Proceedings 10th NUMGE 2023
10th European Conference on Numerical Methods in Geotechnical Engineering
Zdravkovic L, Kontoe S, Taborda DMG, Tsiampousi A (eds)

https://doi.org/10.53243/NUMGE2023-424

Mutual interaction among three nearby shallow foundations

E. Zeolla[1], S. Sica[1],

[1]*Department of Civil Engineering, University of Sannio, Benevento, IT*

ABSTRACT: In dense urban areas, an interaction among closely-spaced buildings could arise through the underlying soil even if the buildings are structurally disconnected. In seismic design of new buildings or safety assessment of existing ones, the structure-soil-structure interaction is completely disregarded and each building is analysed independently of its neighbours. To understand if this practice represents a gross simplification of reality, the paper illustrates a parametric study in which the mutual interaction among three rectangular shallow foundations on an elastic halfspace has been investigated through a continuum-based approach with the FLAC3D software. The objective of the study is to evaluate how the different terms of the soil-foundation impedance matrix could be affected by the presence of nearby structures in dependence of the foundation-foundation spacing.

Keywords: shallow foundations; foundation-soil-foundation interaction; impedance function; crossed-beam foundation; 3D modelling

1 INTRODUCTION

In modern metropolises or in the historic centers of ancient cities, buildings are often built very close together and, consequently, interact through the underlying soil. While it is common practice to design buildings as isolated structures, the seismic response of a structure that is very close to other buildings could significantly differ from that of the same structure considered in isolation. In literature, this phenomenon is called Structure-Soil-Structure Interaction (SSSI) and raises many challenges to the scientific community due to several factors involved and computational effort required to properly model it.

As well-known, when a building is not founded on rock, common soil-structure interaction (SSI) could arise. If soil compliance is further modified by the cross interaction among nearby foundations/structures (SSSI), the usual practice to assess building safety against earthquakes without accounting for this additional interaction could be misleading.

The first studies on SSSI date back to the early 70s of the last century and were mainly aimed at capturing the physics of the problem together with suggesting suitable approaches and methods of analysis (e.g., Warburton et al., 1971; Chang-Liang, 1974; Roesset & Gonzales, 1977).

The studies published to date have shown that for some combinations of foundation-foundation spacing, plan arrangement, input frequency, dynamic features of the structures and of the soil deposit, the cross interaction phenomenon may not be negligible (Padron et al., 2009; Alexander et al., 2013; Knappett et al., 2015; Vicencio and Alexander, 2021, Zeolla et al., 2023).

Due to the SSSI phenomenon, for instance, a dynamic coupling between shear and normal forces may occur even for shallow foundations such as typical of piles (Triantafyilidis and Prange 1989; Triantafyilidis and Neidhart, 1989; Mulliken and Karabalis, 1998).

Furthermore, the nearby foundations exhibit displacement (rotation) components that may be in phase or out-of-phase between them and with respect to the surrounding soil (Betti, 1997).

As reported in Qian and Beskos (1995, 1996) and Qian et al. (1996), the cross interaction is likely to modify the vertical and horizontal stiffness components more significantly with increasing the number of foundations or reducing the distance between them; conversely, the rotational and torsional stiffness components are more sensitive to the oscillation input frequency. These coupling phenomena may be emphasised or attenuated by soil characteristics, with major concerns for densely urbanized areas on soft soils (Karabalis and Mohammadi, 1998; Sbartai, 2016; Bybordiani and Arici, 2019; Wang et al., 2022). Alternatively, for a given subsoil a structure surrounded by many other constructions may have a worse or better response with respect to the same structure considered alone in dependence of building plan arrangement and dynamic properties (Padron et al., 2009; Alexander et al., 2013; Knappett et al., 2015; Vicencio and Alexander, 2021).

Based on the evidences collected so far, the goal of this paper is to unravel the contribution of closer foundations on the compliance matrix computation in order to apply the substructure (impedance) approach. Actually, this latter may still be considered the best solution to handle soil-structure interaction problems in practice.

