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Effect of initial stiffness on the induced horizontal displacements of geotechnical structures built on/in overconsolidated clays

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ABSTRACT: This work intends to study the effect of the initial stiffness on the induced horizontal displacements of two geotechnical structures performed on/in overconsolidated clays: an embankment built on Boston blue clay and a tunnel dug in London clay. The numerical predictions are performed with the MIT-E3 model, which reproduces the small strain nonlinearity and hysteretic stress-strain response observed in unload-reload cycles. Firstly, the results of unload-reload cycle of two consolidation tests were used to validate the constitutive model. Afterwards, the horizontal displacements of the two geotechnical structures predicted numerically for different initial stiffness (changing the κ_0 parameter of the Mit-E3 model) are compared with each other and with field data. The results show that a decrease in the initial stiffness induces an increase in the horizontal displacements. This effect is more significant during the construction of the embankment and immediately after the execution of the tunnel, that is, when the stress paths evolve within the yield surface and the behaviour is governed by a non-linear elastic law.

Keywords: MIT-E3 model; embankment; tunnel; Boston blue clay; London clay

1 INTRODUCTION

In the last few decades, geotechnical engineering has benefited from the continuous technological development in informatics tools that lead to more efficient construction management and the design of geotechnical structures. Nowadays, the increasing complexity of geotechnical challenges requires more reliable numerical predictions, whatever are the boundary and loading as well as soil response. In general, the greater reliability of the constitutive laws of the soil material tend to increase the complexity of the mathematical formulation and the number of parameters involved. Over the last years, several constitutive models have been proposed to reproduce general or specific features of soil's behaviour, such as the MIT-E3 model, initially presented by Whittle (1993) and Whittle and Kavvadas (1994).

The reliable response of the geotechnical works in serviceability conditions, which are distinct of the ultimate conditions, requires the use of constitutive soil models able to reproduce the soil's behaviour for small strain levels and unloading stress paths. This effect is more important in the presence of high overconsolidated soils, where there is a long distance between the current stress point and the yield surface. Consequently, the stress path inside the yield surface is long where the response of the soil is fundamentally non-linear elastic. In these conditions, the initial stiffness and the way the non-linear elastic behaviour is defined plays a very important role in the material's response. The initial stiffness of soils has been widely studied by several authors, having concluded that the initial Young's modulus in undrained triaxial compression depends on: i) stress level; ii) void ratio; iii) consolidation path; iv) lateral stress ratio; v) strain rate; vi) aging (Santagata et al., 2005); vii) fabric; viii) cementation; ix) stress history and x) overconsolidation ratio (OCR) of clays, while in sands and silt mixtures the OCR does not have a significant effect (Ku and Mayne, 2015). On the other hand, some studies have shown a non-linear stress–strain behaviour of soils, even at very small strains (Jardine et al., 1986).

Since the problem of stiffness has already been well characterized from experimental tests, it seems relevant to study the effects of the initial stiffness and the nonlinearity behaviour of the soil on the response of geotechnical structures, mainly when executed on/in oveconsolidated clays, which is the aim of this communication. For this, two geotechnical structures, an embankment built on Boston Blue Clay (BBC) and a tunnel excavated in London clay (LC), are considered and the variation of soil's initial stiffness on the induced horizontal displacements are analysed.

The numerical study is performed on a finite element (FE) code, developed at the University of Coimbra (Venda Oliveira, 2000), which is able to perform elastoplastic analyses with coupled consolidation and creep. All the simulations are performed with the MIT-E3 model, that incorporates a non-linear elasticity law which depends on the initial stiffness. The various values of the initial stiffness are simulated with a change in the initial slope of the unloading/reloading phase in e-ln*p'* space (κ_0), which also affects the variation of the non-linear stiffness with strain. To better understand the influence of these variations, the numerical predictions are compared with the field measurements and the numerical results obtained with a constant and average stiffness (i.e., linear elastic behaviour).

2 MIT-E3 MODEL

2.1 Description of the model and soil parameters

According to Whittle and Kavvadas (1994), the MIT-E3 model is suitable to simulate the behaviour of over consolidated clays with OCR up to eight. The model uses yield surface, which correspond to the normally consolidated state, and has the shape of an ellipsoid (Figure 1) oriented in the consolidation direction (point V).

