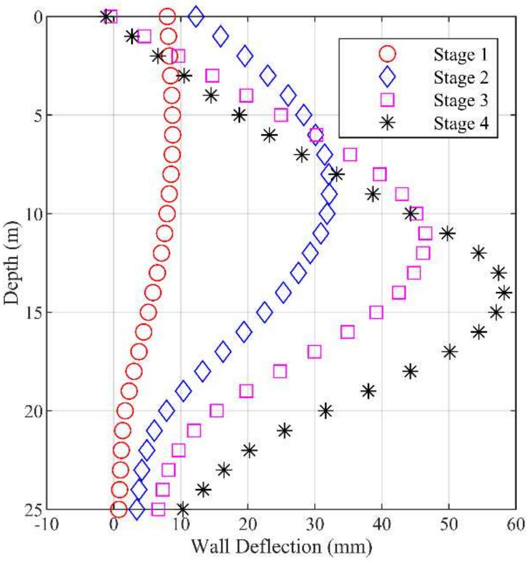

Figure 3. Synthetic wall deflection measurements of Case A.

uncertainties using measurements. In Case A, where no spatial variability is considered. The benchmark values of Young's modulus and friction angle are deterministic and known. For both Cases B and C, where spatial variability is considered, the mean, CoV and scale of fluctuations used are specified in Table 2.

3.2 *Case A*

Case A is considered as the baseline case, which is to validate the Bayesian model updating. In this case, the synthetic field-response measurements are generated using the deterministic model that involves no spatial variability. Figure 3 shows the synthetic wall deflection measurements generated using a deterministic model and the parameter values shown in Table 2. After adding some measurement noises to the data shown in Figure 3, Bayesian model updating is carried out following the three phases shown in Figure 2.

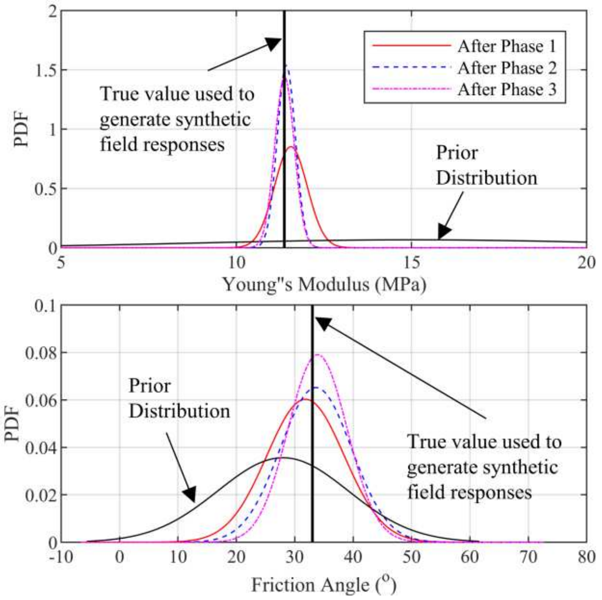

Figure 4. Posterior distributions sampled in Case A.

The posterior distributions are shown in Figure 4. The vertical line indicates the true parameter values (e.g., Table 2) used to generate the synthetic measurements. As shown in the figure, the Bayesian model updating sampled reasonable posterior distributions right after Phase 1, and the mean values of the posterior distributions agree well with the true parameter values.

