Proceedings 10th NUMGE 2023
10th European Conference on Numerical Methods in Geotechnical Engineering
Zdravkovic L, Kontoe S, Taborda DMG, Tsiampousi A (eds)

https://doi.org/10.53243/NUMGE2023-39

Towards the development of a new isotach elastoplastic constitutive model for soft soils

W. Wang[1], A.M. Lester[1], G. Kouretzis[1], J.A. Pineda[1], J.P. Carter[1]

[1]School of Engineering, University of Newcastle, Australia

ABSTRACT: This paper presents some features of a new isotach elastoplastic constitutive model for soft soils, named as the Modified Hunter Clay (MHC) model, developed from the isotach elastoplastic model of Yang et al. (2016), named as the Original Hunter Clay (OHC) model. Both models are capable of reproducing the three key behaviours of destructuration, anisotropy and rate dependency which feature in the mechanical response of soft soils. A key development in the MHC model is the replacement of creep strain rate with viscoplastic strain rate, which results in a simplified understanding and modelling of the time dependent behaviour of soft soils. Additionally, to overcome the difficulty in dealing with different time scales in the OHC model, the MHC model considers creep as a strain rate phenomenon and incorporates a new relationship to describe the evolution of viscoplastic strain rate with time, based on the time resistance concept. This modification eliminates the need to use absolute time, which refers to the geological history of the soil. A single element simulation of the model has been carried out, and promising results are obtained in predicting the 1D compression behaviour of Batiscan clay (Leroueil et al., 1985).

Keywords: isotach; elastoplasticity; constitutive model; soft soils

1 INTRODUCTION

The mechanical response of natural soft soils typically features destructuration of their internal fabric, the evolution of fabric anisotropy, and dependence on strain rate. Various constitutive models have been proposed to account for one or more of these three key behaviours (e.g., Liu and Carter, 2002). Accurate prediction of the mechanical response of natural soft soils is crucial for the safe and economical design of civil infrastructure.

Yang et al. (2016) proposed a hierarchical constitutive model for natural soft soils, named as the Original Hunter Clay (OHC) model in this paper, to take into account all three of the key behaviours. The OHC model is classified as an isotach elastoplastic model, as it is formulated within the framework of conventional elastoplasticity and does not depend on the overstress theory adopted in many other rate dependent models. Additionally, the OHC model incorporates a unique yield stress-strain rate relationship based on the isotach concept for rate dependent deformation of soils (Leroueil, 2006; Suklje, 1957), and a characteristic strain rate-time relationship. The OHC model has its basis in critical state soil mechanics, and in its simplest form reduces to the Modified Cam Clay (MCC) model.

The OHC model was formulated in the triaxial (q:p:v) space based on 1D loading conditions. It was utilised for predicting the mechanical behaviour of soft soils beneath a trial embankment (Yang and Carter, 2018; Kelly et al., 2018). Lester (2020) presented a 3D formulation of the OHC model with some refinements to the original formulation which was subsequently implemented into a finite element code (Lester et al., 2021).

Despite the success of the aforementioned studies, the OHC model still suffers from some shortcomings that can be improved upon in its application to practical problems. The OHC model describes strain rate dependency by relying on a relationship between the so-called creep strain rate and the yield pressure. However, physical evidence suggests that the phenomena of creep and plasticity cannot be treated separately, as only their combined effect is measurable in the laboratory or in the field. Therefore, it is worth exploring other, more robust means to describe strain rate dependency. Numerical implementation of the OHC model also requires consideration of two different time scales. The first is the analysis time, with its origin taken as the beginning of the simulation. The second is the time associated with the creep strain rate, also referred to as the natural time in this paper, with its origin related to the geological and loading history of the soil. However, there are significant uncertainties when it comes to the actual value of the natural time, and settling upon a specific value can involve significant engineering judgement. This introduces a high degree of complexity in applying the OHC model for the simulation of practical problems.

The aim of this paper is to present some features of a new isotach elastoplastic constitutive model for soft soils under development, referred to as the Modified Hunter Clay (MHC) model. A key development in the MHC model is the replacement of creep strain rate with viscoplastic strain rate, which results in a simplified understanding and modelling of the strain rate dependency of soft soils. Additionally, to overcome the difficulty in dealing with different time scales in the OHC model, the MHC model incorporates a new relationship to describe the evolution of viscoplastic strain rate with time based on the time resistance concept. A single element implementation of the MHC model has been carried out, and simulations undertaken to predict the 1D compression behaviour of Batiscan clay, the results of which are presented in this paper.

