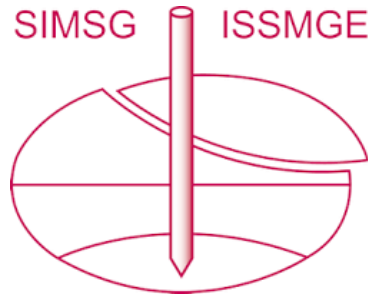


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# Influence of leakage direction and pipe depth on the soil fluidisation using the MPM

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**ABSTRACT:** Soil fluidisation induced by a defective pressurised pipe could generate an underground cavity leading to severe ground collapse. Pipe leakage may fluidise the surrounding soil if sufficient leakage is presented. This study provides insights into the significance of the fluidisation mechanism triggered by a defective pressurised pipe and identifies the critical state that causes the ground subsidence. A two-phase double-point Material Point Method (MPM) model is developed to simulate the behaviour of a soil bed subjected to leakage from a buried pressurised pipe. Additionally, in/outflow boundary conditions (BCs) are applied to prescribe constant velocity inflow of material points into the region. The MPM model is used to study the soil fluidisation mechanism, which involves a change in soil porosity and expansion of the soil bed. Parametric analyses are conducted to investigate the effect of the leakage direction and the buried depth of the pipe on soil-leakage interaction. The results indicate that lateral leakage requires a higher leakage velocity to trigger soil fluidisation than upward leakage. The shallower the pipe depth, the lower the inflow velocity required for the onset and development of fluidisation, but the greater the ground displacement.

**Keywords:** Soil fluidisation; lateral leakage; upward leakage; Material Point Method.

## 1 INTRODUCTION

Every city is maintained operationally using a network of water, energy, and sewer infrastructure. With the expansion of cities, on the one hand, and the advances in technology and engineering, on the other hand, this infrastructure network undergoes continual expansion and transformation. This continuous development of infrastructure systems, comprising surface transportation infrastructure and subsurface utilities, offers substantial opportunities for the further prosperous growth of urbanized cities. An important element of these systems is underground pipelines which are critical components of infrastructure in modern developed cities that are used for the transportation of water, sewage, gas and liquid fuels (Dave and Solanki, 2020). Given their significant role in urban systems, underground pipe networks need to be adequately maintained throughout their designed life span. However, the anticipated and designed life span and the performance of water networks can be adversely impacted by pipeline defects caused by several factors, including but not limited to pipe aging, corrosion, material defects, excavation, external loads, and internal pressure (Fang et al., 2011).

Among various reasons for pipe defects, pipe leakage is perceived to be a serious issue by water-management authorities, i.e., 35% of the transmitted water in the world is wasted as a result of leakage (Hecke, 2020). In the United Kingdom, this problem amounts to 3,113

million litres of water that are lost daily (DiscoverWater, 2022). Besides the loss of valuable fluids during transmission due to damaged pipes, water leakage in buried water distribution mains may result in other severe problems, such as water shortage, subsurface contamination, and ground collapse (Karoui et al., 2018). Soil fluidisation is one of the consequences of water leakage, which occurs when soil particles lose their interlocking forces and move freely within the pore (Richards Jr et al., 1990; Alsaydalani, 2010). This process results in reduction of bearing capacity and, subsequently, leads to surface subsidence, usually in the form of a sinkhole that can significantly damage underground infrastructures and cause significant social and economic drawbacks. Thus, understanding the dynamics and governing processes of underground pipe leakages, including the soil-fluid interaction, is an essential factor for maintaining the infrastructure and consequently preventing the aforementioned disasters.

Numerous laboratory studies have been conducted to investigate the soil fluidising mechanism around leaking pipes (Rogers et al., 2008; Teeluckdharry, 2017). However, difficulties and uncertainties during the data acquisition phase inhibited these studies from determining essential factors for interpreting the soil-fluid behaviour, including the fluidising pressure and soil displacement aspects (Cui et al., 2014). To undertake more effective measures, further studies are required to fill

knowledge gaps of the soil-fluid behaviour, for example, the transition to the post-fluidisation process under the above-mentioned circumstances (Zhu et al., 2018).