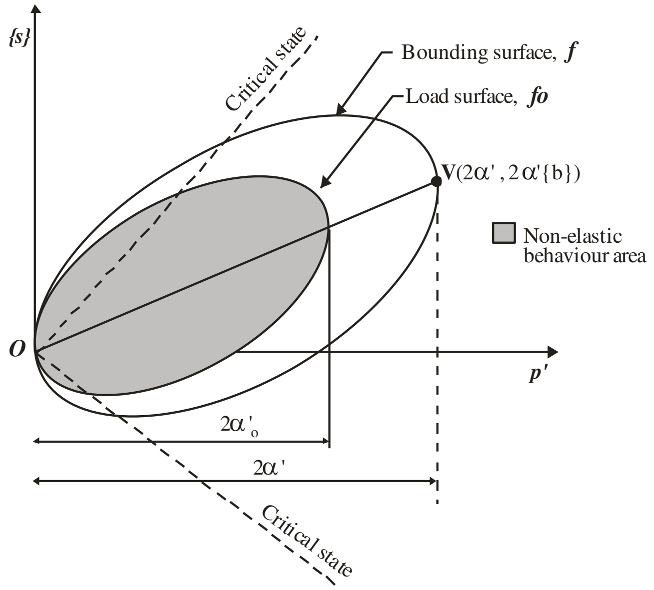


Figure 1. Critical state, bounding and loading surfaces of MIT-E3 model (based on Whittle, 1993; Venda Oliveira and Lemos, 2011).

The MIT-E3 model considers a non-associated flow rule and two hardening rules, that control the movement of the yield surface, namely its size and direction. The hardening rule that defines the orientation of the yield surface controls the rotation of the major axis of the ellipsoid towards the *p'* axis, that is, it permits the change of soil anisotropy (Whittle, 1993). The MIT-E3 model also considers strain softening behaviour (Whittle, 1993, Whittle and Kavvadas, 1994).

To replicate the behaviour of overconsolidated clays, mainly the unloading (illustrated in Fig. 1 by the grey area) and reloading paths, the MIT-E3 model associates two important features, a hysteretic model and a bounding surface plasticity model. The hysteretic model simulates the non-linear elastic behaviour in an unloading–reloading path, inducing a smooth change of stiffness with strain (Figure 2). Thus, the variation of the tangential bulk modulus (K) is defined by equation (1) as a function of the initial slope of the unloading/reloading phase in e-ln*p'* space (κ_0), the current void ratio (e), and the variable δ, which controls the stiffness nonlinearity (equation (2)), where C and n describe the non-linear behaviour at small strain levels, while the variables ξ and ξ_s measure the volumetric and shear stress amplitude, which vary with the distance between the current stress state and the reversal point (Whittle, 1993).

$$K = \frac{1+e}{\kappa_0(1+\delta)} p' \quad (1)$$

$$\delta = C \cdot n \cdot (\ln_e \xi + \xi_s)^{n-1} \quad (2)$$

As it can be seen in Figure 2(a) and (b), the non-linear behaviour during reloading and unloading, respectively, depends on the value of κ_0. When κ_0 increases, the tangential bulk modulus decreases (see equation (2)), which induces a higher variation of the void ratio and, for a reloading path, the confluence to the normally consolidated state at higher stresses (Figure 2(a)).

The bounding surface plasticity model is defined by an external surface, which represents the yielding function at the normally consolidated state, and an internal load surface, which is located at the current stress state and is homothetic to the bounding surface (Figure 1). One of the main advantages of the bounding surface plasticity model is the generation of plastic strains inside the bounding surface, i.e., in overconsolidated stress states, allowing a smooth transition between an overconsolidated state and a normally consolidated state, as Figure 2(a) illustrates. A more comprehensive description of MIT-E3 model is beyond the scope of this work. More details of the model can be read in some published works, namely Whittle (1993), Whittle and Kavvadas (1994), Ganendra (1993), Venda Oliveira (2000, 2005).

MIT-E3 requires the definition of fifteen parameters, that can by divided in two sets. A first set is composed by the parameters that are directly evaluated from experimental data ($e_{\lambda 0}$, λ, v, κ_0, K_0^{nc}, ϕ'_{TC}, ϕ'_{TE}). The second set of parameters comprises those which are estimated from parametric studies in order to adjust the numerical curves to laboratory results (c, ψ_0, C, n, w, S_t, γ, h). The meaning and the adopted values for each parameter for LC and BCC are summarised up in Table 1.

