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Numerical investigations on the influence of initial static shear stress in bi-directional loading of dense sand

A. Csuka[1] , C.E. Grandas-Tavera[2] , R. Cudmani[1]

[1]*Chair of Soil Mechanics and Foundation Engineering, Rock Mechanics and Tunneling, Technical University of Munich, Munich, Germany*

[2]*Chair of Soil Mechanics and Foundations/Geotechnical Engineering, Brandenburg University of Technology Cottbus-Senftenberg, Cottbus, Germany*

ABSTRACT: Advanced constitutive models are required for realistic prediction of the seismic behavior of geotechnical structures, such as dams and embankments, under undrained alternating loading conditions. The constitutive models should be able to capture accurately two important aspects related to the boundary value problem: the influence of a non-zero initial static shear stress and the multi-directional nature of the earthquake loading. Experimental studies show that the deformation characteristics of granular materials under undrained cyclic loading conditions depend on the initial static shear stress, initial void ratio and loading direction. In this paper, two advanced constitutive models from the hypoplasticity family are compared with each other and with experiments regarding to their ability to account for the influence of the initial static shear stress on liquefaction.

Keywords: Hypoplasticity, Triaxial test, Bi-directional cyclic shear, Static shear stress

1 INTRODUCTION

Several major earthquakes have shown that a better understanding of the soil behaviour under multi-directional loading conditions is required for a correct stability assessment of geotechnical structures. The M 7.2 Gujarat earthquake in 2001 caused severe damage in the Indian harbor of Kandla (Gudehus et al. 2004), even though the direction of shaking was parallel to the shoreline. Conventional methods to assess the stability and serviceability of geotechnical structures are unable to predict the observed damage due to the anti-plane loading conditions. A similar behaviour was also observed during the M 6.9 Hyogoken-Nambu earthquake in 1995. The rims of the Port and Rocco Islands in Kobe spread somewhat independently of their direction, while the major inner parts experienced a more uniform settlement and almost no damage.

In a common simplified approach for stability assessments, the expected earthquake-induced “loading“ of the soil, described by the cyclic stress ratio (CSR), is compared to the liquefaction “resistance” of the soil deposit, described by the cyclic resistance ratio (CRR). The CRR is estimated by means of either in-situ tests (e.g. Standard Penetration Test SPT, Cone Penetration Test CPT) or laboratory tests. The resulting CRR is then further modified using correction factors to take into account the overburden stresses (K_σ) and the initial static shear ratio (ISSR) (K_α). For example, if the CRR is determined in the laboratory using a triaxial test at an initial mean pressure of $p'_0 = 100$ kPa and with no ISSR (stress deviator $q = 0$ kPa), the modified CRR value for a given stress state is given by:

$$CRR_{p'_0\neq 100;\alpha\neq 0} = CRR_{p'_0=100;\alpha=0} * K_{p'_0\neq 100} * K_{\alpha\neq 0} \text{ (1)}$$

