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Challenges and opportunities in teaching constitutive models in geotechnical courses

G. Medicus[1], K. Ziotopoulou[2], N. Huvaj[3]

[1]*University of Innsbruck, Austria*
[2]*University of California, Davis*
[3]*Middle East Technical University, Turkey*

ABSTRACT: Constitutive modelling of materials is often perceived as an abstract and challenging topic in engineering education. This is a combination of multiple reasons including the required backgrounds on advanced algebra and geometry, the coupling with system-level numerical modelling, and the abstract nature of certain concepts. In 2021, the authors carried out an international survey (n=192) to assess the challenges and opportunities of teaching and learning constitutive models. In this survey, they asked geotechnical students and lecturers how their courses are designed, which textbooks and tools they use, and what main challenges and opportunities they face in teaching and learning soil constitutive modelling. This paper summarises the survey results, draws conclusions from the commonalities and discrepancies identified in the survey, and presents several existing tools for geotechnical engineering education. The paper focuses on the project *'Animating Soil Models'* (soilmodels.com/soilanim), that aims to facilitate teaching and understanding concepts related to constitutive modelling using visualisations. The content of the project is shared under an open licence and is well received by the community.

Keywords: engineering education; constitutive models; elastoplasticity; hypoplasticity

1 INTRODUCTION

An in-depth understanding of both fundamental soil mechanics and constitutive modelling is required to use material models in a reasonable way in practical applications. Constitutive modelling is often perceived as an abstract and challenging topic in engineering education. This is a combination of multiple reasons including advanced algebra and geometry, the coupling with system-level numerical modelling, and the abstract nature of certain concepts.

This paper presents several innovative, existing tools for geotechnical engineering education. In addition, an anonymous survey was performed to assess the challenges and opportunities related to teaching and learning constitutive models. Its results are presented and conclusions are drawn.

2 SOME EXAMPLES OF AVAILABLE RESOURCES

The challenging nature of constitutive modelling has inspired the creation of multiple freely or commercially available resources, aimed at making the topic more accessible, helping students to learn, and providing more diverse usable resources for instructors. In this section, some of these resources are described. The selection is not intended to be exhausting but instead to provide some examples of resources that are useful for educational purposes. The authors have independently checked the resources and have found them useful for their needs.

2.1 ExCalibre

ExCalibre soilmodels.com/excalibre-en, developed by Kadlíček et al., 2022a, 2022b, is an automatic calibration tool for the models sand and clay hypoplasticity (von Wolffersdorff, 1996 and Mašín, 2013) and the Modified Cam Clay (MCC) model by Roscoe & Burland, 1968, see Figure 1.

Figure 1. ExCalibre (Kadlíček et al., 2022a, 2022b) is available on Soilmodels.com

It simply enables the automatic calibration of the models by uploading experimental data of standard laboratory tests (isotropic and/or oedometric compression tests, drained and/or undrained triaxial compression tests). In addition, there is a laboratory test simulation tool for the MCC model, clay and sand hypoplasticity. Without knowledge in programming, it is possible to simulate drained or undrained triaxial tests, oedometric and isotropic compression tests. Students can perform parameter variations and thus understand the parameters' influence on strength and stiffness predictions. In addition, they can also experientially learn the models' capabilities and limitations.

2.2 *PLAXIS SoilTest*

The element test tool (*PLAXIS SoilTest*, 2011) in the commercial software Plaxis (Brinkgreve et al., 2019) offers the possibility to perform element test simulations with a wide range of implemented as well as user-defined constitutive models. The following tests can be simulated: triaxial tests, oedometer tests, constant rate of strain tests and direct simple shear tests. It is further possible to prescribe arbitrary stress/strain paths to investigate the models' response for unconventional loading paths. The SoilTest tool enables further understanding of constitutive models. In addition, the possibility of parameter variation is of educational purpose in order to better estimate the influence of different parameters on the results.