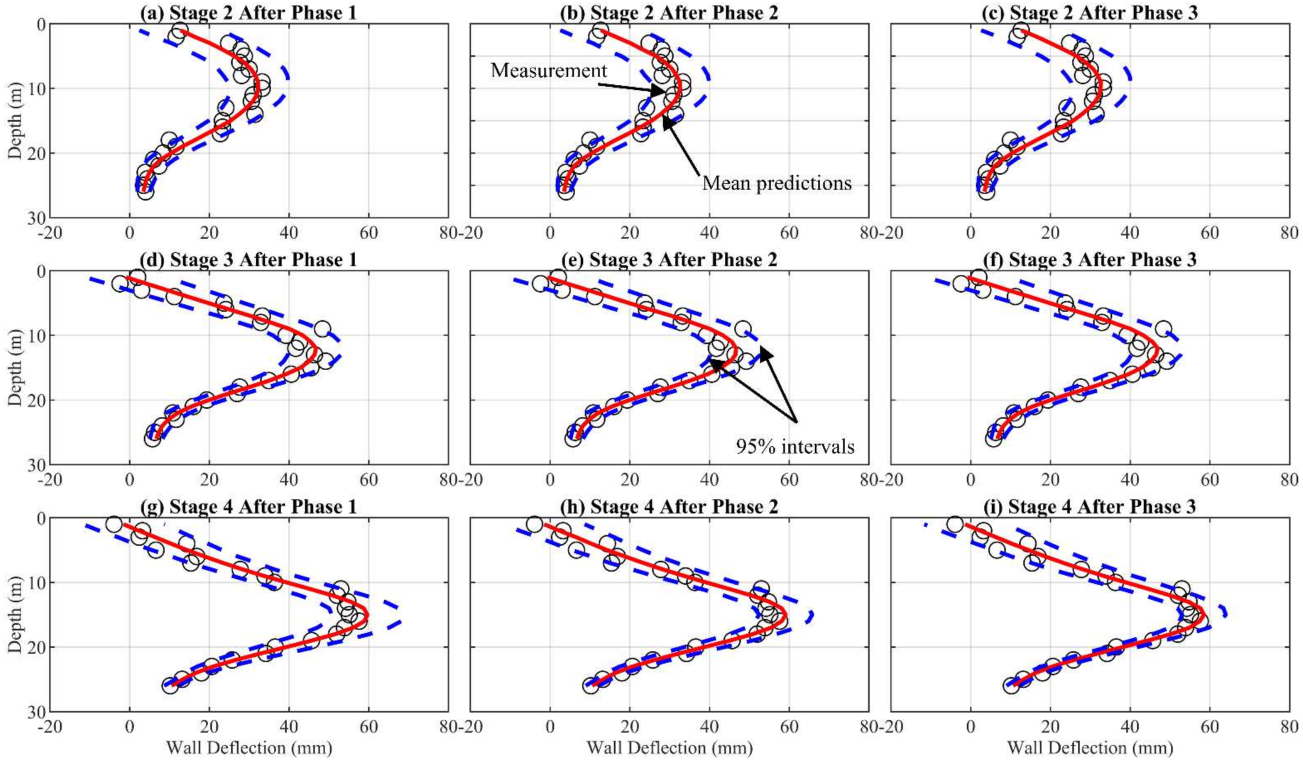

Figure 5. Wall deflection predictions made using the posterior distributions after each phase of model updating (Case A).

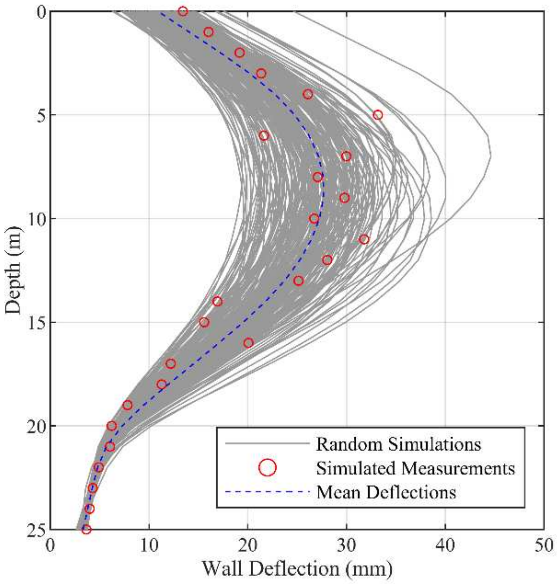

Figure 6. Synthetic wall deflection measurements at stage 2 in Case B.

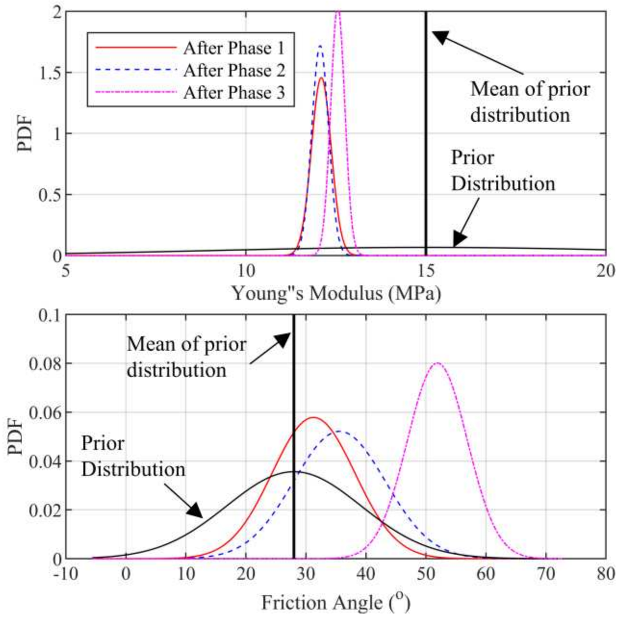

Figure 7. Posterior distributions sampled in Case B.