Experimental values and diagrams for K_σ and K_α factors as a function of the soil state can be found in literature (Harder and Boulanger 1997; Yang and Sze 2011). The variable α is defined depending on the conducted test (cyclic triaxial or simple shear test). In a simple shear test, $\alpha = \tau_s/\sigma'_{v,0}$ is the ratio of the initial shear stress τ_s in the horizontal plane to the overburden stress $\sigma'_{v,0}$. $\alpha = 0$ implies a horizontal soil deposit while α becomes larger in a slope. Similarly, in triaxial test conditions, $\alpha = 0$ for an isotropic consolidated samples and $\alpha \neq 0$ for anisotropic consolidated samples. K_α is a function of α as exemplarily shown in Figure 1. The results presented in Figure 1 are based on triaxial tests. This diagram illustrates that in slopes and inclined soil layers below foundations where $\alpha > 0$, the liquefaction resistance can increase or decrease with respect to the reference value at $\alpha = 0$, depending on the initial relative density of the soil. While there is a consensus on the influence of the ISSR on the liquefaction resistance, and thus on the importance of the K_α factor, it is not clear, whether existing advanced constitutive models are able to account for the influence of K_α on the undrained shear response of soils The need for a better understanding of the K_α factor was stated during the 1996 NCEER and 1998 NCEER/NSF workshops (Youd et al. 2001): “Although curves relating K_α to α have been published (Harder and Boulanger 1997), these curves should not be used by nonspecialists in geotechnical earthquake engineering or in routine engineering practice”. It is seems that in cyclic triaxial tests, the ISSR is detrimental for loose samples and beneficial in dense cases, as can be seen in Figure 1. The importance of the loading type was shown by (Sivathayalan and Ha 2011). The results of their investigations on a medium-dense, subrounded silica sand showed that while K_α increased with α in triaxial testing, the opposite trend was observed for cyclic simple shear testing. In addition, the influence of the direction of shear loading on the liquefaction behaviour was experimentally investigated by Boulanger and Seed (1995) and Kammerer et al. (2004) using bi- and multi-directional cyclic shear tests. The main advantage of multi-directional, simple shear devices is that they reproduce the in-situ stress conditions and the seismic loadings (e.g. multi-directional nature of earthquake loadings) more realistically compared to conventional triaxial devices. The application of the SS-device to a bi-directional loading of a slope is shown in Figure 2. In the initial stress state, a static shear stress acts in the slope dip direction at a horizontal material point inside the slope. Cyclic loading in the plane of the slope is defined as in-plane loading, whereas the loading perpendicular to that as anti-plane loading. The experimental results showed that anti-plane loading leads to a lower CRR than in the case of in-plane loading (Boulanger and Seed 1995).

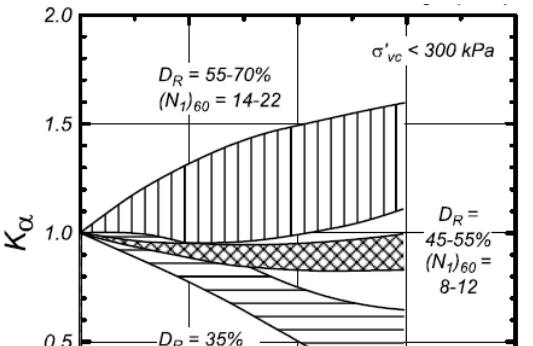

Figure 1. Influence of initial shear stress and density on K_α, from (Ziotopoulou et al. 2014) based on (Harder and Boulanger 1997)

In this paper, we analyse the influence of ISSR in triaxial and bi-directional cyclic shearing for two advanced constitutive models. Since the authors are unaware of any published laboratory data on the same soil sample for the investigated element tests, the analysis is of a qualitatively nature. We also restrict the simulations to the dense case, because according to experimental data, the existence of an ISSR may only be beneficial in a liquefaction analysis for this case. These types of investigations allow the identification of weak points in the constitutive models, which may be the starting point for further improvements.

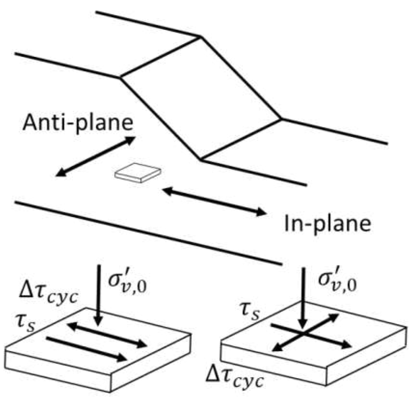

Figure 2. Definition of in-plane and anti-plane loading in bi-directional cyclic shearing