2 CONSTITUTIVE FORMULATION

The following sections provide a detailed description of the MHC model in the triaxial stress space by using the mean effective stress p and deviator stress q, as well as the corresponding volumetric strain ε_p, and deviatoric strain ε_q. For ease of notation, the usual prime symbol is dropped for effective stress components. As the MHC model has been developed based on modification of the OHC model, all features of the MHC model are inherited from the OHC model unless otherwise stated.

2.1 *Elastic behaviour*

The MHC model has its basis in the MCC model, with the elastic stress-strain relations for MHC being the same as those for MCC. This includes the decomposition of total strain into elastic and plastic components, and the dependence of the elastic moduli on the stress state and Poisson's ratio. It is assumed in the MHC model that elastic deformation is independent of destructuration, anisotropy and rate dependency.

2.2 *Destructuration*

Breakdown of the fabric and inter-particle bonding in natural soft soils i.e., destructuration, is accounted for in the MHC model. It is quantified by varying the stress-strain response in the $\upsilon : \ln p$ plane as a function of the accumulated plastic strain. The amount of destructuration corresponds to the difference in compressibility between the natural and reconstituted states of the soil.

As shown in Figure 1, and following the approach of Lester (2020), the destructuration law for the MHC model can be expressed as:

$$\lambda = \lambda_\infty + (\lambda_0 - \lambda_\infty)\, exp\left(-k_d \varepsilon_r^p\right) \quad (1)$$

where λ is the slope of the NCL. This appears in the equation of the NCL as:

$$\upsilon = \upsilon_\lambda - \lambda lnp \quad (2)$$

where υ_λ is the value of υ at a unit value of p; λ_0 and λ_∞ are the maximum and minimum values of λ,

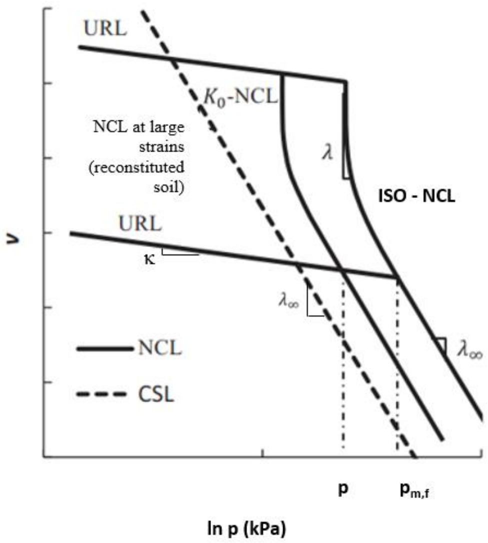

Figure 1. Schematic of compression behaviour for natural soft soils (after Yang et al., 2016)

respectively, representing the compressibility of the soil in its initial state and that obtained at large strains where the soil structure has been erased; k_d is then a material constant controlling the rate of destructuration; ε_r^p is the accumulated plastic strain which is calculated as:

$$\varepsilon_r^p = \int d\varepsilon_r^p = \sqrt{\frac{\left(d\varepsilon_p^p\right)^2 + \left(d\varepsilon_q^p\right)^2}{2}} \quad (3)$$

where ε_p^p and ε_q^p are the plastic volumetric strain and plastic deviatoric strain respectively.

Unloading-reloading behaviour is described by a straight line in the semi-logarithmic compression plane (Figure 1), with the URL written as:

$$\upsilon = \upsilon_\kappa - \kappa lnp \quad (4)$$

where κ is the slope of the URL and υ_κ is the value of υ at a unit value of p along the URL.

2.3 *Anisotropy*

The MHC model accounts for inherent (i.e., pre-existing) anisotropy by adopting rotated and distorted yield and plastic potential surfaces. The shapes of the yield surface and plastic potential in the q:p plane are depicted in Figure 2. The MHC model allows different parameters to be adopted for the yield surface and plastic potential, hence incorporating a non-associated flow rule for realistic modelling of soft soil behaviour.