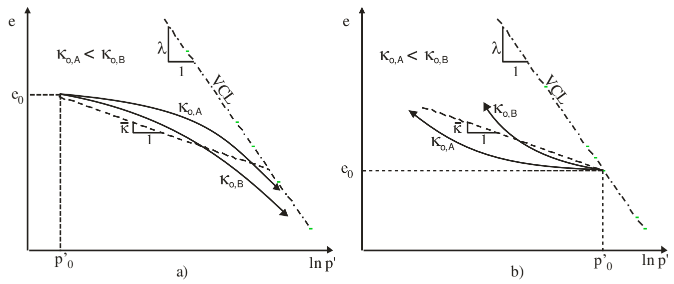


Figure 2. Effect of κ_0 compare with the average value ($\bar{\kappa}$):a) reloading path; b) unloading path.

Table 1. MIT-E3 parameters for BBC and LC

Parameter		Description	BBC	LC
$e_{\lambda 0}$	Evaluated from experimental tests	Void ratio for a K_0 normally consolidated at the reference average stress (p' = 1 kPa)	3.56[(a),(b)]	2.005
λ		Slope of the virgin compression line in *e*-ln*p'* space	0.147[(a),(b)]	0.172
κ_0		Initial slope of the swelling line in *e*-ln*p'* space	0.001[(a),(b)]	0.001
K_0^{nc}		Coefficient of earth pressure at rest for normally consolidated clay	0.50[(a),(b)]	0.62
ν		Poisson's ratio	0.30[(a),(b)]	0.288
ϕ'_{TC}		Critical-state angle of shearing resistance in triaxial compression	26.5[(a),(b)]	22.5
ϕ'_{TE}		Critical-state angle of shearing resistance in triaxial extension	39.5[(a),(b)]	22.5
C	Evaluated from parametric studies	Parameter affecting the non-linear volumetric swelling behaviour (hysteretic elasticity)	22	50[(c)]
n		Parameter affecting the non-linear volumetric swelling behaviour (hysteretic elasticity)	1.60	1.5
w		Parameter affecting the non-linearity at small strains in undrained shear (hysteretic elasticity)	0.07	0.03[(c)]
h		Parameter affecting the irrecoverable plastic strain	0.20	0.10
c		Parameter affecting the undrained shear strength (geometry of bounding surface)	0.86	0.80
S_t		Parameter affecting the degree of strain softening	4.5	3.9
γ		Parameter affecting the generation of pore pressures induced by shear in overconsolidated clay	0.5	0.5
ψ_0		Parameter affecting rotation of bounding surface	100	100

[(a)] Borja et al. (1990) [(b)] Venda Oliveira and Lemos (2011) [(c)] Araújo Santos (2009)

The parameters used are based on the values firstly proposed by Whittle (1993), having been complemented by Borja et al. (1990) and Venda Oliveira e Lemos (2011) for the BBC and Araújo Santos (2009) for the LC. In addition to the parameters summarized in Table 1, values of 0.034 (Whittle and Kavvadas, 1994) and 0.06 (Avgerinos et al., 2016) were also considered for the parameter $\bar{\kappa}$ (mean slope of the discharge line in the *e*-ln*p'* space), which was used in numerical simulations of the geotechnical structures, to emphasize the importance of the effect of non-linearity on their response.

The ability of MIT-E3 to simulate the behaviour of BBC and LC under 1D consolidation was previously tested before simulating the geotechnical structures. The results of a constant rate strain (CRS) consolidation test (Ghantous, 1982) and an oedometer test (Gasparre, 2005) carried out on BBC and LC, respectively, are simulated. Axisymmetric FE analysis is considered to simulate the conditions imposed for the consolidation tests. Comparing the results (Figure 3), it can be seen that MIT-E3 is able to replicate the non-linear behaviour of BBC and LC observed during the unloading and reloading paths, as well as the plastic strains close to the end of the reloading path, allowing a smooth transition from overconsolidated to normally consolidated states.