2 ELEMENT TEST SIMULATIONS

The first constitutive model considered is the hypoplasticity model of Wolffersdorff (1996) with inter-granular strain extension (Niemunis and Herle 1997), referred to in the following as the “vW-model”. Its ability to successfully simulate liquefaction phenomena in boundary value problems has been shown in several applications (e.g. vibratory pile installation (Chrisopoulos and Vogelsand 2019)). The second constitutive model is the hypoplastic model with a historiotropic yield surface proposed by Grandas et al. (2020), referred as the “Hist-model”. This model can track the influence of long-lasting preloading effects on liquefaction. In the following, these features are compared with experimental data to check the ability of the models to account for the influence of ISSR on liquefaction behaviour. A detailed formulation of the constitutive equations can be found in the references given above. Due to the lack of any available experimental database for a single material that contains all of the laboratory tests considered in this paper, we use the parameters of the Karlsruhe Sand provided for the vW-model in (Wichtmann et al. 2019) and for the Hist-model in (Grandas et al. 2020). Comparisons of the results of numerical simulations and experiments can also be found in these references.

In the numerical simulation, we adopt an initial density of $D_r = 0.85$. The reason for adopting a high density is that, according to Figure 1, the soil element in a slope should show a larger liquefaction resistance in comparison to the soil element below the horizontal ground surface, as the former has a larger ISSR. A larger liquefaction resistance does not necessarily mean that the slope has a higher resistant to liquefaction during an earthquake, because the induced shaking can be amplified by the geometry of the slope.

In order to identify the triggering liquefaction, we use the pore pressure criterion, defined as the point where the cyclic variation of pore pressure, r_u, where

$$r_u = \Delta p_w / p'_0 \tag{2}$$

becomes stationary. Δp_w represents the pore pressure's increment and p'_0 is the initial mean effective stress. One reason to adopt this criterion is that most constitutive models stop increasing shear strain amplitude in cyclic simple shear tests (Ziotopoulou et al. 2014) and the usual criterion based on achieving a target amplitude cannot be applied to the numerical results. This is also the case for the two models considered here. A pore-water pressure criterion also seems to be more objective because during the cyclic mobility phase (stationary pore water pressure variation), fabric changes occur which are still not accounted for by most of the advanced constitutive models and are currently subject of investigation.

The factor α is defined depending on the testing procedure as follows

$$\alpha = q_0 / 2p'_0 \quad \textit{in triaxial test} \tag{3}$$

$$\alpha = \tau_s / \sigma'_{v,0} \quad \textit{in cyclic shear test} \tag{4}$$

where q_0 is the initial deviatoric stress. Both values of α are comparable, but not the same.

The applied loading is described by the CSR and is defined as follows

$$CSR = \Delta q / 2p'_0 \quad \textit{in triaxial test} \tag{5}$$

$$CSR = \Delta \tau / \sigma'_{v,0} \quad \textit{in cyclic shear test} \tag{6}$$

Figure 3 presents the influence of ISSR in triaxial testing for the two models. For two initial α values, α = 0 and α = 0.3, the amplitude of CSR was chosen so that liquefaction occurs after 10 cycles. In the vW-model, the required loading for the case α = 0 is CSR = 0.25, and the value increases to CSR = 0.325 for α = 0.3. In the second model, the loading required to reach liquefaction is CSR = 0.175 for α = 0.0 and the value also increases to CSR = 0.275 for the larger initial deviatoric stress. Although the Hist-model generally predicted a larger accumulation of pore-water pressure compared to the experiments, it should be noted that the Hist-model can describe the butterfly stress-path during cyclic mobility more accurately than the vW-model, where the pore pressure stops accumulating at around 20 kPa. This is a well-known drawback of the vW-model. Both models predicted similar accumulated axial strains at the onset of liquefaction, but the magnitude is different (Figure 4). The strains reached with the vW-Hypo model are around 1%, whereas for the Histo-model they are around 2.5%. Both values are usual for a strain-based definition of triggering liquefaction.