As more measurements are considered in the model updating exercise, the standard deviation of the posterior distributions of both parameters decreases compared to the prior distributions, indicating the effectiveness of the Bayesian model updating in using field measurements for back-analyses.

Figure 5 shows the predictions made using the posterior distributions. The mean predictions agree reasonably well with the measurements for all cases. With the posterior distributions obtained after Phase 1 of the model updating, the wall deflections at stage 4 can already be reasonably predicted. In addition, the 95% confidence intervals reasonably bound the measurements, indicating that the Bayesian model updating also reasonably estimates the uncertainties associated with system behaviour.

3.3 *Case B*

The effects of spatial variability on the Bayesian model updating are investigated. Synthetic measurements are first generated. Following the parameter values shown in Table 2, 200 Monte-Carlo realizations are

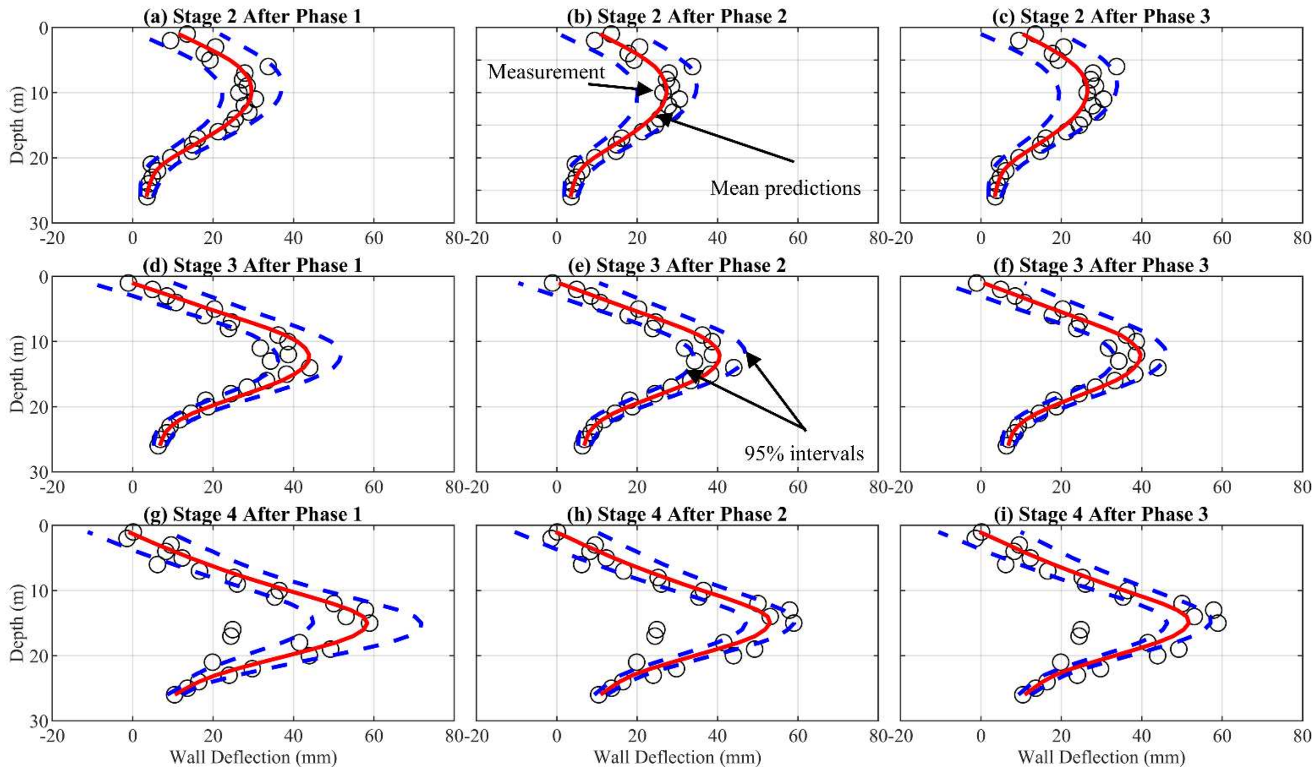

Figure 8. Wall deflection predictions made using the posterior distributions after each phase of model updating (Case B).