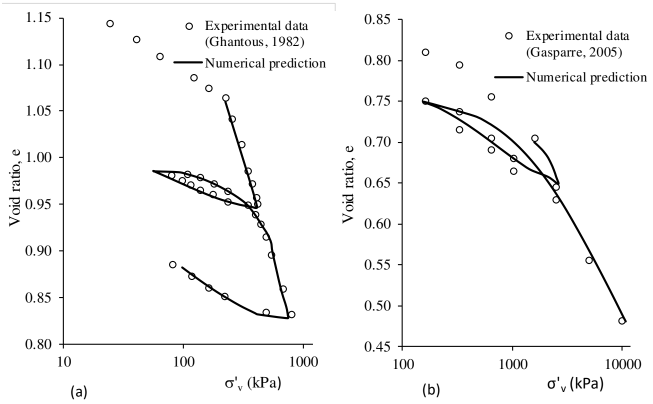


Figure 3. Comparison of experimental and numerical results: a) CRS test on BBC; b) oedometer test on LC

3 MODELLING OF A TUNNEL AND AN EMBANKMENT

3.1 Cases study description

The study analyses the effect of the variation of the initial stiffness on the behaviour of two geotechnical structures built in/over overconsolidated clays, where it is expected to occur a stress path in a non-linear elastic behaviour.

The first structure to be simulated is an embankment located on the I-95 motorway on the BBC. The soil profile and the mesh used in the FE analysis at plane-strain conditions are represented in Figure 4(a). The mesh is composed of 676 nodal points and 203 quadrilateral isoparametric elements, each with eight nodes. The process of consolidation is simulated by FE elements with twenty nodal degrees of freedom, which make it possible to couple the analysis of fluid flow and deformation. Each of these elements allows the calculation of the displacements at eight nodes and the excess pore pressure at four corner nodes. In terms of boundary conditions, vertical borders only permit vertical displacements, while the bottom boundary is restrained in both directions. Concerning the hydraulic boundary conditions, only the top boundary, where the water table is located, is permeable. The remaining boundaries are impermeable.

The soil profile under the embankment is composed of three layers. The firs layer is a soft peat, while the second layer corresponds to a silty sand. The BBC are the bottom soil, being divided in six layers (layers 3 to 8). Under the embankment, the peat was replaced with a dense granular fill. An elastic law is assumed for the embankment's material, with Young's modulus (E') varying between 5.7 MPa and 78 MPa, which is intended to replicate the increasing stiffness of the embankment's material from the top to the bottom layers, due to the post construction confining stresses (Venda Oliveira and Lemos, 2011). The response of the silty sand is simulated by the Modified CamClay model. Details on the parameters used for these layers can be consulted in Venda Oliveira and Lemos (2011).

The BBC are simulated with the MIT-E3 model, whose parameters are summarized in Table 1, and different degrees of overconsolidation in depth are assumed, ranging from 4.5 (layer 3) to 1 (layer 8). Finally, regarding the permeability of silty sand and BBC, it was considered that there is an anisotropy of permeability, with a ratio between the horizontal and vertical permeability coefficients of 4 (Borja et al. 1990), with the latter varying with the void ratio (equation (3)), where the initial void ratio (e_0) and the initial permeability coefficient (k_{v0}) are 3.3 and 1.1x10^{-3} m/day, respectively. The C_k parameter is assumed to be 1.2 (Tavenas et al., 1983).

$$k_v = \mathrm{k_{v0}} \times 10^{\frac{e-e_0}{C_k}} \tag{3}$$

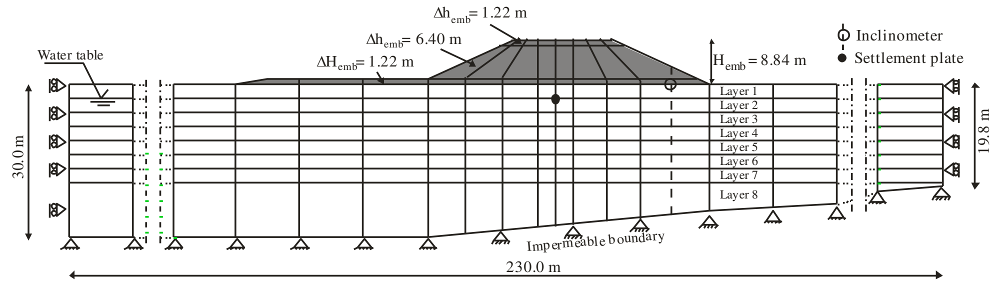
The instrumentation of the embankment comprises an inclinometer and a settlement plate. Nonetheless, in the present work, the study of the response of the embankment is based, only, on the horizontal measurements of an inclinometer tube located close to the toe of the embankment slope. Regarding the construction process, the embankment was built in three stages (reaching 1.22, 7.62 and 8.48 m) and with three consolidation phases, two intermediate and one final.