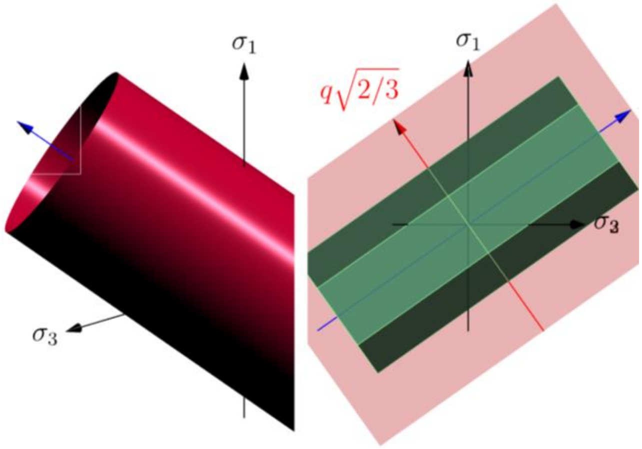


Figure 2. Von Mises and Tresca yield/failure surfaces visualised on the 'Animating Soil Models' project's page: [Link]

2.3 *YouTube channel 'Advanced Geomechanics'*

The lectures of Dr. Nicolas Espinoza (University of Texas at Austin, USA) of the Course *Advanced Geomechanics* are available online on YouTube (Espinoza, 2020). The explanations provided are detailed, complete and insightful. The covered topics are: (i) review on continuum mechanics, (ii) constitutive models, (iii) mechanics of the saturated porous solid (iv) inelasticity: failure criteria, plasticity-yield surface, post-peak behaviour, and (v) mechanics of open mode fractures. The topics enable the dissemination and thus understanding of these advanced topics at a flexible pace.

2.4 *Animating Soil Models*

The open education resource "Animating Soil Models" soilmodels.com/soilanim (Medicus, 2021/2022) aims to increase the understanding of soil constitutive modelling using animations and interactive graphics to improve the visual aspect of teaching and learning. The visualisations are shared for open education under the open licence CCBY. The project focuses on models that include concepts from critical state soil mechanics: clay hypoplasticity (Mašín, 2013) and elastoplastic models such as the Modified Cam Clay Model (Roscoe & Burland, 1968) and SANISAND plasticity (Manzari and Dafalias, 1997; Yang et al., 2022) are visualised. The project was initiated and mainly developed by Dr. Gertraud Medicus (University of Innsbruck, Austria). Visualisations related to SANISAND were carried out in cooperation with Dr. Mahdi Taiebat (The University of British Columbia, Canada).

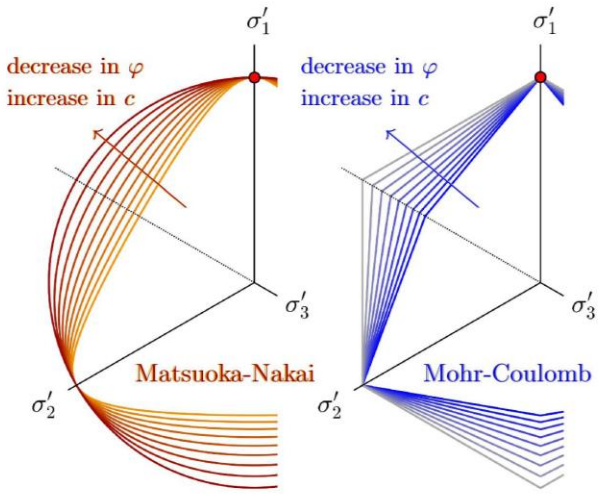


Figure 3. How do the parameters c and φ influence the shapes of the failure surfaces Matsuoka-Nakai and Mohr-Coulomb? [Link]

Constitutive models often include 3D surfaces with sections that are difficult to conceive for students. Figure 2 shows the yield surfaces von Mises and Tresca. On the project's page you can find other yield/failure surfaces. The figures are linked to interactive plots that have been created using the software asymptote.sourceforge.io which allows embedding 3D vector WebGL graphics within HTML files: it is possible for the students to rotate the surfaces themselves and thus understand different projections/intersections, in order to visualise stress invariants, plane stress predictions, or different planes in principal stress state as p'-q plane or the deviatoric plane. In addition to the visualisations, it is possible to download commented, simple MATLAB scripts for some 3D surfaces. For the equation-based explanation, the reader is referred to Griffiths (1990); Griffiths and Huang (2009).