At the same time, considerable efforts have been made to develop numerical simulations of the soil fluidisation around a leaking pipe through different approaches. This includes Finite Element Method (FEM) analyses (for example, the work by Zhu et al. (2018)) investigating the effects of various parameters e.g. leakage pressure, crack sizes, and soil layering on the flow regime. Previous experiments have revealed that the fluidisation initiation is associated with a localised cavity formation around the leakage zone that exhibits large displacements. Standard FEM technique is not capable of fully modelling the fluidisation mechanism due to mesh distortion and tangling in large deformation problems (Wang et al., 2015). Accordingly, alternative methods are more suitable for the simulation of large deformations. For example, Cui (2014) investigated the inhomogeneities of granular particle behaviour in the soil fluidisation around a leaking pipe using coupled Discrete Element Method-Lattice Boltzmann Method (DEM-LBM). However, DEM is computationally expensive for real-scale pipe leakage problems. Therefore, a thorough understanding of the soil fluidisation around a leaking pressurised pipe should be established through an efficient numerical method that is applicable to simulate such a multi-phase large deformation problem.

In this study, the soil fluidisation mechanism induced by a leaking buried pressurised water pipe is studied using the Material Point Method (MPM), capable of modelling large deformations in multi-phase geotechnical problems (Bandara and Soga, 2015; Yerro et al., 2015; Ceccato et al., 2018; Li et al., 2022). The MPM is a particle-based method that was developed by Sulsky et al. (1994) in which the time-dependent continuum bodies are represented using a set of Lagrangian Material Points accompanied by a conventional finite element computational mesh. The Lagrangian MPs represent a part of a continuum rather than individual solid particles. The MPs move through a fixed Eulerian mesh that is used to solve the main governing equations (i.e., momentum balance). These MPs convey the stress, strain, mass, energy, momentum, and other material parameters of the continuum.

Recently, Monzer et al. (2022) investigated the capabilities of two MPM two-phase approaches to simulate the soil fluidisation mechanism around a leaking pipe. The results show that the single point approach is limited to identifying the onset of fluidisation due to the numerical complexity of maintaining water leakage and the inability of the constitutive model to represent the transition from solid to liquid state of the porous media. The two-phase double-point (2P-DP) MPM approach is proven to be capable of capturing the evolution of soil fluidisation until it reaches the soil surface. The application of 2P-DP MPM approach was further expanded

by Monzer et al. (2023) to study the effect of different soil and pipe parameters on the soil fluidisation mechanism around a leaking pipe. However, these studies have not focused on the effect of leakage direction and pipe depth that might significantly affect the soil fluidisation mechanism.

The two-phase double-point (2P-DP) MPM approach, coupled with the use of inflow/outflow boundary conditions, is used to capture the initiation and evolution of soil fluidisation around a leaking pipe. The paper is organised as follows. First, the description of the developed MPM pipe leakage model is presented. Then, the results are presented and discussed, where the effect of the leakage direction and the buried depth of the pipe are investigated in terms of the soil fluidisation mechanism. Finally, the conclusions are summarised at the end. All the simulations are conducted adopting an in-house version of the open-source Anura3D MPM software (Anura3D, 2022).

## 2 NUMERICAL MODEL

The soil fluidisation induced by a leaking pipe is studied using the two-phase double-point (2P-DP) formulation (Bandara, 2013; Abe et al., 2014; Martinelli, 2016; Cao and Neilsen, 2021) that uses two sets of MPs to represent the solid phase and the liquid phase separately; these are so-called solid material points (SMPs) and liquid material points (LMPs). The LMPs can represent the free water as well as the pore water. The water leakage is triggered by inducing a constant water flow through the orifice, and the in/outflow boundary conditions (BCs) established by Zhao et al. (2019) are incorporated. In the 2P-DP formulation developed by Martinelli (2016), a maximum porosity threshold  $n_{\max}$  (i.e., critical porosity) is defined to distinguish between the solid-like and liquid-like state of the solid-liquid mixture. A mixture with a porosity lower than  $n_{\max}$  is considered saturated soil (solid-like state), where the SMPs are controlled by the effective stresses. In this case, constitutive models associated with the solid skeleton are used. When the porosity is higher than  $n_{\max}$ , the mixture fluidises (i.e., liquid state), the effective stresses at the SMPs become zero, and the liquid-like behaviour of the mixture is described by the Navier-Stokes equation.