Assuming compressive stresses are positive, the equations for the yield and plastic potential curves in the q:p plane, denoted by f and g respectively, can be expressed as:

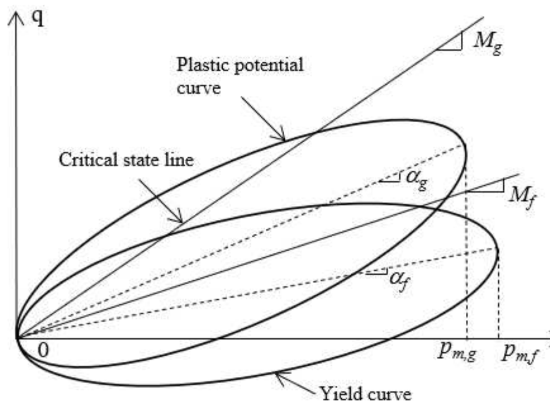

Figure 2. MHC yield and plastic potential curves in the q:p plane (after Yang et al., 2016)

$$f = \left(q - \alpha_f p\right)^2 - p\left(M_f^2 - \alpha_f^2\right)\left(p_{m,f} - p\right) = 0 \quad (5)$$

$$g = \left(q - \alpha_g p\right)^2 - p\left(M_g^2 - \alpha_g^2\right)\left(p_{m,g} - p\right) = 0 \quad (6)$$

where $p_{m,f}$ is the maximum value of p and quantifies the yield surface size; α_f is the slope of the major axis of the yield surface and represents its inclination in the q:p plane; M_f is a factor controlling the shape of the yield surface; $p_{m,g}$ and α_g have the same meanings as $p_{m,f}$ and α_f, but applied to the plastic potential curve; M_g is the slope of the critical state line (CSL).

The plastic dilatancy (d_g^p) defining the ratio of plastic strain increments can be derived as follows:

$$d_g^p = \frac{d\varepsilon_q^p}{d\varepsilon_p^p} = \frac{2(\eta - \alpha_g)}{M_g^2 - \eta^2} \quad (7)$$

where η is the stress ratio which is defined as:

$$\eta = \frac{q}{p} \quad (8)$$

The MHC model also accounts for induced anisotropy, where changes in the anisotropy of the soil fabric develop during loading. This is achieved via the introduction of a rotational hardening law, which is described in Section 2.6.2.

2.4 *Formulation of strain rate dependency relationship in the MHC model*

Different from the OHC model, where strain rate dependency relies on the creep strain rate, an attempt was made in the MHC model to incorporate viscoplastic strain rate ($\dot{\varepsilon}_{vp}$). This strain rate includes the combined effects of plasticity and creep with respect to time, and is directly measurable. The introduction of $\dot{\varepsilon}_{vp}$ into the MHC model simplifies the understanding and modelling of the strain rate dependency of soils.

According to Watabe et al. (2012), the effect of strain rate on soil behaviour can be described using an isotach relationship relating preconsolidation pressure to viscoplastic strain rate. Such a relationship can be derived, for instance, from long-term consolidation tests and/or constant rate of strain (CRS) tests. The constitutive expression that relates the preconsolidation pressure p_c to the viscoplastic strain rate $\dot{\varepsilon}_{vp}$ is:

$$ln\frac{p_c - p_{cL}}{p_{cL}} = c_1 + c_2 ln\dot{\varepsilon}_{vp} \quad (9)$$

where p_{cL} is the lower limit of the preconsolidation pressure at an infinitesimal strain rate; c_1 and c_2 are isotach parameters determined from experimental data; $\dot{\varepsilon}_{vp}$ may be defined as $d\varepsilon_{vp}/dt$, and the viscoplastic strain ε_{vp} may be calculated as the difference between the total strain ε and the elastic strain ε_e.

Differentiating Equation (9) yields:

$$\frac{dp_c}{d\dot{\varepsilon}_{vp}} = \frac{c_2}{\dot{\varepsilon}_{vp}}\left(\frac{p_c - p_{cL}}{p_{cL}}\right) \quad (10)$$

Equation (10) is a new relationship adopted in the MHC model that quantifies the effect of the viscoplastic strain rate on the preconsolidation pressure. Defining p_{c0} as the preconsolidation pressure for a reference $\dot{\varepsilon}_{vp}$ of 10^{-7} s^{-1}, Watabe et al. (2012) reported that $p_{cL}/p_{c0} = 0.7$ for Osaka Bay clay samples taken from various depths. They also considered this value of p_{cL}/p_{c0} appropriate for other clays on the basis of experimental data.

