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Prediction of hydraulic conductivity characteristics of slurries using inverse analysis

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ABSTRACT: The finite strain consolidation theory governs the self-weight consolidation of slurries such as mine tailings and dredged clays. It is crucial to understand the consolidation behavior of slurries to design safe and efficient slurry storage facilities. The theory considers compressibility and permeability as functions of void ratio. Experiments for determining the hydraulic conductivity function are time-consuming and require complicated setups. This work explores an inverse analysis approach for predicting the hydraulic conductivity parameters of slurries when the compressibility function is known. The finite difference method is used to solve the governing equation in the forward analysis and the Artificial Bee Colony (ABC) algorithm is used for the backward analysis. Excess pore pressure versus time data for the base of the consolidating slurry sample is the major input for inverse analysis. The method is tested using synthetic input data for two slurries. Predicted functions show good agreement with results obtained from the literature. This method offers a robust and relatively faster approach for the estimation of hydraulic conductivity function parameters of slurries.

Keywords: Finite strain consolidation; Hydraulic conductivity function; Inverse analysis.

1 INTRODUCTION

Every year slurry wastes such as mine tailings and dredged clays are produced in large volumes all over the world. The storage and disposal of these materials pose a challenge due to the volume of production, consistency of the slurry, low permeability, and slow settlement. Once deposited, these slurries undergo excessive settlement under self-weight. Layers of slurry are deposited one after the other, allowing sufficient time in between to avoid excess pore pressure build-up within the deposit. Dredged clays are used in multi-million dollar land reclamation projects (Lee et al., 1987). Mine tailings are stored in large tailings storage facilities (TSF) that occupy a large area of land. The database of the World Information Service on Energy (WISE, 2023) provides an overview of the catastrophic TSF failures since 1960. It is crucial to understand the settlement behaviour of slurries to ensure the safe and cost-effective disposal of these materials.

The settlement behaviour of slurries is governed by the finite-strain consolidation theory (Gibson et al., 1967). The theory accounts for the considerable settlement of slurries under self-weight, by taking hydraulic conductivity and compressibility of the settling material as functions of void ratio. The relative motion of solid particles and fluid particles is also considered in this theory. The experimental determination of hydraulic conductivity function using seepage-induced consolidation (Abu-Hejleh et al., 1996; Imai, 1979; Robinson et al., 2003), and constant rate of strain tests (Scully et al.,1974; Somogyi et al., 1984; Znidarcic et al., 1986), etc. are time-consuming due to the low permeability of the slurry.

The estimation of hydraulic conductivity function parameters by inverse analysis is an interesting area of research. In several studies, inverse analysis was performed using the settlement data from self-weight consolidation tests (Vasudev and Bharat, 2022; He et a., 2017; Qi et al., 2020; Qi and Simms, 2020). The turbidity of the slurry mixture, however, can cause mistakes when determining the precise fluid-solid interface. In such cases, advanced instrumentation is required for digitalizing the continuous monitoring of the solid-fluid interface. In self-weight settlement tests in settling columns, the excess pore water pressure profile is generally recorded. Typically, piezometers and pore pressure transducers are used for this (Barthelomuseen et al., 2002; Sorta, 2015). These measurements can be easily monitored digitally, eliminating any potential for human error in recording the values.

This study focuses on back-predicting the hydraulic conductivity function parameters using the excess pore pressure dissipation data measured at the base of samples of settling slurries . The process essentially involves two steps, the forward analysis and the backward analysis. The forward analysis step solves the governing equation for finite strain consolidation taking material properties of the slurries as input, using the finite difference method. The Artificial Bee Colony (ABC) algorithm is used for the backward analysis. In this step, the objective function values are calculated based on the excess pore pressure dissipation data given as input and the data simulated using forward analysis. After sufficient iterations, the hydraulic conductivity function parameters are optimised. The study assumed that the compressibility function is known and does not vary with time. MATLAB codes were written for the forward and backward analysis. The inverse analysis method was tested on synthetic excess pore pressure dissipation data generated for two slurries. The hydraulic conductivity function was assumed to take the power form while the compressibility function was considered to be in modified power form. The inverse analysis technique using ABC was observed to be efficient in predicting the multiplier and power values of the hydraulic conductivity functions. The settlement curves and the excess pore pressure dissipation curve simulated using the precited parameters were observed to match well with the observed settlement behaviour as well as synthetic pore pressure dissipation behavior.