Figure 3 shows a static figure of an animation related to Matsuoka-Nakai and Mohr-Coulomb failure surfaces. The linked animation illustrates how the shapes of the yield surfaces change with increasing cohesion and decreasing friction angle. Axisymmetric triaxial compression is kept constant for the mean effective stress that corresponds to the displayed deviatoric plane. For any other deviatoric direction, a change of φ and c affects the strength predictions. For φ = 0° with c≠0, Matsuoka-Nakai turns into von Mises and Mohr-Coulomb turns into Tresca. Figure 4 shows a visualisation of stress invariants. The interactive figure enables users to change the location of the principal stress state (red bullet in Figure 4) in the deviatoric plane as well as in principal stress space. The related stress invariants are computed.

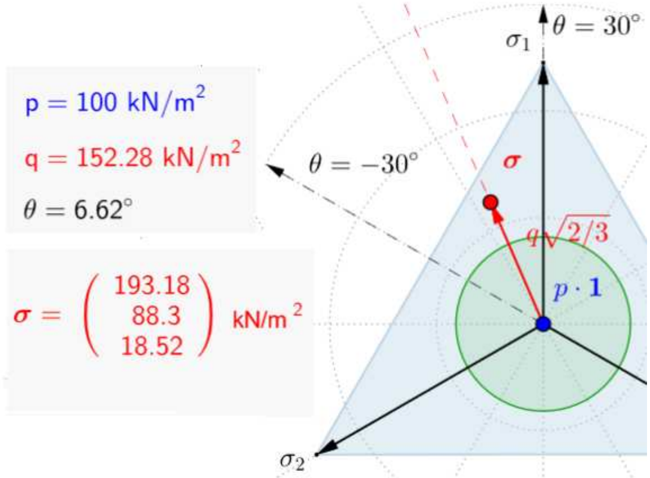


Figure 4. Visualisations of stress invariants with the help of an interactive plot: [Link]

Concepts from Critical State Soil Mechanics are included in the mathematical formulations of e.g. the Modified Cam Clay Model (MCC, Roscoe & Burland 1968) and clay hypoplasticity (Mašín 2013). The Modified Cam Clay Model is an elastoplastic hardening model, assuming associated flow.

Figure 5 shows a simulation of normally consolidated drained triaxial compression test. An animation is linked in the figure's caption. Other overconsolidation ratios, as well as undrained compression tests can be found online. Figure 6 shows the state boundary surface (SBS) of the MCC model. The interactive figure allows users to vary the parameters that control the shape of the SBS. This interactive graphic has been created using the software GeoGebra (Hohenwarter et al, 2013).

Clay hypoplasticity includes similar concepts as the MCC model as a stress-dependent CSL and NCL. Critical stress states according to Matsuoka-Nakai are included in clay hypoplasticity. Figure 7 shows the asymptotic state boundary surface of clay hypoplasticity in *e*-*p'*-*q* space as well as in principal stress space. Different interactive figures enable to rotate the graphics and to investigate a parameter variation.

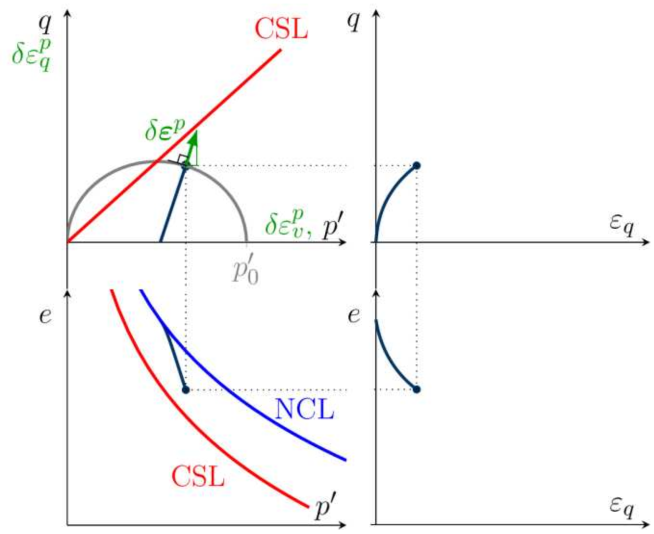


Figure 5. Example Modified Cam Clay Model of a normally consolidated drained triaxial compression test: [Link]

Figure 3 shows a static figure of an animation related to Matsuoka-Nakai and Mohr-Coulomb failure surfaces. The linked animation illustrates how the shapes of the yield surfaces change with increasing cohesion and decreasing friction angle. Axisymmetric triaxial compression is kept constant for the mean effective stress that corresponds to the displayed deviatoric plane. For any other deviatoric direction, a change of φ and c affects the strength predictions. For φ = 0° with c≠0, Matsuoka-Nakai turns into von Mises and Mohr-Coulomb turns into Tresca. Figure 4 shows a visualisation of stress invariants. The interactive figure enables users to change the location of the principal stress state (red bullet in Figure s4) in the deviatoric plane as well as in principal stress space. The related stress invariants are computed.