By a 3D continuum approach solved through the finite difference method (f.d.m.), a simple scheme of three shallow rigid foundations on a homogeneous linear elastic halfspace has been analysed. In the parametric study, the distance among three nearby foundations has been varied and the different components of the soil-foundation impedance matrix were computed. In the final part of the paper, the three footings were connected through orthogonal beams to create a unique foundation system. The goal is to understand if the footing-footing interaction may also arise among the multiple footings of a crossed-beam shallow foundation.

2 PROBLEM STATEMENT

2.1 *Impedance function*

In the substructure approach, soil compliance is modelled by endowing the foundation with springs and dashpots representing the stiffness (real part) and damping (imaginary part) components of the impedance function, respectively. This function is frequency dependent and links the *i-th* component of the harmonic force vector to the *j-th* component of the foundation displacement vector. Several analytical impedance functions have been derived under the assumption of a single rigid foundation placed on or embedded in the halfspace or layer-over-halfspace scheme (Gazetas, 1983, 1991; Pais and Kausel, 1988). Later on, other coefficients accounting for foundation flexibility (Apsel and Luco, 1987; Pitilakis and Karatzetzou, 2015), soil heterogeneity and nonlinearity were provided.

The real and imaginary part of the impedance functions are expressed as the product of a static (ω→0) contribution, K_{ij} and C_{ij}, and dynamic coefficients, $k_{ij}(a_0)$ and $c_{ij}(a_0)$:

$$\overline{K}_{ij} = k_{ij}(a_0)K_{ij} + i\omega c_{ij}(a_0)C_{ij} \tag{1}$$

where K_{ij} and C_{ij} depend on the soil shear modulus, G, the Poisson ratio, ν, and a characteristic dimension of the foundation, B. The dynamic coefficients $k_{ij}(a_0)$ and $c_{ij}(a_0)$ depend on B and V_s, both condensed into the dimensionless frequency, $a_0 = \omega B/V_s$. This latter is linked to the the input frequency (ω=2πf) and the soil stiffness through the shear wave velocity, V_s.

2.2 *Numerical model and analysis procedure*

The numerical study was performed in FLAC3D (Itasca 2004) considering the simple soil configuration of a homogeneous halfspace and rectangular foundations of base width, 2B, and a length, 2L (Figure 1). Fixed boundaries were set at the bottom and side surfaces of the soil volume under static conditions, while quiet boundaries were switched on in dynamic analyses. The distance, S/B, between the adjacent foundations was varied between 0.5 and 4 in the static analysis, and between 0.5 and 2 in the dynamic case. The foundation was not included in the numerical model, as done in previous literature studies (Gazetas, 1991), but its presence was considered by applying the same velocity field to all nodes along the foundation footprint to simulate the case of a rigid foundation (EI → ∞). In the static case, a constant velocity was applied to compute the translational stiffnesses (in x, y or z), whereas rotational stiffnesses were obtained by applying a variable velocity with a linear distribution with respect to the x and y axes of the foundation (zero in the centre). Reference could be made to Zeolla et al. (2023) for more details. In the dynamic case, the applied velocity function was harmonic (as well as the imposed displacement and the induced stress state). The input frequency f=ω/2π was set equal to the odd values between 1 and 13 Hz, to cover a frequency band likely to occur in the engineering field. Impedance functions in the frequency domain were obtained by performing Fourier transforms of the force and displacement functions. The soil-foundation flexibility was first computed (ratio of the displacement to the contact forces) and then the impedance matrix as its inverse. The adopted soil unit weight γ, bulk modulus K, and shear modulus G, are given in Table 1. A very low value of the soil stiffness (soft soil) was chosen to emphasise all interaction effects.

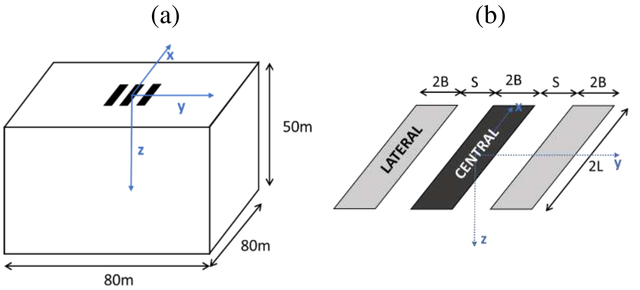

Figure 1. Reference scheme for three footings on half-space (a) and geometric features (b).