2 MATERIALS AND INPUT PROPERTIES

The input properties necessary for the inverse analysis include the specific gravity, the initial void ratio of the slurry, the initial height of the slurry deposit, and compressibility function parameters. The known hydraulic conductivity function parameters of the slurries were used initially to generate the synthetic excess pore pressure dissipation data. The compressibility function used in this study is of the modified power form given by Equation (1)

$$e = A(\sigma' + B)^C \quad (1)$$

where A (1/kPa), B (kPa), and C are constant coefficients, e is the void ratio and σ' is the vertical effective stress (kPa). The hydraulic conductivity is considered to take the power form representation given by Equation (2)

$$k = Me^P \quad (2)$$

where k (m/s) is the hydraulic conductivity, M (m/s) is the multiplier and P is the power.

The large strain consolidation behaviors of two clay slurries were considered in this study. The Antwerpen river soil slurry was the first data set simulated. Barthelomuseen et al. (2002) analysed the soil from the Antwerpen Soil collected from the Schelde river in Belgium to understand the influence of the forms of constitutive functions used in the prediction of finite strain settlement values. This soil is silt with intermediate plasticity, with a 39% liquid limit and a 28% plastic limit. It has a specific gravity of 2.72. Excess pore pressure dissipation data of a settling column test on this slurry having an initial height of 0.215 m, and uniform void ratio distribution of value 2.17 was simulated for the present study.

The characteristics of Kingsford clay were used to simulate the second artificial data set. It is a phosphatic clay of specific gravity 2.71, having 155% and 230% plastic limit and liquid limit values respectively. The single drained test on the Kingsford Clay, in a tank, proposed by McVey et al. (1986), with a small surcharge of 0.224 kPa was simulated in this study. An initial depth of 6 m and a uniform void ratio of 18.8 was considered for simulating the dataset. The constitutive function details are mentioned in Table 1.

Table 1. Constitutive function parameters used to simulate the synthetic excess pore pressure dissipation data

Material	Compressibility function parameters			Hydraulic conductivity function parameters	
	A	B	C	M	P
Antwerpen Soil	1.41	6E-3	-0.085	1.68E-9	8.26
Kingsford Clay	12.19	0	-0.290	1.41E-11	4.11

3 FORWARD ANALYSIS

The 1D finite strain consolidation theory proposed by Gibson et al. (1967) governs the settlement behaviour of slurries. The slurries consolidate under self-weight resulting in variation of void ratio profile with time. The theory accommodates this variation by considering the hydraulic conductivity and compressibility as functions of void ratio. The modified Darcy‘s law, relating the gradient of excess pore pressure with the relative velocity of pore fluid and soil skeleton, was used in the derivation of this theory.

The explicit finite difference solution of the Gibson‘s differential equation was given by Cargill (1982), as shown in Equation (3). The material coordinate system was used to account for the moving datum or the moving solid-fluid interface.

$$e_{i,j} = e_{i,j-1} - \frac{\tau}{\gamma_w}\left[\frac{e_{i+1,j-1} - e_{i-1,j-1}}{2\,\delta}\left(\frac{\alpha(e_{i+1,j-1}) - \alpha(e_{i-1,j-1})}{2\,\delta} + (\gamma_s - \gamma_w)\,\beta(e_{i,j-1})\right) + \alpha(e_{i,j-1})\left(\frac{e_{i+1,j-1} - 2\,e_{i,j-1} + e_{i-1,j-1}}{\delta^2}\right)\right] \quad (3)$$