simulated. Figure 6 shows the 200 wall deflection profiles at excavation stage 2. The standard deviation of wall deflections at all measurement depth is then calculated. For each measurement depth, the mean wall deflection, which is then perturbated with the quantified uncertainty, is taken as the synthetic measurement. Figure 6 also shows the synthetic measurements at excavation stage 2. The same procedures are then repeated for all the four stages to generate the full set of field-response measurements. In this way, the effects of spatial variability are manifest in the measurements to resemble realistic field-response measurements. Deterministic model is used in the Bayesian model updating following the conventional engineering practices where soil properties are spatially homogeneous. Following this configuration, we would be able to understand how the spatial variability has influenced the Bayesian model updating in practice.

Figure 7 shows the posterior distributions of the two parameters. Biases in the posterior distributions are observed. For example, the posterior mean of the Young's modulus is lower than the mean of the prior distribution, which is the true statistic. Similarly, the posterior mean of friction angle is much larger than the true statistic. These observations may suggest that the conventional Bayesian model updating in practice could have identified biased posterior distributions owing to the omission of spatial variabilities.

Figure 8 shows, in a similar manner as in Figure 5, the predicted wall deflections. It can be observed that although the mean predictions agree reasonably well with the synthetic measurements at stages 2 and 3, wall deflections at stage 4 are underpredicted at some depths. Most importantly, the 95% confidence intervals failed to bound all the measurement data for the three stages, which indicates that the Bayesian model updating consistently underestimates the uncertainties in the system. This is explained by the fact that uncertainties related to the spatial variability are not considered in the deterministic model used for the Bayesian model updating.

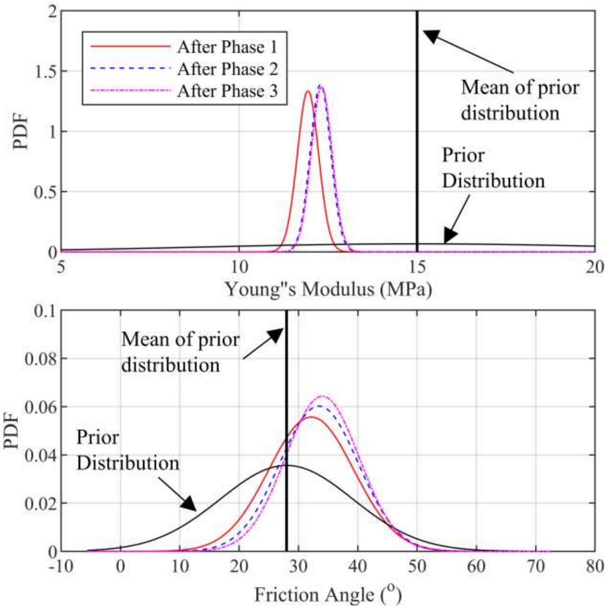

Figure 9. Posterior distributions sampled in Case B.