The mesh size, Δ, was established according to the relation $\Delta < V_S/8f_{max}$ (Kuhlemeyer and Lysmer, 1973), being f_{max} the maximum frequency of the imposed harmonic oscillation. An isotropic linear visco-elastic constitutive law was assigned to the soil. The overall numerical procedure and its validation against the available analytical solutions (Gazetas, 1991) for a single foundation were reported in Zeolla et al. (2021, 2022 and 2023).

Table 1. Adopted soil parameters

	γ [kN/m^3]	K [MPa]	G [MPa]	Vs [m/s]
Halfspace	19	12.2	5.6	54

3 RESULTS

3.1 Static stiffnesses

Table 2 provides the low-frequency (static) dimensionless stiffnesses associated to the swaying and rocking mode, computed for different values of the foundation-foundation spacing, S, divided by the footing half-width, B. The swaying stiffnesses have been scaled with respect to the shear modulus, G, and B, while the rocking ones have been scaled with respect to G, and B^3. For the sake of comparison, in the first column of Table 2 the dimensionless stiffnesses corresponding to the single foundation case have also been reported.

It may be observed that in case of very close foundations, the translational stiffness components are much lower than those computed for an isolated foundation. Since surrounded by two neighbouring foundations (left and right), the central foundation experiences the lowest stiffness (superposition principle). When the foundation-foundation spacing is very small (e.g., S=0.5B), a percentage difference with respect to the single foundation value of more than 50 % for the central foundation and more than 40 % for the lateral ones is reached. If the distance increases, (e.g., S=4B) the above differences decrease but still are not negligible, being around 40 % for the central foundation and 30 % for the lateral ones.

For rocking around the x-axis (Figure 1), the neighbouring foundations exert a sort of mutual constraint on the central one, which causes an increase of its rotational stiffness. Regardless of the footing position (central or lateral), the interaction effect on the rocking motion decreases more rapidly with the distance.

Table 2. Dimensionless static stiffnesses for central and lateral foundation

	Dimensionless static stiffnesses								
	Single	**Central foundation**				**Lateral foundation**			
	-	**S/B=0.5**	**S/B=1**	**S/B=2**	**S/B=4**	**S/B=0.5**	**S/B=1**	**S/B=2**	**S/B=4**
$\mathbf{K_{zz}}$	17	8	8	9	10	10	10	11	12
$\mathbf{K_{yy}}$	12	5	5	6	7	7	7	7	8
$\mathbf{K_{xx}}$	11	5	5	6	7	7	7	7	8
$\mathbf{K_{ry}}$	25	13	14	16	18	16	17	18	19
$\mathbf{K_{rx}}$	23	28	23	21	18	25	21	19	19

3.2 Dynamic coefficients

In the dynamic field, the interaction problem becomes more complex as the impedance functions depend on the input frequency, in addition to the other factors already discussed above. To avoid further complexity in the modeling procedure, the three foundations were assumed to be loaded equally and synchronously.

The different input frequencies generate contact stresses below the foundations, depending on the position and distance among the foundations (Figure 2). The stress distribution is not the same for the three foundations; the two lateral footings show a different and asymmetrical distribution with respect to the central one. When the frequency is low, the distribution under the footprint is rather uniform, while at higher frequencies (shorter wavelengths) there is a concentration of stresses along the edges of the footings. This phenomenon results in a floating of the dynamic coefficients around the solution found for the single foundation, as can be seen in Figure 3. The impedances of the lateral foundations (continuous lines) are close to those of the single foundation (dashed black line) for low values of the dimensionless frequency a_0. The same response is observed for the central foundation, except for K_{zz}. At higher frequencies, significant differences are observed in the translational coefficients, especially for the central foundation. The dimensionless frequency at which the presence of the neighbouring foundations influences the dynamic stiffness coefficients decreases as S/B increases. Actually, longer wavelengths associated to lower frequencies, cause very close foundations (e.g., S/B=0.5) to move together with minimal differences in their response with respect to the case of single foundation.

4 MULTIPLE CONNECTED FOUNDATIONS

This section investigates the behaviour of a foundation system consisting of crossed-beam footings arranged as shown in Figure 4. The examined scheme, although simplified, may represent the typical foundation system of a building. In this case, the foundation-foundation interaction was evaluated inside a unique foundation system.