2.1 Triaxial test simulations

Most of the available experimental investigations of the influence of ISSR on the liquefaction potential are based on positive α factors (compression side). This anisotropic stress describes the state of a material point at the crest of a slope. At the base of the slope, a more realistic stress state is that of an extension triaxial test. According to Eq. (3), α becomes negative for a triaxial extension. Few experimental data presented in the literature show that with negative α values, the liquefaction resistance of dense sand decreases with decreasing values of α (Pan and Yang 2018). This means that the liquefaction resistance at the base of a slope should be smaller than at the crest and smaller than for horizontal ground. This experimental observation should be reproduced by constitutive models used for liquefaction assessment.

The response of the investigated constitutive models in cyclic triaxial tests is presented exemplarily in Figure 5. The numerical simulations are carried out with an initial mean pressure of $p'_0 = 100\ kPa$ and different α values, ranging from -0.2 in extension to 0.3 in compression, are considered. In the vW-model, the CSR values required to trigger liquefaction clearly increase with increasing α. The difference between the CSR curves is relatively small for α factors between -0.2 and 0.0. The results of the Hist-model show CSR decreasing with increasing α value from -0.2 to 0.0 and then increasing with increasing α value.

2.2 Bi-directional cyclic shear test simulations

The main advantage of a multi-directional cyclic shear test is that it can better reproduce the rotation of the principal stresses related to the actual soil motion during an earthquake. However, due to the complexity of the testing device and its operation, the use of a multi-directional cyclic simple shear test is not established in current engineering practice.

In the simulations, we consider the initial vertical effective consolidation stress $\sigma'_{v,0} = 100\ kPa$. K_0 conditions are assumed for the horizontal stresses and the initial static shear stress is defined so that α varies from 0 to 0.3. The CSR curves for the in-plane loading are shown in Figure 6. Both models display a general trend of increasing the liquefaction resistance with increasing α. This behaviour matches qualitatively the experimental data found in literature (Boulanger and Seed 1995).

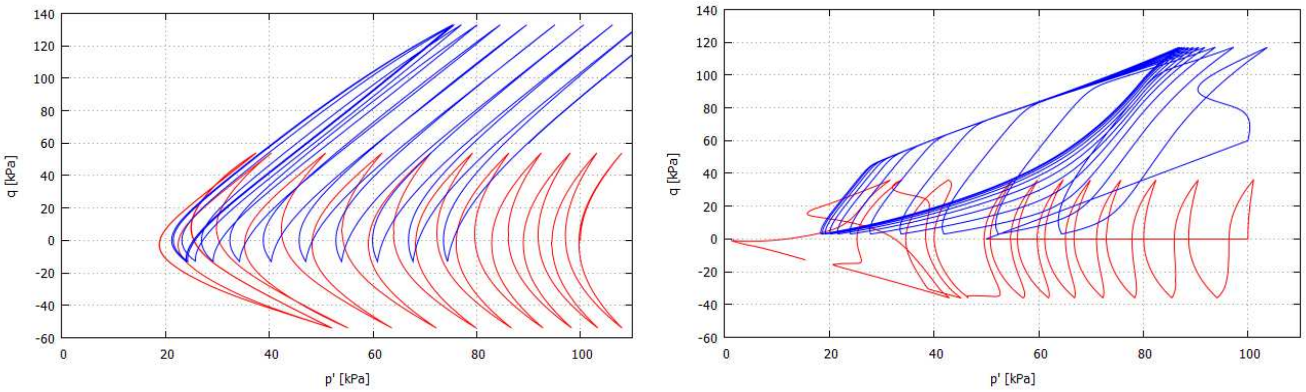

Figure 3. Influence of initial shear stress in p'-q plane triaxial testing left) vW-model, right) Hist-model,