In the above equation, e is the void ratio, δ is the space-step considered in the material coordinates obtained by dividing the volume of solids by the number of points considered for the void ratio profile including the top and bottommost points, τ is the time-step, $e_{i,j}$ is the void ratio of the point in i^{th} space-step at the j^{th} time step, γ_w is the unit weight of water and γ_s is the unit weight of solids. The terms α and β can be expanded as shown in Equations (4) and (5).

$$\alpha(e) = \left(\frac{k(e)}{1+e}\right)\left(\frac{d\sigma'(e)}{de}\right) \tag{4}$$

$$\beta(e) = \frac{d}{de}\left(\frac{k(e)}{1+e}\right) \tag{5}$$

where $k(e)$ and $\sigma'(e)$ are hydraulic conductivity and compressibility functions respectively.

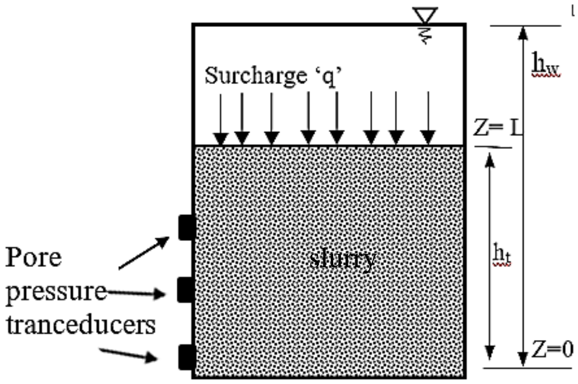

Figure 1. Slurry undergoing consolidation with pore pressure measurement

The void ratio profile was determined at each time step and the corresponding settlement value was calculated using Equation (6) given below

$$S(z,t) = \int_0^z [1 + e(z,0)]dz - \int_0^z [1 + e(z,t)]dz \tag{6}$$

where z is the height from the base in the material coordinate system, $e\ (z,\ t)$ is the void ratio at any point at z distance from the base at time t, and $S\ (z,\ t)$ is the settlement of the point at height z from the base in comparison to its initial height at $t=0$. The excess pore pressure (u_e) profile of the slurry sample is computed using Equation (7) as

$$u_e(z,t) = u_t(z,t) - u_s(z,t) \tag{7}$$

where the total pore pressure (u_t) and the static pore pressure (u_s) are calculated using Equations (8) and (9) respectively.

$$u_t(z,t) = \sigma(z,t) - \sigma'(z,t) \tag{8}$$

$$u_S(z,t) = \gamma_w[h_w - \xi(z,t)] \tag{9}$$

In Equation (8) $\sigma(z,t)$ is the total stress at any point at any given time. In Equation (9) h_w is the height as shown in figure (1) from base to the top, while $\xi(z,t)$ is the convective coordinate corresponding to the material coordinate point z, given by Equation (10)

$$\xi = \int_0^z [1 + e(z,t)]dz \tag{10}$$

A MATLAB code was created that simulates the excess pore pressure dissipation curve for the base of two slurry samples using the input parameters mentioned in Section 2 and Table 1. The boundary conditions considered were an impervious base and free drainage at the top. The vertical boundaries were assumed to be impervious.

4 BACKWARD ANALYSIS

The process of consolidation is a combination of settlement and dissipation of excess pore pressure developed due to surcharge or self-weight. When the pore fluid leaves the pore spaces the material undergoes volume change. The grain-to-grain stress transfer increases which is essentially an increase in the effective stress. Simultaneously, dissipation of excess pore pressure occurs, and the total stress at any point remains unchanged. The speed of drainage of pore fluid is dependent on the hydraulic conductivity of the material. Low hydraulic conductivity causes slower draining. Therefore, the pore pressure dissipation rate is also dependent on the hydraulic conductivity function.