Two numerical models have been developed to investigate the effect of leakage direction and pipe depth on the fluidised regime and soil behaviour. A pipeline with a diameter of 50 mm is embedded at a burial depth ( $H_p$ ) of 150 mm below the ground surface in a saturated soil bed. Two cases are studied considering the locations of the orifice located at the crown (upward leakage) and springline (lateral leakage) of the pipe with the orifice size ( $\phi$ ) fixed at 10 mm. The two-dimensional MPM models of both cases are presented in Figure 1, where the pipe is replaced by an equivalent square with

a 50 mm long side. The model designed to study the impact of the upward leakage where the orifice is located at the crown of the pipe is presented in Figure 1a. A similar model is designed to investigate the effect of the lateral leakage, shown in Figure 1b. The parameters for both soil and water used in the model are derived from the experiment performed by Alsaydalani (2010), listed in Table 1, and consistent with Monzer et al., (2022, 2023). The Mohr-Coulomb elastic-perfectly plastic constitutive model (MC) is employed to represent the behaviour of the soil, while the standard Newtonian compressible constitutive model is used to describe the characteristics of water.

Throughout each simulation, LMPs in the inflow element (Figure 1, in green) are assigned a prescribed velocity ( $v_i$ ). If an inflow element is no longer filled with LMPs, new LMPs are added at the Gauss point location to refill the inflow elements. The outflow elements at the top of the model (Figure 1, in pink) maintain a constant free water table by automatically removing LMPs that enter this zone. The free water region (Figure 1, in blue) represents elements initially filled with only LMPs. The saturated soil zone (Figure 1, in light brown) is represented by assigning both SMPs and LMPs. The model consists of 3,520 three-node triangular non-structured elements. The mesh is refined around the pipe to accurately capture the orifice size and the inflow process. The minimum element size is 0.0025m, that gradually increases to 0.033m towards the edges of the model. Six material points (three LMPs and three SMPs) per element are initially assigned to the saturated soil domain, while six LMPs are considered for the free water and inflow elements. The displacements of the soil and liquid are restricted in the normal direction and free in the longitudinal direction. The earth pressure coefficient at rest ( $K_0$ ) procedure is used to initialise the effective stresses, and pore pressures are initially hydrostatic.

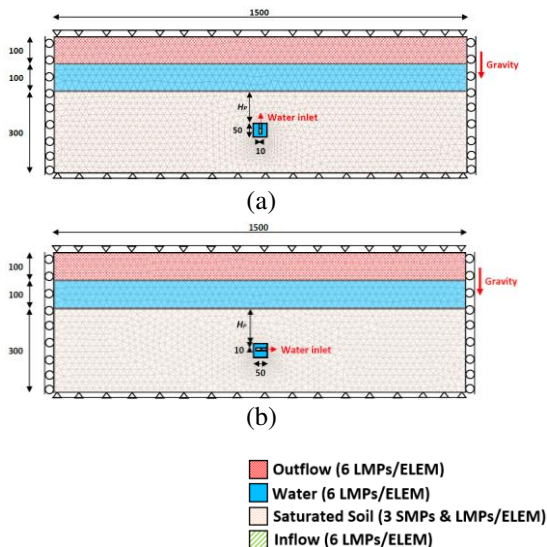


Figure 1. Geometry of soil bed subjected to (a) upward leakage and (b) lateral leakage (unit: millimetres)

Table 1. Material properties of the silica sand and water used in the model (Alsaydalani 2010).

Property	Value
Initial porosity ( $n_0$ ) [-]	0.45
Density soil ( $\rho_s$ ) [ $kg/m^3$ ]	2660
Water density ( $\rho_l$ ) [ $kg/m^3$ ]	1000
Water bulk modulus ( $K_l$ ) [ $kPa$ ]	50000
Water viscosity ( $\mu_d$ ) [ $kPa \cdot s$ ]	$10^{-6}$
$K_0$ -value ( $K_0$ ) [-]	0.44
Effective Poisson ratio ( $\nu'$ ) [-]	0.3
Young's modulus ( $E'$ ) [ $kPa$ ]	69000
Effective cohesion ( $c'$ ) [ $kPa$ ]	1.0
Effective friction angle ( $\phi'$ ) [ $^\circ$ ]	34
Soil grain diameter ( $D_p$ ) [ $mm$ ]	0.9
Maximum soil porosity ( $n_{max}$ ) [-]	0.50

### 3 RESULTS AND DISCUSSION

This section provides and discusses the findings of the investigation into the impact of leakage direction and pipe depth on the mechanism of soil fluidisation.