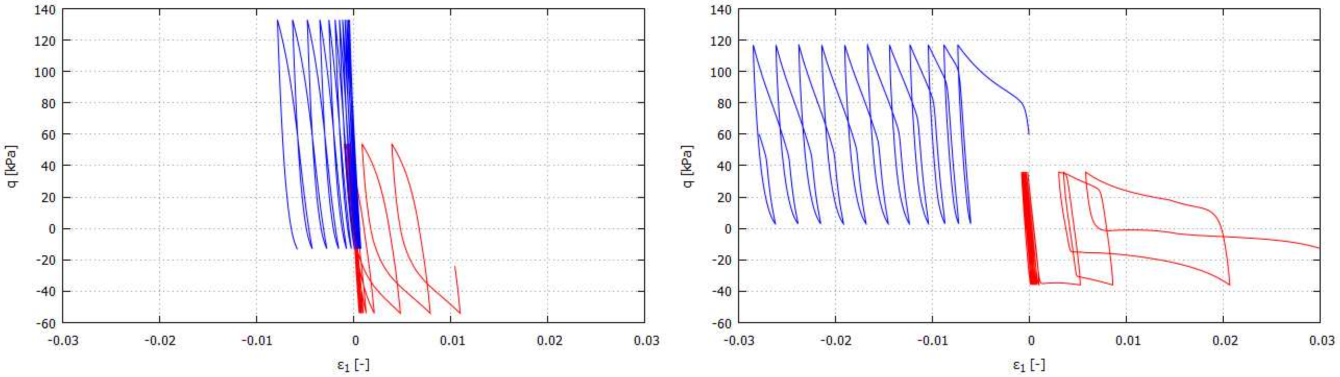

Figure 4. Influence of initial shear stress in q-ε_1- plane in triaxial testing left) vW-model, right) Hist-model,

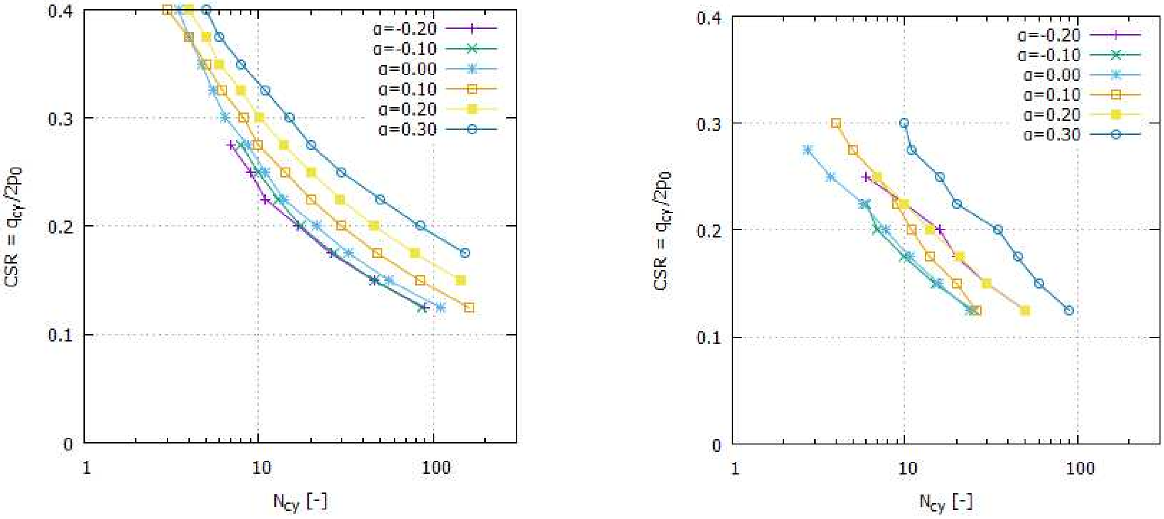

Figure 5. Influence of initial shear stress on the cyclic stress ratio in triaxial testing: left) vW-model, right) Hist-model

The results for the anti-plane loading conditions are presented in Figure 7. A general trend of a decrease in liquefaction resistance with an increasing α can be observed. This behaviour also qualitatively matches the experimental data found in literature (Boulanger and Seed 1995).