In the backward analysis step, a given set of pore pressure dissipation data for the base of the sample is compared with the same simulated using the forward analysis step by substituting the hydraulic conductivity function parameters (*M* and *P*) predicted by the ABC algorithm. The root mean square error (RMSE) between the predicted curve and the input data is minimised while varying the hydraulic conductivity parameters in successive iterations. At the minimum root mean square error, the predicted excess pore pressure dissipation curve matches the best with the input excess pore pressure data and the corresponding hydraulic conductivity function parameters are the optimal parameter values. The objective function to calculate the RMSE for the optimisation is given by Equation (11):

$$RMSE = \sqrt{\left(\frac{1}{N}\right)\sum_{j=1}^{N}\left(U_o^i - U_p^i\right)^2} \tag{11}$$

In the above equation U_o^i is the excess pore pressure data for the t_i^{th} time, U_p^i is the excess pore pressure data predicted using the parameters from the algorithm using the forward analysis for the t_i^{th} time and N is the number of input excess pore pressure data points available. The termination criteria for the algorithm was the maximum number of iterations.

The Artificial Bee Colony (ABC) (Karaboga, 2005) algorithm is a stochastic swarm intelligence algorithm that simulates the behaviour of a swarm of honey bees searching for an optimal food source. In the search space, a swarm of virtual bee simulations flies about at random. When they arrive at the desirable solution, they communicate with one another. When the solution is poor quality, it is discarded, and the bees start over with a new food source. The algorithm's governing parameters are the bee colony's size and the number of iterations.

The swarm of bees generated virtually consists of three groups namely employed bees, onlooker bees, and scout bees. The number of food sources available is equal to the number of employed bees. A food source is the set of parameters that are to be optimised. These values are initially generated randomly, within the upper and lower bounds provided for the parameters. Each food source is associated with one employed bee at any given time. The employed bees dance to share information about the food source which is observed by the onlooker bees. The onlooker bees then decide whether or not to choose that food source based on the quality of the food source which is evaluated based on the fitness value of the food source. The fitness value is calculated from the objective function value corresponding to that food source. The employed and onlooker bees together exploit the food sources. The scout will be randomly moving around in the domain searching for a probable food source and is responsible for exploration. Each food source is modified in successive iterations and if the modifications lead to improvement of the solution, then the improved food source replaces the old food source. Else the old one is retained. The algorithm has a parameter that limits the number of trials for improving a food source. If this number is exceeded the food source is abandoned and the corresponding employed bee becomes a scout bee and starts searching for a new food source randomly.

5 RESULTS AND DISCUSSION

Synthetic excess pore pressure data for the base of Antwerpen Soil (Barthelomuseen et al., 2002) and Kingsford Clay (McVey et al., 1986) were generated by adding a maximum of 5% white noise to the excess pore pressure values simulated using the data given in Table 1 and the required material properties. The code for inverse analysis was written in MATLAB. The input properties for the code include the specific gravity, initial height, initial void ratio distribution, finite difference time and space step values, and the maximum number of iterations. The code was run for three cycles to check the repeatability of the solutions obtained. The maximum number of iterations was set as 300 for both cases and was the stopping criteria for each cycle.

The optimal hydraulic conductivity function parameters predicted are given in Table 2. These are the best-fit values from three runs of the code. The average RMSE from 3 runs of the code is also reported in Table 2.