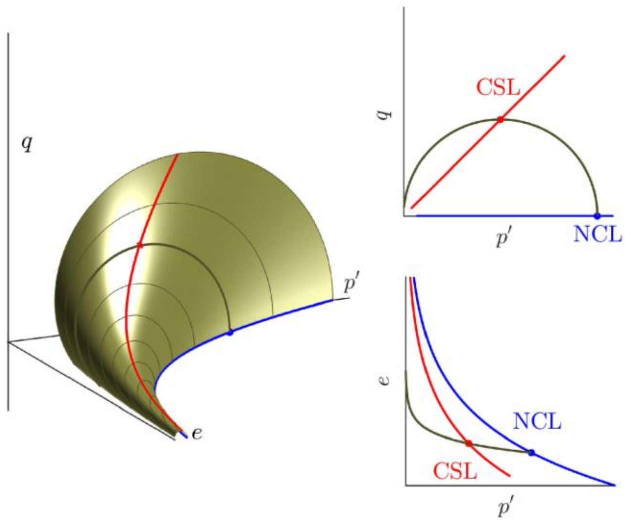


Figure 6. State boundary surface (SBS) of the MCC model. The linked interactive graphic allows to vary the parameters that control the shape of the SBS. [Link]

3 SURVEY AND RESULTS

In order to assess the practices, experiences, and challenges of the geotechnical community an online survey was performed and disseminated to a broad audience via the authors' social media pages and the United States University Council on Geotechnical Education and Research emailing list. The survey was answered by 192 participants from 63 countries, having different levels of teaching experience on constitutive models ranging from "no teaching experience (42%)", 1-4 years of experience (28%), 5 or more years (30%). The top three highest numbers of respondents were from the USA, Turkey and UK with 19, 9 and 5 percent of respondents, respectively. The survey's results are summarised in this section.

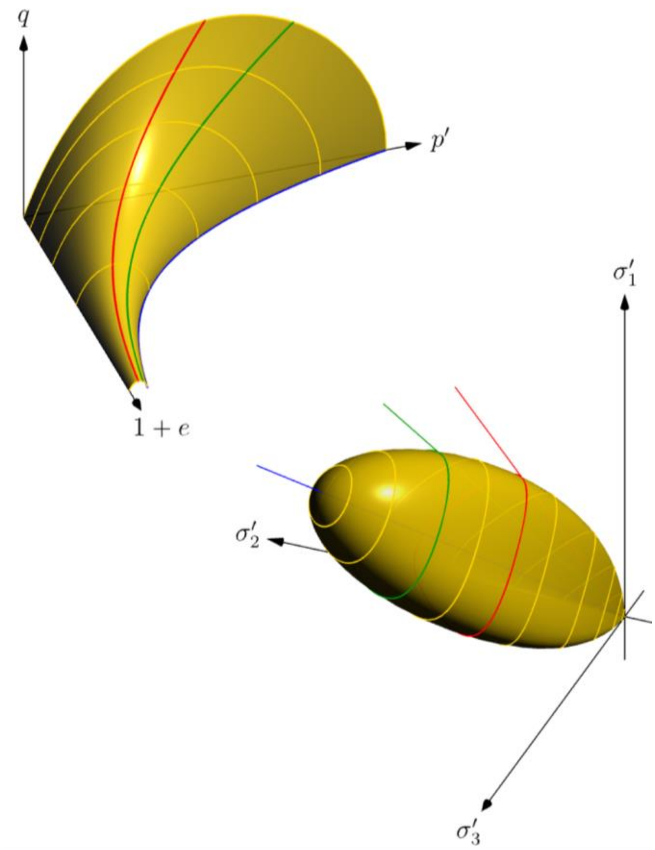


Figure 7. Asymptotic state boundary surface (ASBS) of clay hypoplasticity in p'-q-e space (left) and principal stress space (right) [Link]

Based on the survey, instructors find it rather challenging to teach constitutive models compared to other topics in (geotechnical) engineering: On a scale of 1 (*Not challenging at all*) to 5 (*Very challenging*), the mean value among lecturers was 3.7.