The results presented here for the vW-model support the findings of Gudehus et al. (2004). In numerical simulation of the behaviour slopes under in-plane and anti-plane shaking, they observed larger deformations at the crest for anti-plane in comparison with in-plane shaking. They concluded that in-plane analysis, as usually conducted in engineering practice, is not necessarily on the safe side for seismic loading.

2.3 K_α factors

The variation of K_α with α can be computed on the basis of the CSR curves. K_α is defined as

$$K_\alpha = CRR_{\alpha\neq0} \,/\, CRR_{\alpha=0} \tag{6}$$

where CRR is the cyclic resistance ratio and is defined as the CSR value for a given number of cycles. A number N = 10 cycles is ususally adopted. The results for K_α based on the CSR-curves in Figures 5, 6 and 7 are plotted in Figure 8. The results show that the numerical results for in-plane loading lie in the range given by Harder and Boulanger (1997) for dense sands. The behaviour is softer for anti-plane loading and falls into the range of loose to medium-dense sand. The ratio between the in-plane CRR and anti-plane CRR is presented in Figure 9. It matches qualitatively the data from Boulanger and Seed (1995) for a medium-dense sand.

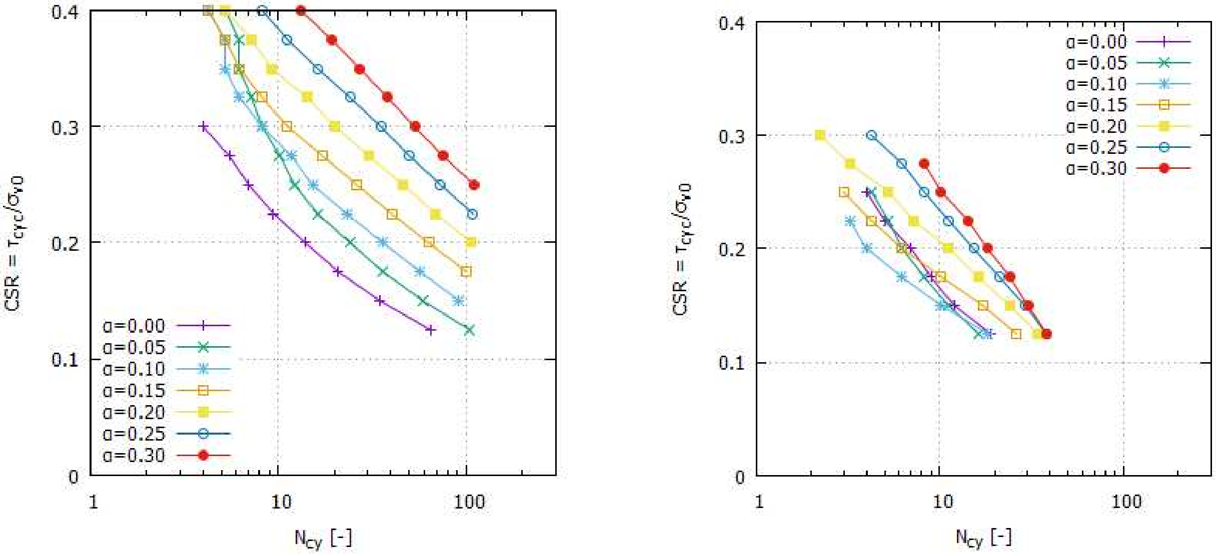

Figure 6. Influence of initial shear stress on the cyclic stress ratio for in-plane loading: left) vW-model, right) Hist-model model