2.5 *Evolution of viscoplastic strain rate with time in the MHC model*

In the OHC model, Yang et al. (2016) proposed a characteristic creep strain rate versus time relationship estimated from curve fitting of long term creep test results. The relationship relies on natural time to describe the evolution of the creep strain rate. However, determining values of natural time is difficult as they are dependent on the geological and loading history of the soil. To circumvent this problem, the MHC model incorporates a new relationship describing the evolution of viscoplastic strain rate according to the time resistance concept. Janbu (1969) defined the time resistance of a soil as:

$$R = \frac{increment\ in\ time}{increment\ in\ strain} = \frac{dt}{d\varepsilon} \quad (11)$$

Figure 3 defines the time resistance R and resistance number r_s (the slope of the time resistance versus time curve) using the results of an incremental loading oedometer test. The curve is divided into three zones:

Zone A corresponds to the early stages of primary consolidation, Zone B is a transition zone where the curve changes from parabolic to quasi-linear, while Zone C corresponds to the stage where the excess pore pressure has become zero i.e., secondary consolidation.

With reference to Figure 3, the following relationship can be established:

$$r_s = \frac{dR}{dt} = \frac{d\left(\frac{1}{\dot{\varepsilon}}\right)}{dt} \tag{12}$$

Further mathematical manipulation leads to an expression for the increment of strain rate as:

$$d\dot{\varepsilon} = -r_s \dot{\varepsilon}^2 dt \tag{13}$$

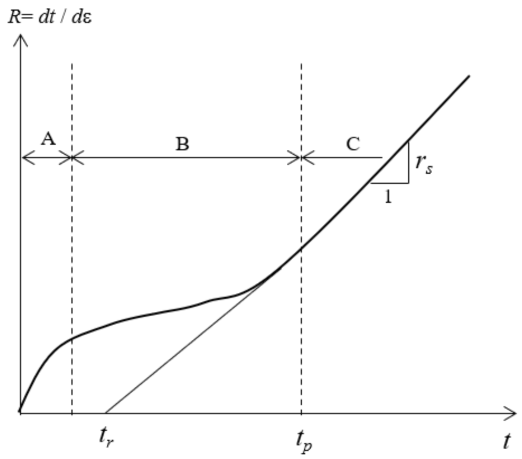

Figure 3. Time resistance R and resistance number r_s from an incremental loading oedometer test (after Janbu, 1969).

In the context of 1D incremental loading tests, the strain rate observed during the secondary consolidation stage is the viscoplastic strain rate. Thus, a relationship between the time resistance number r_s and the coefficient of secondary consolidation in terms of strain, $c_{\alpha\varepsilon}$, can be established as:

$$c_{\alpha\varepsilon} = \frac{2.3}{r_s} \tag{14}$$

Combining Equations (13) and (14) results in an expression that describes the evolution of viscoplastic strain rate under 1D loading conditions:

$$d\dot{\varepsilon}_{vp} = -2.3 \frac{\dot{\varepsilon}_{vp}{}^2}{c_{\alpha\varepsilon}} dt \tag{15}$$

Equation (15) allows the MHC model to capture time-dependent soil behaviour in a much more straightforward manner than the OHC model. In fact, the change in $\dot{\varepsilon}_{vp}$ in the MHC model is determined directly from the time increment, as well as the current values of $\dot{\varepsilon}_{vp}$ and $c_{\alpha\varepsilon}$. This will greatly simplify numerical modelling of 1D problems involving variation of the imposed strain rate, and boundary value problems where strain rates vary across the analysis domain.

2.6 Hardening laws

Two hardening laws i.e. volumetric and rotational hardening, are built into the MHC model, and are used to describe the evolution in size and inclination of the yield and plastic potential surfaces with plastic straining.