The second geotechnical structure modelled is a section of the Jubilee Line of the London Underground at St. James's Park (Figure 4(b)). The line comprises two twin tunnels measuring 4.85 m in diameter, both fully excavated in the LC. In this analysis, only the westbound tunnel (WB) is considered. As it was the first to be built, both its construction and the following consolidation period, which lasted 250 days (Nyren et al., 2001), are not affected by any other external loading. Geologically, three distinct layers can be identified. A surface layer of alluvium rests on top of a layer of gravel (Thames River Terrace Gravel). From a depth of 9 m onwards, on may find the LC. The water table is located between the alluvium and the gravel layers (Nyren, 1998). The instrumentation is vast. However, in this analysis only the measurements of inclinometer 4 are considered (Figure 4(b)).

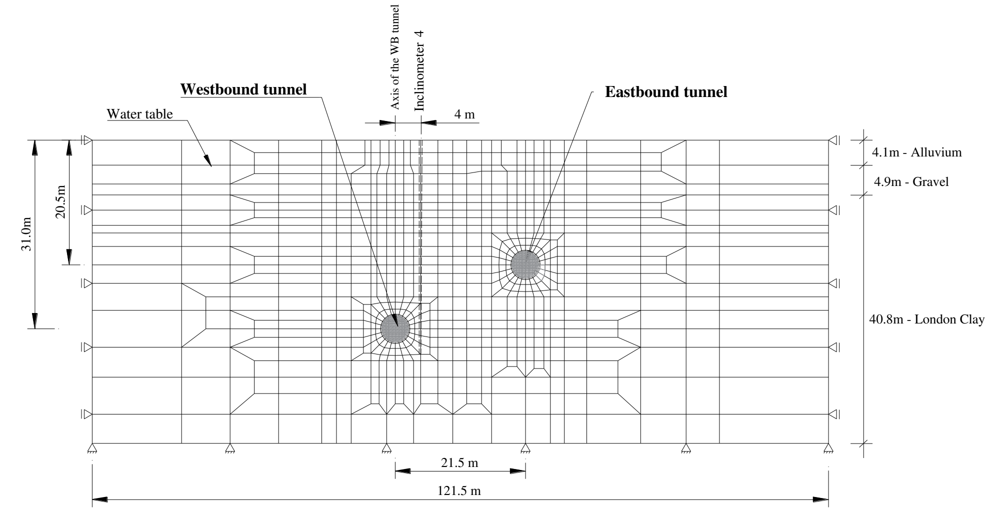
The finite element mesh used (Figure 4(b)) is identical to the one used in other published works (Addenbrooke et al., 1997; Addenbrooke and Potts, 2001; Avegerinos et al., 2016), consisting of 1,010 eight-node quadrilateral finite elements, totalling 2,963 nodal points. The elements used to simulate the materials under the water table are hybrid elements (identic to the elements used for the embankment), which allow simulating the phenomenon of consolidation. In the lower boundary, conditions of zero displacements are imposed, while in the lateral boundaries, only horizontal displacements are restricted. The outer boundaries are defined as impermeable, with the only permeable boundary coincides with the water table. The tunnel lining and the alluvium are modelled as linear elastic materials, while the Thames River Terrace are assumed to behave as an elastic-perfectly plastic law associated with the Mohr-Coulomb failure criterion. The soil parameters used in these materials coincide with those used in other works, namely, Addenbrooke et al. (1997) and Addenbrooke and Potts (2001). The LC are modelled using the MIT-E3 model (Table 1), with a K_0 ranging from 1.5 at the top of the layer to 1.2 at a depth of 25 m. K_0 = 1 is assumed for depths greater than 25 m.

Around the tunnel (up to 1.5 times its diameter), a zone of reduced K_0 of 0.5 is considered, as considered by Grammatikopoulou (2004). Permeability anisotropy was considered in LC, with k_h = 2.8x10-11 m/s and k_v = 1.1x10-10 m/s (Dixon and Bromhead, 1999; Chandler et al., 1990). A stress relief coefficient (α) of 0.322 is assumed, in order to reach a loss volume of 3.2% (Nyren, 1998). Finally, in the numerical simulation, water percolation into the interior of the tunnel is allowed, according to observations reported by Nyren (1998).