The static stiffnesses were computed both numerically and analytically by using the classical closed-form solutions available in literature for a rigid, massless foundation placed on a homogeneous halfspace (Gazetas, 1991). In neither literature nor technical standards there are specific indications for computing soil-foundation compliance matrix for this type of footing configuration. Actually, the static terms of the compliance matrix can be estimated analytically in different ways, depending on the interconnection constraint between the crossed beams.

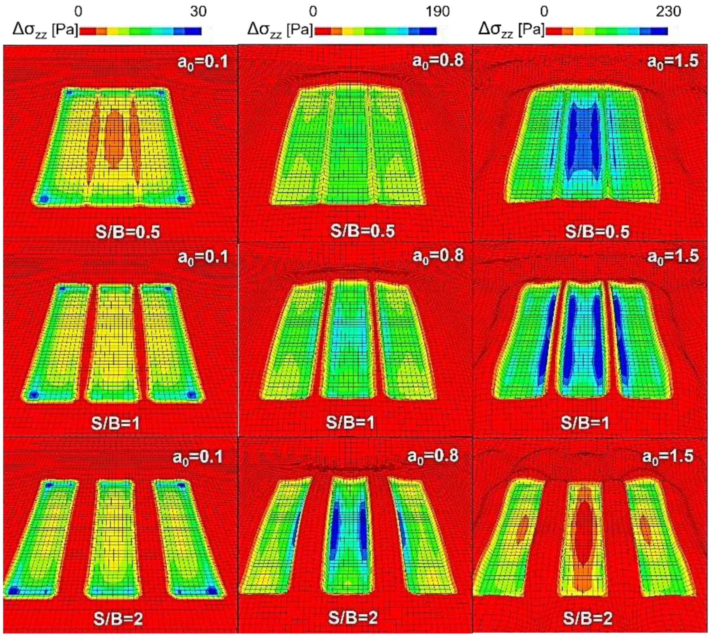

Figure 2. Contour of vertical contact stresses for different values of the oscillation input frequency and clear distance S/B.