2.6.1 Volumetric hardening law

The change in size of the yield surface with plastic straining is controlled by a volumetric hardening law. It comprises a conventional strain rate-independent volumetric hardening as well as a strain rate dependent (isotach) hardening, and can be expressed as:

$$p_{m,f} = \frac{\partial p_{m,f}}{\partial \varepsilon_p^p} d\varepsilon_p^p - \frac{\partial p_{m,f}}{\partial \dot{\varepsilon}_{vp}} < -d\dot{\varepsilon}_{vp} > \tag{16}$$

where $\frac{\partial p_{m,f}}{\partial \varepsilon_p^p} = \frac{v p_{m,f}}{\lambda - \kappa}$ (17)

and $\frac{\partial p_{m,f}}{\partial \dot{\varepsilon}_{\mathrm{vp}}} = \frac{\partial p_c}{\partial \dot{\varepsilon}_{\mathrm{vp}}}$ is given by Equation (10).

The Macaulay brackets < > are applied to the change in viscoplastic strain rate $d\dot{\varepsilon}_{\mathrm{vp}}$ such that only a decrease in this strain rate can induce an isotropic reduction in the size of the yield surface.

2.6.2 Rotational hardening law

Following the approach adopted in the OHC model, the MHC model uses a rotational hardening law to quantify the change in the inclination of both the yield and plastic potential surfaces, in terms of the anisotropic variables α_f and α_g. The purpose of this is to capture the effect of induced anisotropy on the soil fabric. The adopted rotational hardening law is based on the concept of an equilibrium state which is attained by the soil fabric under an imposed stress ratio η. Changes in the inclination of the yield surface α_f and plastic potential surface α_g are calculated as:

$$d\alpha_{f/g} = h_{f/g}{}^{\omega_{f/g}} \left(\alpha_{E,f/g} - \alpha_{f/g}\right) d\varepsilon_r^p \tag{18}$$

where the subscript f/g denotes either f or g, depending on whether the equation is being applied to the yield surface or plastic potential; $\omega_{f/g}$ denotes the rotation rates of the yield surface and plastic potential:

$$\omega_{f/g} = \left|\frac{\alpha_{f/g} - \alpha_{E,f/g}}{M_{f/g}}\right|^{\left|\frac{\eta - \alpha_{E,f/g}}{M_{f/g}}\right|} \tag{19}$$

where $\alpha_{E,f/g}$ denotes the equilibrium inclinations $\alpha_{E,f}$ and $\alpha_{E,g}$ of the yield surface and plastic potential, respectively, which depend on the stress ratio η. Readers are referred to Yang et al. (2016) for details on how $\alpha_{E,f}$ and $\alpha_{E,g}$ may be calculated.

3 SINGLE ELEMENT SIMULATION

The MHC model has been implemented into the University of Newcastle's in-house single Gauss point program SASSE. This implementation has allowed numerical simulations of laboratory tests performed on Batiscan Clay (Leroueil et al., 1985) to be carried out, and comparison of the results with predictions of the OHC model.

Batiscan clay is a sensitive Canadian clay which has been extensively tested by Leroueil et al. (1985) and others. The test data were used by Yang et al. (2016) in assessing the performance of the OHC model. As the MHC model has its basis in the OHC model, values for some of the material parameters calibrated by Yang et al. (2016) were adopted in the simulations using the MHC model undertaken here. Material parameters for the simulations are presented in Table 1. It is noted that the simulations are confined to 1D loading conditions, and as such, the imposed stress anisotropy on the soil and hence the values of α_f and α_g remain constant throughout the analysis.

Table 1. Material parameters for simulation of Batiscan clay

Symbol	Description	Unit	Value
μ	Poisson's ratio	-	0.25
κ	Slope of URL	-	0.061
M_f	Yield Surface shape factor	-	1.2
M_g	Plastic potential shape factor	-	1.2
λ_0	Initial slope of NCL	-	12
λ_∞	Slope of NCL at large plastic strain	-	0.037
k_d	Destructuration rate	-	23
α_f	Yield Surface inclination	-	0.345
α_g	Plastic potential surface inclination	-	0.345
$\alpha_{E,f,max}$	Maximum yield surface inclination	-	0.39
e_0	Initial void ratio	-	2.15
$p_{f,0}$	Initial yield surface size	kPa	66
$c_{\alpha e}$	Coefficient of secondary consolidation	-	0.05
$\dot{\varepsilon}^*_{vp}$	Reference viscoplastic strain rate	s^{-1}	$1.07X10^{-7}$
c_1	Isotach parameter	-	1.45
c_2	Isotach parameter	-	0.14

The material parameters in Table 1 were determined from best fitting of the one-dimensional compression curve corresponding to a viscoplastic strain rate of 1.07 x 10^{-7} s^{-1}. Therefore, a reference strain rate of 1.07 x 10^{-7} s^{-1} has been chosen for modelling. It is noted that other reference strain rates can be adopted with properly calibrated material parameters.