Similar to that, the mean value related to the question "*Do you find it (did you find it) challenging to learn about constitutive models with traditional teaching methods (such as lecturing etc.) compared to other topics in (geotechnical) engineering?*" was 3.9. The level of difficulty of constitutive modelling courses in relation to other courses of engineering studies was as well rated 3.9 (1: *I find it very easy*; 5: *I find it very difficult*).

Teaching and learning constitutive models requires multiple disciplines as continuum mechanics, tensor algebra, experimental soil mechanics and the knowledge of numerical methods. The main reasons given for the challenges of teaching and learning constitutive models were: the lack of the required backgrounds on advanced algebra and geometry, the coupling with system-level numerical modelling, and the abstract nature of certain concepts.

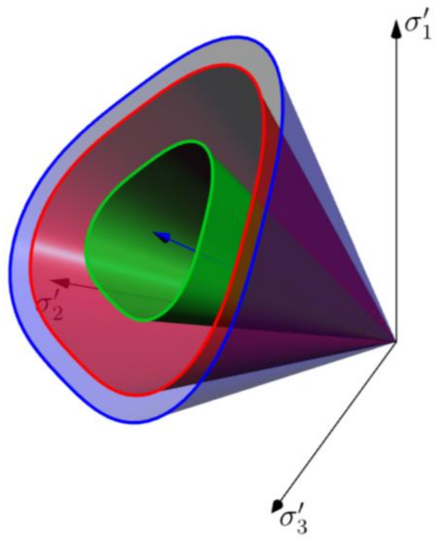


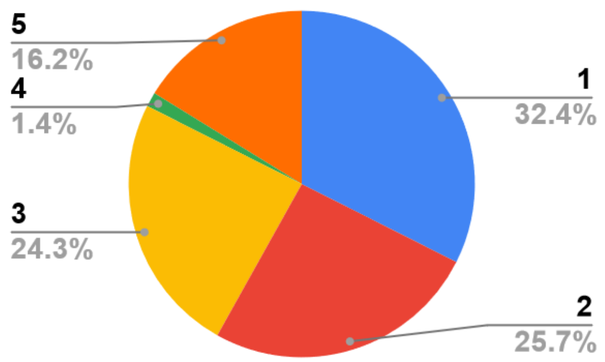
Figure 8. The dilatancy, critical state, bounding surfaces of SANISAND plasticity are visualised. More interactive visualisations and a video can be online: [Link]

The participants were asked which textbooks they use in their courses for teaching and learning about constitutive models. In what follows, a summary is provided of the books that were most frequently mentioned (in alphabetical order):

- Atkinson, J. and Bransby, P.L. 1978. *The Mechanics of Soils: An Introduction to Critical State Soil Mechanics.* McGraw-Hill Book Company
- Atkinson, J. 2007. *The Mechanics of Soils and Foundations.* CRC Press
- Budhu, M. 2011. *Soil Mechanics and Foundation*. 3rd Edition, John Wiley & Sons, Inc., Hoboken
- Muir Wood, D. 1991. *Soil Behaviour and Critical State Soil Mechanics.* Cambridge University Press
- Muir Wood, D. 2004. *Geotechnical Modelling.* CRC Press
- Ortigao, A. 2020. *Soil Mechanics in the Light of Critical State Theories: An Introduction.* CRC Press
- Pietruszczak, S. 2010. *Fundamentals of plasticity in geomechanics.* CRC Press

- Potts, D.M. and Zdravković, L. 2001. *Finite Element Analysis in Geotechnical Engineering: Volumes 1 and 2: Theory and Application.* Thomas Telford
- Puzrin, A.M. 2012. *Constitutive Modelling in Geomechanics,* Springer Berlin, Heidelberg
- Schofield, A. N. and Wroth, C. P. 1968. *Critical State Soil Mechanics*, McGraw-Hill