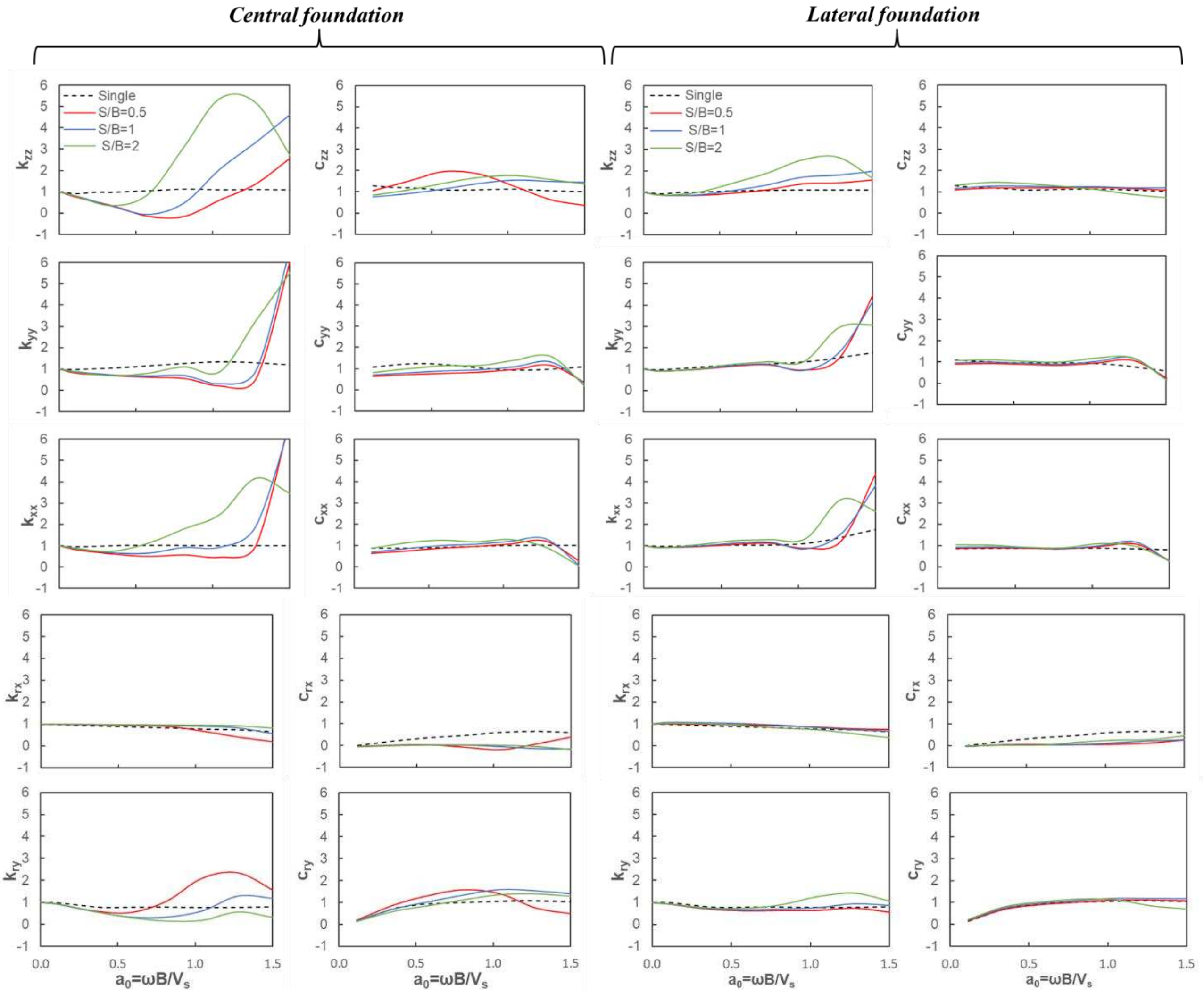

Figure 3. Dynamic stiffness and damping coefficients with a_0 for the central and lateral foundation.

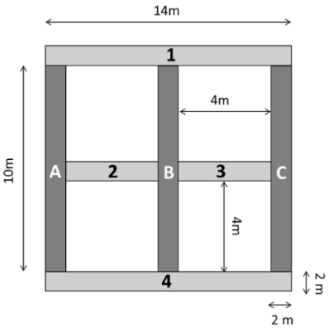

Figure 4. Geometric scheme of the crossed-beam foundation system

Firstly, the entire footprint of the foundation was considered (2Bx2L=14mx14m, total area of 196 m^2 with moment of inertia around the x and y axes equal to 3201 m^4). Secondly, by assuming a perfect rigid connection between the crossed beams, a rigid rectangular hollow section was considered with external dimensions 2Bx2L=14mx14m and internal dimensions of the 4 cavities 4mx4m (area of $16m^2$ and moment of inertia around the x and y axes equal to 21 m^4). Thirdly, the contribution of all single beams (2Bx2L=2mx10m for A, B, C, 2Bx2L=2mx14m for 1 and 4 and 2Bx2L=2mx4m for 2 and 3) can be summed according to the superposition principle; therefore, the translational stiffnesses of the 7-beam system are the sum of the individual contributions. Likewise, the rocking stiffness is the sum of the individual rotational stiffnesses plus the moment of transport of the vertical terms with respect to the axis of rotation of the entire foundation, as suggested by the old seismic standard ATC-3 (1984) for buildings.

The stiffnesses resulting from the three different approaches are shown in Table 3. The numerical results are assumed to be the most reliable as they include the three-dimensional nature of the problem and the true interaction effects between the different elements of the foundation. The literature closed-form equations provide for stiffness values close to the numerical ones only if a full square section corresponding to the entire footprint (size $2Bx2L=196m^2$) of the foundation is adopted. In contrast, the stiffness superposition principle, with the contribution of the single beams, could be very misleading, as it provides a significant overestimation of the translational static terms of the impedance matrix. The hollow section, on the other hand, underestimates the vertical swaying component and the rocking ones.