The results shown in Figures 7 and 8 may look con-

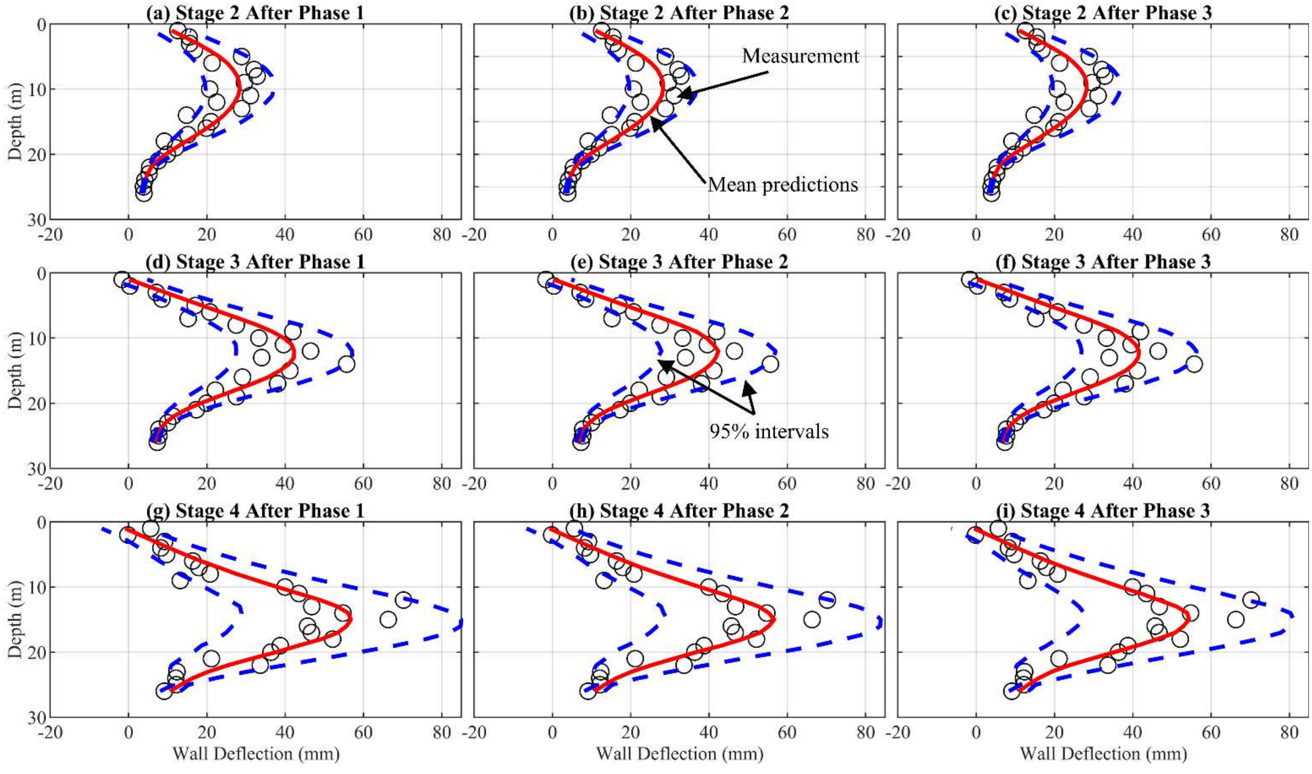

Figure 10. Wall deflection predictions made using the posterior distributions after each phase of model updating (Case C).

tradictory. While there are biases in the posterior distributions, the predictions are not always biased. This is the result of "parameter compensation" (Sakurai et al., 2003; Juang et al., 2013). In this case, there is a systematic bias between the random-field finite element model used to generate the synthetic measurements and the deterministic model used for the Bayesian model updating. Therefore, the model updating must compensate for the systematic bias in order to match the measurements, which eventually results in biases in the posterior distributions. This is not desirable. Although wall deflections are seemingly well predicted in Figure 8, the material parameter values are not correct, which may yield wrong predictions if the information is used in other projects. This explains why the predictions at stage 4 are not well predicted. The systematic biases at stage 4 are different from those at other stages; therefore, the posterior distributions are more reasonable at stages 2 and 3 but not at stage 4.

3.4 *Case C*

The uncertainties related to spatial variability are considered in this case. With reference to Figure 6, for each measurement depth, the standard deviation of the wall deflections is calculated based on the 200 simulations. By assuming a normal distribution with a mean of zero and the calculated standard deviation, the uncertainty related to the spatial variability can be incorporated into the model updating through the covariance matrix, C, in Equation (2).

Figure 9 shows the posterior distributions after incorporating the quantified uncertainty. As can be seen in the figure, biases in the identified posterior distributions still exists although the bias in the posterior distributions of the friction angle are lower compared to the results shown in Figure 7. The consideration of the uncertainty of spatial variability could potentially explain this observation. However, the systematic bias in the deterministic model used is still not adequately addressed, and parameter compensation still occurs.

As can be seen in Figure 10, the 95% confidence intervals can now reasonably bound the measurements. This is due to the use of an additional uncertainty term in the model updating, which improves the overall estimation of the variations in the system. However, it cannot be denied that the posterior distributions are biased against the true statistics.

4 CONCLUSION

In summary, through a simple synthetic excavation example, the effects of spatial variability on the Bayesian model updating using field-response measurements are investigated. The random-field finite-element model is used to generated synthetic measurements, which are believed to resemble realistic system behaviour. The results highlight that the Bayesian model updating based on a deterministic model with assumed homogeneity may result in biases in the posterior distributions, and the variability in the system behaviour may be underestimated. These observations further suggest that there is a need to revamp the conventional back-analysis practices and consider the effects of spatial variability for improved and rigorous back-analyses.

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