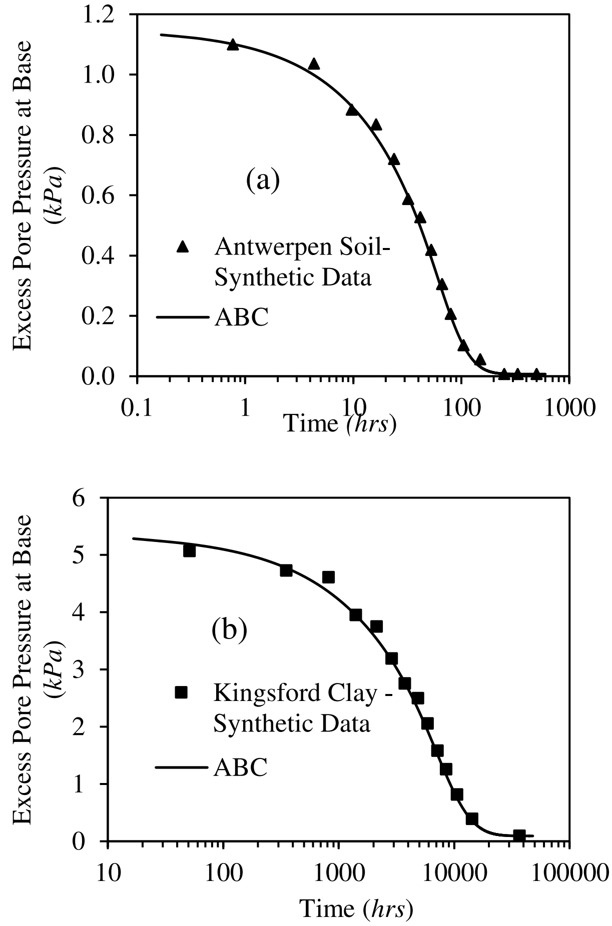

Figure 2. Predicted base excess pore pressure compared with the simulated data used as input for (a) Antwerpen Soil (b) Kingsford clay

The predicted hydraulic conductivity parameters were used to simulate the excess pore pressure data for the base using the forward analysis code. Material properties as mentioned in section 2 were given as input along with the predicted parameters and the compressibility function as given in Table 1. The simulated excess pore pressure dissipation curves for both cases matched well with the synthetic data that were used as input for inverse analysis as shown in Figure 2. A maximum variation of 3.5% was observed between the predicted excess pore pressure data and the synthetic data for the case of Antwerpen Soil, at around 4 hours. A slight under-prediction was observed. For Kingsford Clay, a maximum under-prediction of 4.6% was observed at around 820 hours. For both cases, the maximum variation is within the tolerance limit. These discrepencies might be due to the white noise added to obtain the synthetic data set or due to error in locating the corresponding data point on the simulated predicted excess pore pressure dissipation curve. When the predicted settlment curves were compared with the corresponding experimental data from literature, as shown in Figure 3, the maximum variation observed was less than 1% for both the cases.

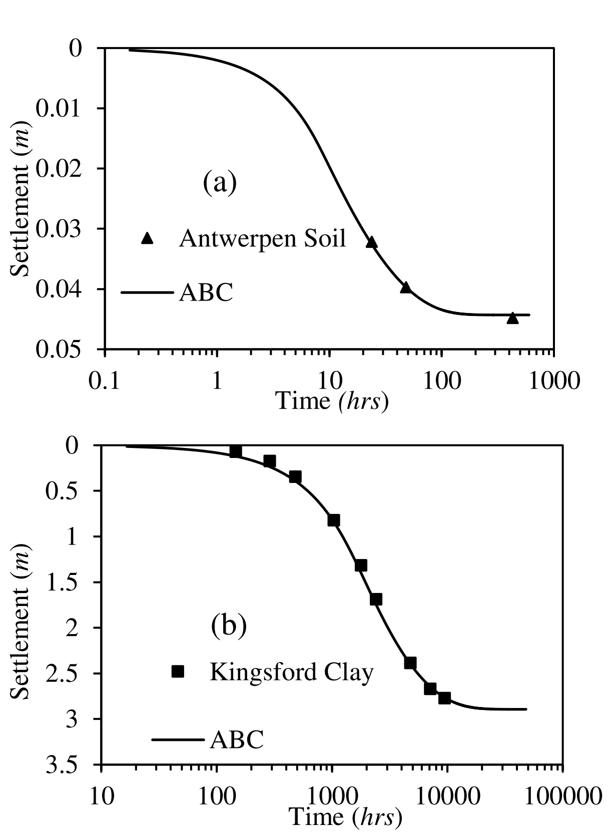

Figure 3. Settlement curve simulated using predicted hydraulic conductivity parameters compared with the experimental data for (a) Antwerpen Soil (b) Kingsford clay

The ABC algorithm predicted almost same optimal parameter values for all the three runs of both the slurries. The Antwerpen soil case reached a common RMSE value within 200 iterations in all three runs. The run 3 was the fastest to reach the optimal value, within 20 iterations. In the case of Kingsford Clay, all three runs reached the common RMSE within 250 iterations. Both runs 2 and 3 reached the lowest RMSE value within 25 iterations. The number of food sources used was set to 40 for both the cases which in turn is equal to the number of employed bees. Limit for the number of trials for improving a food source was set as 50 beyond which the food source will be abandoned.