#### 3.1 Effect of leakage direction

The porosity field at the onset and surface fluidisation for the two scenarios at different inflow velocities ( $v_i$ ) is compared in Figure 2. An inflow velocity ( $v_{io}$ ) of 0.25 m/s at a 10 mm orifice in a soil bed subjected to an upward leakage is required to initiate the soil fluidisation (Figure 2a). For lateral leakage, the soil fluidisation is initiated at a higher inflow velocity ( $v_{io}$ ) of 0.04 m/s (Figure 2c). Soil fluidisation initiates under a lower inflow velocity in a soil bed subjected to an upward leakage. Similarly, the inflow velocity ( $v_{is}$ ) required to develop fluidisation reaching the bed surface increases with lateral leakage. The inflow velocity inducing the surface fluidisation ( $v_{is}$ ) increases from 0.05 m/s to 0.09 m/s as the leakage direction changes from upward (Figure 2b) to lateral leakage (Figure 2d). Thus, the inflow velocity required for the onset ( $v_{io}$ ) and surface ( $v_{is}$ ) fluidisation increases linearly with the change of leakage direction from crown to springline. In addition, the area of the fluidised zone increases as the leakage direction changes from upward to lateral flow. Hence, orifice location is an essential factor in soil fluidisation, and determining the leakage location can effectively improve the interpretation of the fluidisation mechanism.

Alsaydalani (2020) experimentally investigated the impact of lateral water flow from an impermeable vertical surface on the failure mechanism of a granular bed using the Particle Image Velocimetry (PIV) technique. The following sequences of failure were observed in horizontal leakage under an increasing flow rate: (i) outward movement (horizontally and vertically upwards) of soil particles away from the orifice, (ii) cavity formation in the vicinity of the orifice, (iii) cavity enlarge-

ment, and (iv) surface fluidisation. Consistent behaviour is observed in the MPM simulation presented in Figure 2c and Figure 2d. As the inflow rate increases, the fluidised zone expands horizontally and vertically upwards through the soil bed. This expansion primarily occurs horizontally in the leakage direction. However, as the fluidised zone develops away from the orifice, it gradually propagates upwards to reach the soil surface.

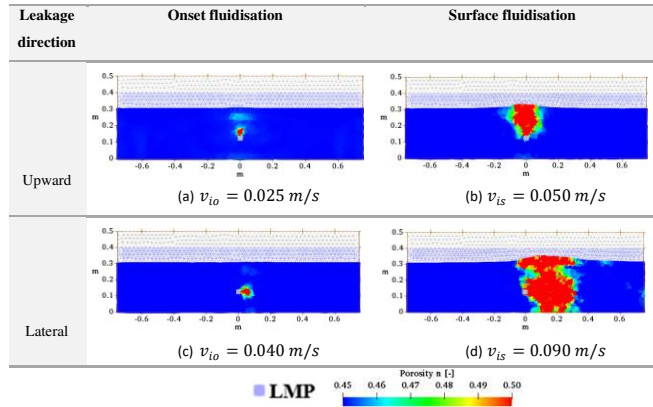


Figure 2. Soil porosity at the onset and surface fluidisation in soil bed subjected to upward (a, b) and lateral (c, d) leakage at the different inflow velocities

The influence of leakage direction on the soil bed expansion at the onset of fluidisation is also investigated. Figure 3 plots the change in bed-height expansion ratio as the inflow velocity increases with different leakage directions. Water can easily flow upwards through the soil subjected to an upward leakage, and soil-bed fluidisation is initiated at a lower inflow velocity, as shown in Figure 3. This higher water velocity results in an increase in the expansion ratio. However, upward flow is more difficult through soil subjected to lateral water leakage. In an upward leakage, the expansion ratio is higher at the same leakage velocity. The final surface heave as the fluidisation reaches the ground level in lateral leakage is higher than the surface heave that occurs in upward leakage. It is inferred from this section that the change in the orifice location from the crown (upward leakage) to the springline (lateral leakage) of the pipe plays a significant role in the development of the soil fluidisation triggered by a leaking pipe.