Figure 4 presents a stress-strain plot comparing four CRS K_0 tests on Batiscan clay with simulation results obtained using the MHC and OHC models in SASSE. It is noted that the MHC model predicts the behaviour observed in the laboratory tests quite well.

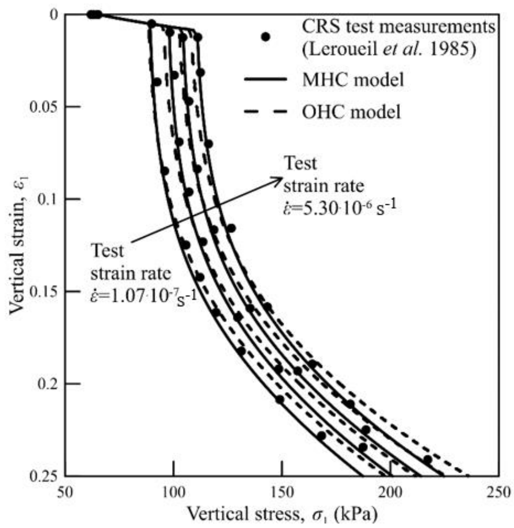

Figure 4. Comparison of measured and simulated CRS K_0 compression tests on Batiscan Clay

Almost parallel CRS K_0 compression curves are produced from the simulations, illustrating the isotach effect on soil behaviour. The slope of the compression curve past yielding is initially steep, and gradually becomes shallower with increased loading as a result of destructuration. There are some minor differences between the results obtained with the OHC and MHC models, and these are attributed to the influence of the different strain rate dependency relationships adopted in the two models. Nevertheless, the aim of the changes introduced in the MHC model is not to improve model predictions, but to facilitate implementation and use of the model in practical problems.

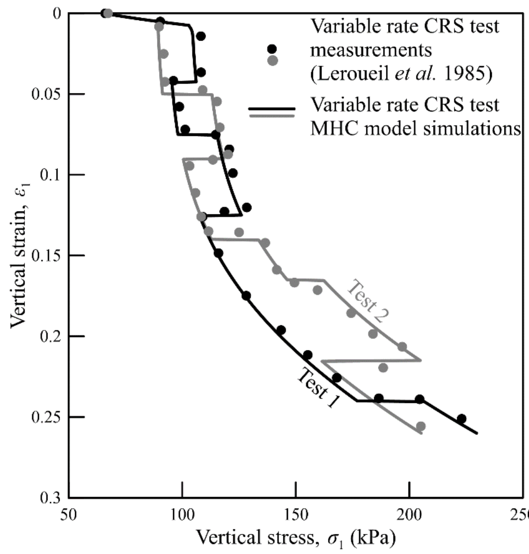

Figure 5. Comparison of measured and simulated CRS K_0 compression tests on Batiscan Clay with changing strain rates

Transitional CRS compression tests, in which the strain rate is changed at various strain levels, were conducted on Batiscan clay by Leroueil et al. (1985). The laboratory tests were simulated with SASSE using the MHC model, and the results are presented in Figure 5. It is observed that the transitions between the isotach lines corresponding to the different strain rates are well predicted by the MHC model. It is noted that the OHC model can also be adopted to simulate transitional CRS compression tests. However, this involves additional user intervention with manual input of different applied strain rates required, and this creates an extra level of complexity during numerical modelling.

Long-term creep tests on Batiscan clay were also carried out by Leroueil et al. (1985). For validating the proposed relationship between viscoplastic strain rate and time presented in Equation 15, numerical predictions of two creep tests on Batiscan clay were undertaken using the MHC model. One had a final vertical stress of 139 kPa, and the other a final vertical stress of 151 kPa. The results of the simulations presented in Figure 6 demonstrate good agreement between simulations and experimental results.

It is noted that the single element simulations carried out with SASSE are unable to model the dissipation of excess pore water pressure. A computer program with the capability of solving boundary value problems involving dissipation of excess pore water pressure would be more appropriate for modelling the entire experimental procedure, particularly for the long-term creep tests. Such modelling will be undertaken in future studies. Nevertheless, the comparison of the SASSE simulations against experimental data demonstrates that the MHC model exhibits great potential in predicting the behaviour of soft soils, while simplifying the analysis procedure compared with the OHC model.