Table 3. Dimensionless static stiffnesses of a crossed-beam foundation system

Dimensionless static stiffnesses of crossed-beam foundation system							
		Gazetas (1991)			Percentage variation (analytical vs FDM)		
	FDM-	Superposition	Square full section	Square hollow section	Superposition	Square full section	Square hollow section
K_{zz}	53	113	45	38	53%	-17%	-41%
K_{yy}	35	103	37	31	66%	5%	-13%
K_{xx}	35	92	37	31	62%	5%	-13%
K_{ry}	224	193	182	179	-16%	-23%	-25%
K_{rx}	215	211	182	179	-2%	-18%	-20%

In short, the cross-interaction between the individual parts of the foundation system leads to a reduction in the total stiffness of the foundation, especially for the swaying modes. This can also be seen simply by comparing the stiffnesses of the three longitudinal footings in the two configurations (with and without rigid connection), listed in Table 4.

Table 4. Dimensionless static stiffnesses of the longitudinal footings

	Without connection (S/B=4)		With connection		Percentage variation between the two schemes	
	A	B	A	B	A	B
K_{zz}	12	10	9	5	26%	51%
K_{yy}	8	7	6	4	27%	44%
K_{xx}	8	7	6	3	29%	50%
K_{ry}	19	18	12	8	37%	56%
K_{rx}	19	18	63	1	-234%	97%

5 CONCLUSIONS

The cross interaction between three closely-spaced shallow foundations placed on a linear elastic halfspace was numerically investigated through a 3D continuum approach.

For the analysed scheme of three independent shallow foundations, it emerged a sensible reduction of the static stiffnesses for all degrees of freedom of the foundation. The smaller the distance between the footings, the greater the reduction (over 50%) with respect to the case corresponding to the single foundation case. The dynamic stiffness coefficients may be assumed equal to those of an isolated foundation only in the low-frequency range of oscillation; conversely, significant differences were observed at higher frequencies.

When the foundations are rigidly connected to generate a crossed-beam foundation system, it is not trivial computing the stiffness components of the whole system from those of the single foundation. From the analyses carried out, it appears that the analytical solution that best represents the static stiffnesses of the connected footing system is the scheme of a unique slab that englobe all the footings, as properly indicated in some technical guideline (NIST, 2012; FEMA, 2020).

6 ACKNOWLEDGEMENTS

The work has been developed in the framework of the 2022–2024 ReLUIS-DPC research programme funded by the Italian Civil Protection Department, as a contribution to the geotechnical Work Package 'Soil-Foundation-Structure Interaction' (Task 16.3) by the UniSannio-Sica Research Unit.

7 REFERENCES

Alexander, N. A., Ibraim, E., & Aldaikh, H. 2013. A simple discrete model for interaction of adjacent buildings during earthquakes. Computers & Structures, 124, 1-10.

Apsel, R. J., & Luco, J. E. 1987. Impedance functions for foundations embedded in a layered medium: an integral equation approach. Earthquake Engineering & Structural Dynamics,15(2), 213-231.

ATC-3, 1st edition 1978, 2nd edition 1984. Applied Technology Council, Tentative Provisions for the Development of Seismic Regulations for Buildings, National Science Foundation and National Bureau of Standards, Washington, DC.

Betti, R. 1997. Effects of the dynamic cross-interaction in the seismic analysis of multiple embedded foundations. Earthquake engineering & structural dynamics, 26(10), 1005-1019.

Bybordiani, M., & Arici, Y. 2019. Structure-soil-structure interaction of adjacent buildings subjected to seismic loading. Earthquake Engineering & Structural Dynamics, 48(7), 731-748.

Gazetas, G. 1983. Analysis of machine foundation vibrations: state of the art. International Journal of soil dynamics and earthquake engineering, 2(1), 2-42.

Gazetas, G. 1991. Formulas and charts for impedances of surface and embedded foundations. Journal of geotechnical engineering, 117(9), 1363-1381.

Gonzalez, J. J., & Roesset, J. M. 1977. Dynamic interaction between adjacent structures. Research report, 1976--1977 (No. PB-287079). Massachusetts Inst. of Tech., Cambridge (USA). Constructed Facilities Div.

Itasca Consulting Group, Inc. 2004. FLAC3D User Manual: Version 5.0. USA.

Karabalis, D. L., & Mohammadi, M. 1998. 3-D dynamic foundation-soil-foundation interaction on layered soil. Soil Dynamics and Earthquake Engineering, 17(3), 139-152.