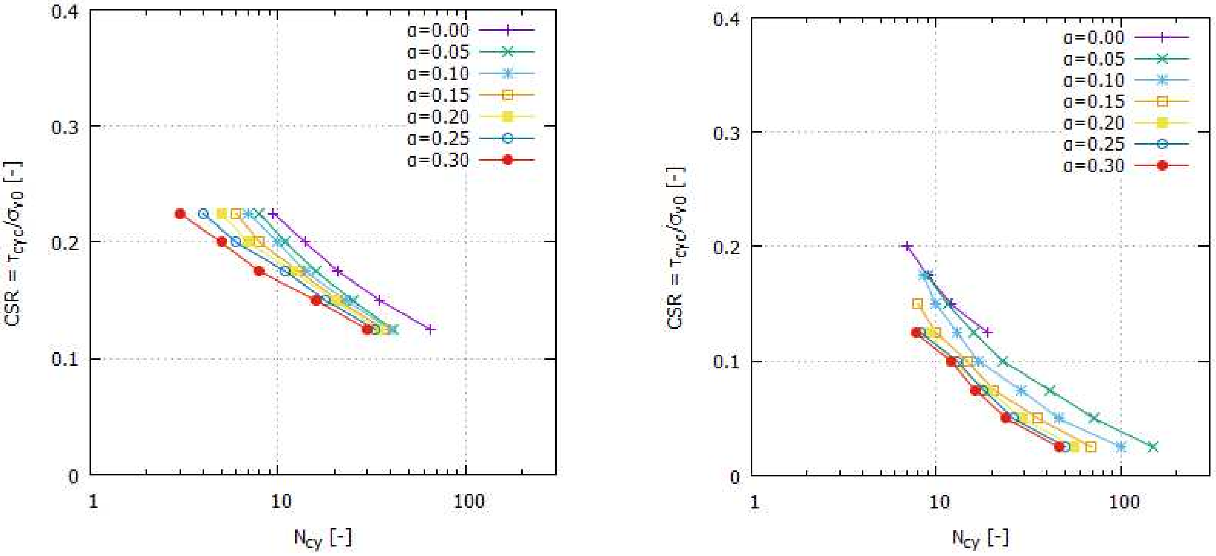

Figure 7. Influence of initial shear stress on the cyclic stress ratio for anti-plane loading: left) vW-model, right) Hist-model

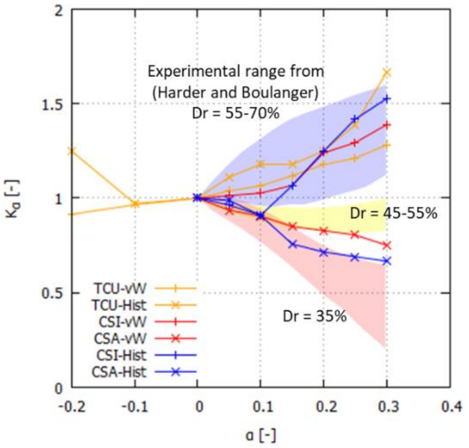

Figure 8. Influence of loading condition on K_α

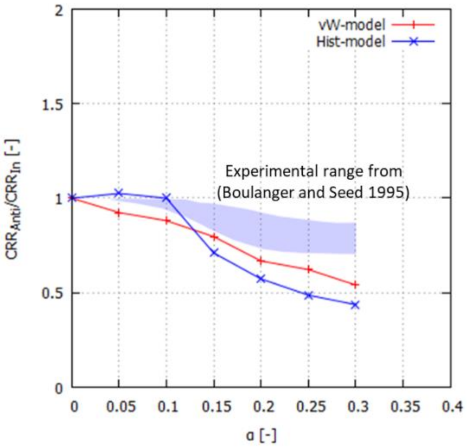

Figure 9. Variation of the ratio anti-plane to in-plane liquefaction resistance with α

3 CONCLUSION

This paper investigates the influence of ISSR on liquefaction susceptibility for two advanced hypoplastic models. In general, the results qualitatively match the experimental data found in literature. Encountered weak points are the starting point for further developments. The available experimental data is insuficient to consistently validate the ability of constitutive models to predict the influence of ISSR on liquefaction. The development of such a experimental database would be greatly benefical for further numerical developments. The relationship between the results at element test level and boundary value problems, e.g. infinite slope conditions, shall be further investigated.

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