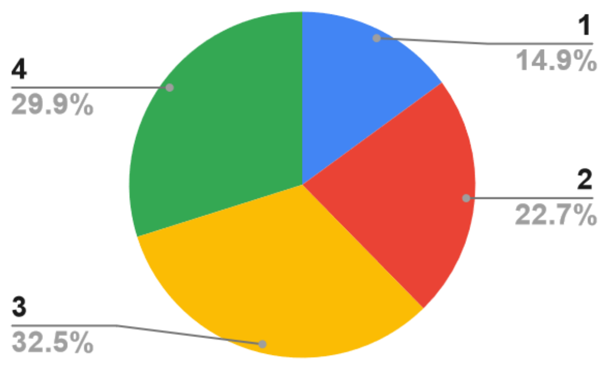
In addition, the open education tool *'Animating Soil Models'* was evaluated within the survey. Generally, almost all participants find it useful to have open education tools to visualise concepts of constitutive models**.** The quality of the project *'Animating Soil Models'* was rated high from the large majority of participants. However, students as well as lecturers see certain challenges in the usage of such online tools (see Figure 9). More than half of the participants expressed a concern that the potential users lack coding or fundamental theoretical background to use the project's content. About a quarter would need more guidance about how to integrate into teaching, or where to start learning.



1: I (or my students) do not have the coding background that might be necessary.
2: I (or my students) do not have the fundamental theoretical background on concepts in constitutive models.
3: I (or my students) will need guidance about how to integrate into teaching, or where to start learning.
4: I do not want to spend time on improving the way I teach, or my lecture materials.
5: I do not know how to create homeworks / tutorials / term projects using these tools.

Figure 9. What challenges do you see in using such open access online tools in teaching / learning?

Additionally, in the survey incentives were addressed for open learning resources within the geotechnical community. Questions asked include, *'Do you consider the pressure to publish as a barrier for open education and open science?'*, see Figure 10. The results show that about one-third of the researchers rate the pressure to publish as a significant barrier to 'Open Education' and 'Open Science'. Only 13% do not see the pressure to publish as a barrier. This raises the following questions: what incentives do we need for academics to devote time to develop open education tools, or to integrate such tools into their courses? The large majority of participants expect that if open education tools would have a higher priority in academic assessment or evaluation processes, more open education projects would be available, see Figure 11.



1: Not a barrier
2: Somewhat of a barrier
3: Moderate barrier
4: Significant barrier

Figure 10. Do you consider the pressure to publish as a barrier for open education and open science?

4 SUMMARY AND CONCLUSIONS

Static lecture is not as beneficial as using animations and interactive graphics in teaching constitutive models. Newer technology and the ability to code offer unique opportunities to reshape the way of traditional teaching. Two pilot demonstrations and uses of these tools in courses clearly improved students' understanding of these topics. Although not measured, the feedback and performance of students were markedly improved compared to past offerings of the same courses. A community survey established the interest of the community in such tools. However, since universities often lack the incentives to create such tools, the number of open education tools is limited. This paper provided some viable and quality options of free or commercial tools and books that should be assisting in the teaching of constitutive models.

5 ACKNOWLEDGEMENTS

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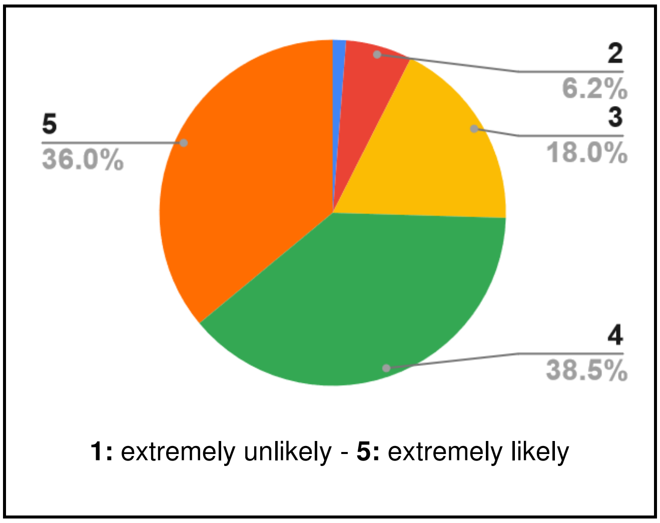

Figure 11. If providing open education tools would have a higher priority in academic assessment/evaluation processes of universities/researchers, do you estimate that more open education projects would be available?

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