### 3.2 Effect of pipe depth

The effect of pipe depth on soil fluidisation is studied by varying the pipe burial depth ( $H_p$ ) for different leakage directions, upward and horizontal flow.

#### 3.2.1 Upward leakage

Four simulations with different embedded depths of pipe of 50, 100, 200, and 300 mm are modelled with an orifice located at the crown (upward leakage) of the pipe. The change in the inflow velocity required for the onset ( $v_{io}$ ) and surface ( $v_{is}$ ) fluidisation at a 10 mm orifice with the change of the pipe depth is shown in Figure

4. The 300 mm burial depth resulted in a greater fluidised zone than the shallower burial depths, as shown in Figure 4. This agrees with the experimental study conducted by Pike et al. (2018) that observed the intensity of the soil movement is increased in a pipe with greater cover depth. As the pipe depth increases from 50 mm to 300 mm, the inflow velocity inducing the soil fluidisation ( $v_{io}$ ) increases linearly from 0.01 m/s to 0.04 m/s. Similarly, the inflow velocity required to expand the fluidisation zone reaching the soil surface ( $v_{is}$ ) increases considerably from 0.02 m/s to 0.10 m/s, corresponding to a pipe depth of 50 and 300 mm, respectively. Therefore, the shallower the pipe depth, the lower the water velocity required to induce fluidisation, and the smaller the influence on the surrounding soil in the fluidisation developing stage. However, the deeper the pipe burial depth, the higher the inflow velocity needed to initiate fluidisation, and the greater the fluidised zone.

The variation in surface heave versus the inflow velocity in a soil bed subjected to an upward leakage with different pipe depths is illustrated in Figure 5. The greater the pipe depth, the lower the bed expansion at the same water velocity. As the pipe depth increases, higher leakage velocity is required to induce soil fluidisation. The expansion ratio at the onset of fluidisation is not significantly affected by the depth of the pipe. However, as the burial depth increases from 50 mm to 100 mm, the final expansion ratio at surface fluidisation increases by 6%. Beyond these values, the final surface heave is not significantly affected by the pipe depth.

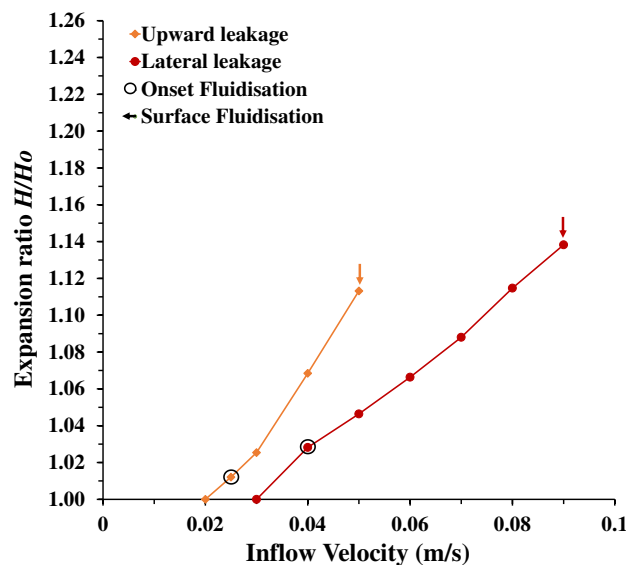


Figure 3. Change of soil bed expansion ratio is soil bed subjected to different flow directions