Figure 4 shows the comparison of hydraulic conductivity function predicted and the experiemntal data from corresponding literature. Figure 4(a) shows the experimental data for Antwerpen soil in comparison with the predicted hydraulic conductivity function. The predicted function was able to represent the general trend of the experimental data set. The predicted hydraulic conductivity function for the Kingsford Clay was compared with the hydraulic conductivity function fitted to constant strain rate test data found in the literature in Figure 4(b). Good correlation was seen in the comparison.

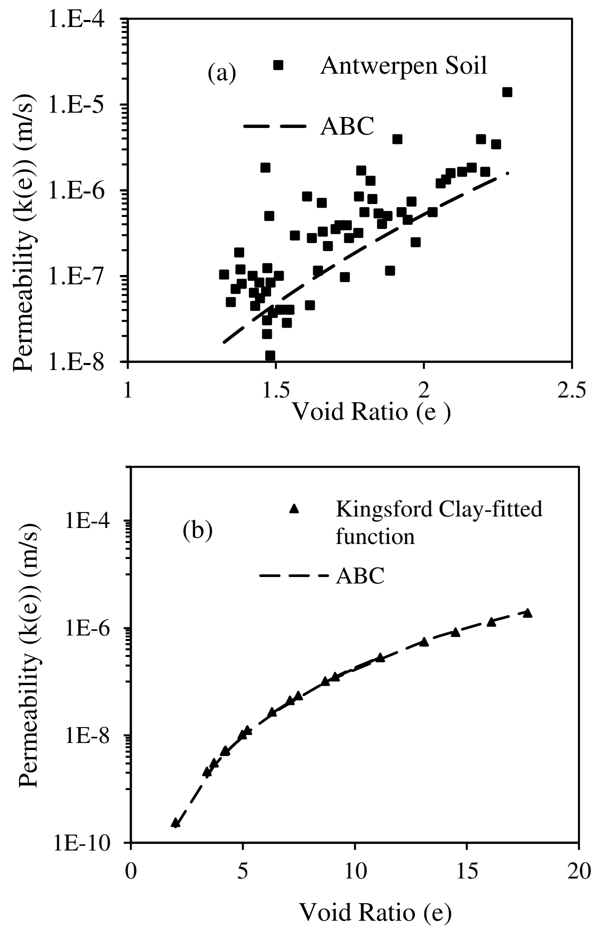

Figure 4. Hydraulic conductivity function predicted using the inverse analysis for (a) Antwerpen Soil (b) Kingsford clay

Table 2. Hydraulic conductivity function parameters predicted by inverse analysis

Material	Mean RMSE	Predicted Values	
		P	**M**
Kingsford Clay	0.122	3.634	1.62×10^{-11}
Antwerpen Soil	0.0018	8.348	1.61×10^{-09}

6 CONCLUSION

The finite strain consolidation behaviour of slurries need to be thoroughly understood to manage and dispose slurry wastes like mine tailings and dredged clays safely and economically. Compressibility and hydraulic conductivity functions control the dissipation of excess pore pressure and thereby controls the speed of consolidation. Experimental determination of the constitutive functions is highly time consuming and need tailor-made set-ups. In the present study, an inverse analysis technique using an optimisation algorithm to back-predict the hydraulic conductivity function parameters from the excess pore pressure dissipation data for the base of the sample is proposed for the first time. The compressibility function is known and is unvarying with time. The forward analysis was carried out using explicit finite difference method while the backward analysis used ABC algorithm.

The predicted hydraulic conductivity function parameters successfully simulated the excess pore pressure dissipation curve for the base of the slurries considered. The large strain settlement behaviour of slurries considered were simulated using the predicted parameters with commendable accuracy. The proposed technique is a robust and a relatively quick method which can be used in combination with simple settlement column tests equipped with a single pore pressure transducer at the base level of the sample.

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