In the landfill simulation, 5 analyses were performed with values of κ_0 = {0.0001; 0.0005; 0.001; 0.003; 0.005}. A sixth calculation was performed with $\bar{\kappa}$ = 0.034. In turn, in the tunnel simulation, an identical number of analyses were performed for the values of κ_0 = {0.0005; 0.00075; 0.001; 0.0015; 0.002} and $\bar{\kappa}$ =0.06. In the reference cases, κ_0 = 0.001 was considered, as shown in Table 1.



a)



b)

Figure 4. Geological profile and FE mesh: a) embankment on BBC; b) tunnel in LC.

3.2 Results and discussion

Starting by analysing the embankment numerical simulation, Figure 5 shows that, in general, parabolic diagrams are predicted numerically, which present some similarities with the field behaviour, namely in terms of the maximum value of the horizontal displacement.

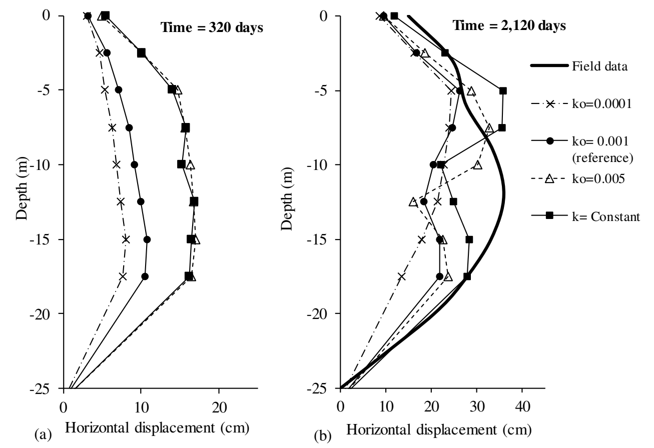


Figure 5. Horizontal displacement for 320 and 2,120 days under the embankment.

For t = 320 days (Figure 5(a)), an increase of stiffness (lower κ_0) induces lower horizontal displacements. Also, the effect of the change in initial stiffness tend to decrease with time. The results obtained with a $\bar{\kappa}$ and κ_0 of 0.001 and 0.005, respectively, for a consolidation time of 2,120 days (Figure 5(b)), show an inflection in the diagram of the horizontal displacements at between 5 and 10 m of depth, which is not measured in the field. This inflection occurs in the upper layers of BBC, which are initially overconsolidated. Such behaviour seems to arise from the change in stiffness between the BBC and the top layers, since this inflection is more significant with the increase in the difference in stiffness between the BBC and the top layers (Venda Oliveira and Lemos, 2007).

The horizontal displacements profile in depth measured right after WB tunnel construction is illustrated in Figure 6. The reference case (κ_0 = 0.001) provides the best prediction of the horizontal displacements, mainly in terms of the maximum value. Additionally, an increase in the κ_0 value also induces an increase in the horizontal displacements. The use of a constant stiffness ($\bar{\kappa}$) leads to a much higher displacement.

4 CONCLUSIONS

The present study analyses the effect of the initial stiffness on the horizontal displacement of two geotechnical structures built on/in overconsolidated clays (BBC and LC) when modelled using FE with the MIT-E3 model. The variation of the initial stiffness and the nonlinear elastic behaviour is simulated by adopting different values for κ_0.

This study demonstrates the ability of MIT-E3 in reproducing the hysteretic behaviour that occurs in unloading-reloading paths and the smooth transition from overconsolidated to normally consolidated state of clays. When modelling an embankment and a tunnel, the decrease in stiffness lead to an increase of the horizontal displacements. The effect of the change in initial stiffness on the displacements predicted numerically around the tunnel is more significant than under the embankment. This fact is associated with the extension of the yield area in the domain, which is less significant in the case of the tunnel than in the embankment. This difference is due to unlike stress path under the structures: compression under the embankment and extension in the top and bottom areas of the tunnel.

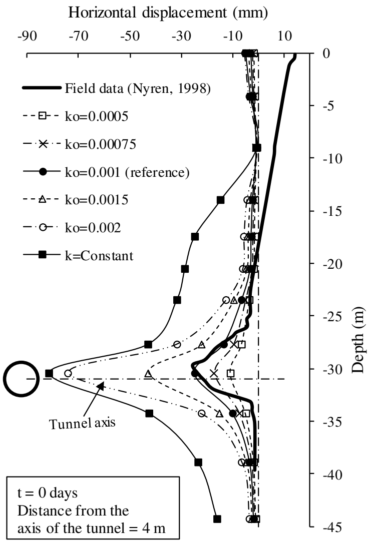


Figure 6. Horizontal displacement profile after WB tunnel construction

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