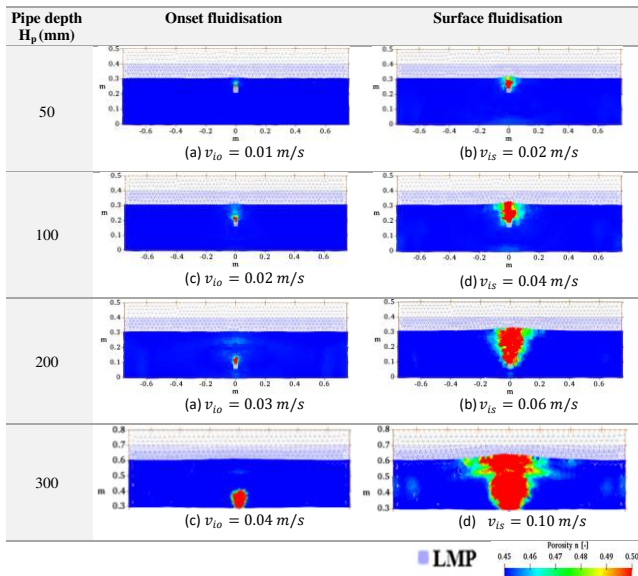


Figure 4. Soil porosity in the soil bed subjected to an upward leakage from a pipe with different burial depths

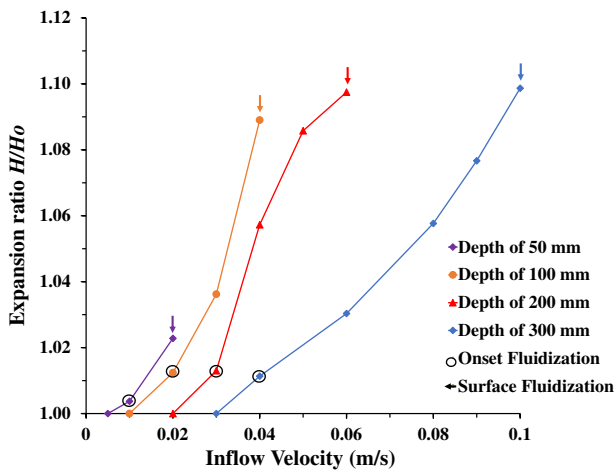


Figure 5. Change of soil bed expansion ratio through a pipe located at different depths

### 3.2.2 Lateral leakage

The effect of burial depth on the soil fluidisation triggered by lateral leakage from an orifice located at the springline of the pipe is shown in Figure 6 for different pipe depths: 50, 100, and 150 mm. The depth of the modelled pipe is limited to 150 mm to avoid the effect of the lower boundary condition during the development of fluidisation. It can be observed that the deeper the pipe, the higher the leakage velocity required to initiate fluidisation. In the presented simulations, an inflow velocity ( $v_{io}$ ) of 0.02 m/s is required to induce the soil fluidisation in a pipe buried 50 mm below the ground level (Figure 6a). This value increases linearly to 0.04 m/s with the increase of pipe depth from 50 mm to 150 mm (Figure 6e). The deeper the buried depth of the pipe, the slower the development of the fluidised zone that occurred at higher water velocity. However, the greater the pipe depth, the greater the fluidised zone induced by lateral leakage.

The influence of the pipe depth on the expansion ratio versus the inflow velocity is plotted in Figure 7. Change of soil bed expansion ratio through a pipe located at different depths. Soil-bed fluidisation is initiated at a lower water velocity through the soil bed subjected to lateral leakage from a pipe located at a shallower depth. The deeper the pipe depth, the lower the surface heave at the same leakage velocity. As the soil fluidisation is fully developed, the expansion ratio is higher in the soil bed with greater pipe depth. For example, the expansion ratio increases by 8% as the pipe depth increases from 50 mm to 150 mm. Thus, the greater the depth of the pipe, the greater the effect on the soil bed. This shows the significant impacts of the leakage location and pipe depth on the fluidisation process.

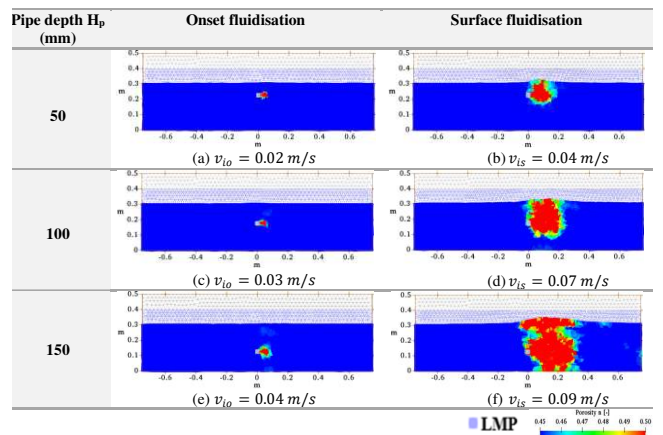


Figure 6. Soil porosity at the SMPs in soil bed subjected to a lateral leakage from a pipe with different burial depths