Knappett, J. A., Madden, P., & Caucis, K. 2015. Seismic structure–soil–structure interaction between pairs of adjacent building structures. Geotechnique, 65(5), 429-441.

Kuhlemeyer, R. L., & Lysmer, J. 1973. Finite element method accuracy for wave propagation problems. Journal of the Soil Mechanics and Foundations Division, 99(5), 421-427.

Liang, C. 1974. Dynamic response of structures in layered soils (Doctoral dissertation, Massachusetts Institute of Technology).

Lin, H. T., Roesset, J. M., & Tassoulas, J. L. 1987. Dynamic interaction between adjacent foundations. Earthquake engineering & structural dynamics, 15(3), 323-343.

Mulliken, J. S., & Karabalis, D. L. 1998. Discrete model for dynamic through-the-soil coupling of 3-D foundations and structures. Earthquake Engineering & Structural Dynamics, 27(7), 687-710.

Padron, L. A., Aznarez, J. J., & Maeso, O. 2009. Dynamic structure–soil–structure interaction between nearby piled buildings under seismic excitation by BEM–FEM model. Soil dynamics and earthquake engineering, 29(6), 1084-1096.

Pais, A., & Kausel, E. 1988. Approximate formulas for dynamic stiffnesses of rigid foundations. Soil dynamics and earthquake engineering, 7(4), 213-227.

Pitilakis, D., & Karatzetzou, A. 2015. Dynamic stiffness of monumental flexible masonry foundations. Bulletin of Earthquake Engineering, 13(1), 67-82.

Qian, J., & Beskos, D. E. 1995. Dynamic interaction between 3-D rigid surface foundations and comparison with the ATC-3 provisions. Earthquake engineering & structural dynamics, 24(3), 419-437.

Qian, J., & Beskos, D. E. 1996. Harmonic wave response of two 3-D rigid surface foundations. Soil Dynamics and Earthquake Engineering, 15(2), 95-110.

Qian, J., Tham, L. G., & Cheung, Y. K. 1996. Dynamic cross-interaction between flexible surface footings by combined bem and fem. Earthquake engineering & structural dynamics, 25(5), 509-526.

Sbartai, B. 2016. Dynamic interaction of two adjacent foundations embedded in a viscoelastic soil. International Journal of Structural Stability and Dynamics, 16(03), 1450110.

Triantafyllidis, T., & Neidhart, T. 1989. Diffraction effects between foundations due to incident Rayleigh waves. Earthquake engineering & structural dynamics, 18(6), 815-835.

Triantafyllidis, T., & Prange, B. 1989. Dynamic subsoil-coupling between rigid, circular foundations on the halfspace. Soil Dynamics and Earthquake Engineering, 8(1), 9-21.

Vicencio, F., & Alexander, N. A. 2021. Method to evaluate the dynamic structure-soil-structure interaction of 3-D buildings arrangement due to seismic excitation. Soil Dynamics and Earthquake Engineering, 141, 106494.

Wang, J., Guo, T., Xie, Y., & Du, Z. 2022. Parametric analyses of dynamic interaction between three-dimensional soil and frame structure group under earthquake loadings. Bulletin of Earthquake Engineering, 20(15), 8305-8324.

Warburton, G. B., Richardson, J. D., & Webster, J. J. 1971. Forced vibrations of two masses on an elastic half space.

Zeolla, E., De Silva, F., & Sica, S. 2021. Dynamic cross-interaction between two closely-spaced shallow foundations. In COMPDYN Proceedings (Vol. 2021). National Technical University of Athens.

Zeolla, E., de Silva, F., & Sica, S. 2022. Dynamic Interaction Between Adjacent Shallow Footings in Homogeneous or Layered Soils. In Proceedings of the 4th International Conference on Performance Based Design in Earthquake Geotechnical Engineering (Beijing 2022) (pp. 1257-1264). Cham: Springer International Publishing.

Zeolla, E., De Silva, F & Sica, S. 2023. A simplified approach to account for through-soil interaction between two adjacent shallow foundations. Bulletin of Earthquake Engineering. https://doi.org/10.1007/s10518-023-01621-1