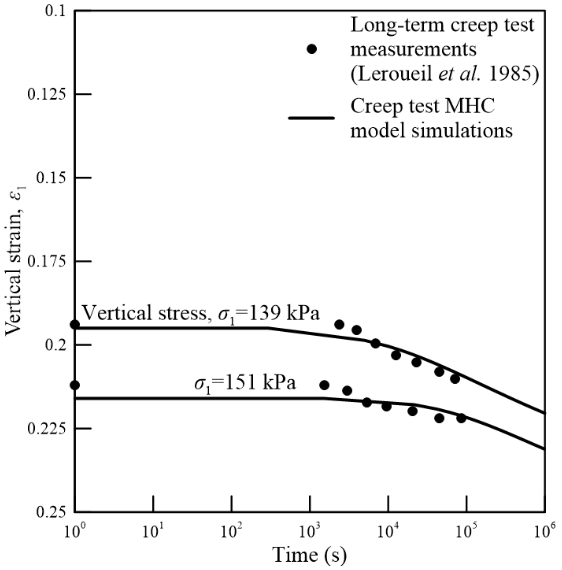

Figure 6. Comparison of creep test results for Batiscan clay with simulations

4 CONCLUSION

This paper has presented some features of a new isotach elastoplastic constitutive model for soft soils under development, i.e., the MHC model. The model is capable of reproducing three key mechanical behaviours of soft soil, namely destructuration, anisotropy and rate dependency. Key developments in the MHC model include the replacement of creep strain rate with viscoplastic strain rate for modelling the strain rate dependency of soft soils, and a new relationship to describe the evolution of viscoplastic strain rate with time based on the time resistance concept. Implementation of the MHC model was carried out in a single Gauss point program, and satisfactory results were obtained in predicting the 1D compression behaviour of Batiscan clay. Future studies will involve refinement of the model to capture shear behaviour of soft soils, and extension of the model to three-dimensional stress space. This will be informed by the results of specialised laboratory tests on truly undisturbed soft clay samples.

5 REFERENCES

Janbu, N., 1969. The resistance concept applied to deformations of soils, *Proc. of the 7th Intl. Conference on Soil Mechanics and Foundation Engineering*, 193-196

Kelly, R., Sloan, S., Pineda, J.A., Kouretzis, G. and Huang, J. 2018. Outcomes of the Newcastle symposium for the prediction of embankment behaviour on soft soil. *Computers and Geotechnics*, 93, 9-41.

Leroueil, S., Kabbaj, M., Tavenas, F., and Bouchard, R. 1985. Stress-strain-time rate relation for the compressibility of sensitive natural clays. *Geoetchnique*, 35, No.2: 159-180.

Leroueil, S. 2006. The Isotache approach. Where are we 50 years after its development by Professor Šuklje? 2006 Prof. Šuklje's Memorial Lecture. *Proceedings of the XIII Danube- European Conference on Geotechnical Engineering*, (2): 55-88, Ljubljana, Slovenia.

Lester, A. M. 2020. *Advanced Numerical Methods for Predicting the Behaviour of Soft Clay Soils*, Ph.D. Thesis, University of Newcastle.

Lester, A.M, Kouretzis, G.P., Pineda, J.A., Carter, J.P. 2021. Finite element implementation of an isotach elastoplastic constitutive model for soft soils. *Computers and Geotechnics*, 136, 104248.

Liu, M. D., Carter, J. P. 2002. A structured cam clay model. *Canadian* Geotechnical *Journal*, 39, 1313–1332.

Sukjie, L. 1957, The analysis of the consolidation process by the isotache method, *The 4th International Conference on Soil Mechanics and Foundation Engineering,* pp 200-206., Butterworths Scientific Publications, London, England.

Yang C., Carter, J.P., Sheng D., Sloan, S.W. 2016. An isotach elastoplastic constitutive model for natural soft clays, *Computers and Geotechnics,* 77:134-155.

Yang, C., Carter, J. P. 2018. 1-D finite strain consolidation analysis based on isotach plasticity: Class A and Class C predictions of the Ballina embankment. *Computers and Geotechnics*, 93, 42-60.