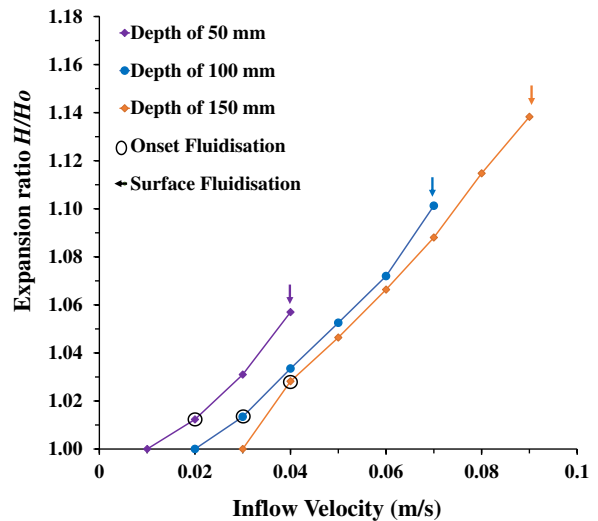


Figure 7. Change of soil bed expansion ratio through a pipe located at different depths

## 4 CONCLUSIONS

In this study, the soil fluidisation triggered by a leaking water pipe is modelled using the two-phase double-point (2P-DP) MPM approach coupled with the use of inflow/outflow boundary conditions. The developed model captures the initiation and development of soil

fluidisation until it reaches the soil surface. The MPM model is used to investigate the impact of leakage direction and pipe depth on the soil fluidisation mechanism. The occurrence of soil fluidisation is dependent on the leakage direction. A lower leakage velocity is required for the soil fluidisation induced by upward leakage compared to lateral leakage. Furthermore, the inflow velocity required for the onset and progression of fluidisation increases with the increase in pipe depth. It can be observed that a lower surface heave is attributed to leakage from a pipe buried in a deeper depth. These results proved the significance of the leakage direction and pipe depth on the fluidisation mechanism triggered by a defective pressurised water pipe and support the identification of the critical state that causes the ground subsidence. In this paper, the pipe is simply modelled as a source of leakage in the soil-fluid interaction. However, the pipe may experience external soil loads, which could cause additional leaks. Thus, future work should consider the soil-pipe interaction effects to obtain a more representative model of the pipe leakage problem. It is also important to note that additional effort is required to more precisely address the transition between soil and fluid by utilizing advanced constitutive models.

## 5 ACKNOWLEDGEMENTS

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## 6 REFERENCES

- Abe, K., Soga, K. and Bandara, S. (2014) Material Point Method for Coupled Hydromechanical Problems. *Journal of Geotechnical and Geoenvironmental Engineering*.
- Alsaydalani, M. (2010) *Internal fluidisation of granular material*. University of Southampton.
- Alsaydalani, M.O. (2020) Failure Mechanism of a Granular Bed Induced by a Horizontal Water Jet Using Particle Image Velocimetry. *KSCE Journal of Civil Engineering*, 24 (6): 1696–1705.
- Anura3D (2022) *Anura3D MPM Research Community*. Available at: <http://www.anura3d.com/>.
- Bandara, S. and Soga, K. (2015) Coupling of soil deformation and pore fluid flow using material point method. *Computers and geotechnics*, 63: 199–214.
- Bandara, S.S. (2013) *Material point method to simulate large deformation problems in fluid-saturated granular medium*. University of Cambridge.
- Cao, C. and Neilsen, M. (2021) Dam breach simulation with the material point method. *Computation*, 9 (2): 8.
- Ceccato, F., Yerro, A., Martinelli, M. (2022) Modelling soil-water interaction with the material point method. Evaluation of single-point and double-point formulation. In *Numerical methods in geotechnical engineering IX*.
- Cui, X., Li, J., Chan, A., and Chapman, D. (2014) Coupled DEM-LBM simulation of internal fluidisation induced by a leaking pipe. *Powder Technology*, 299–306.
- Dave, M.M. and Solanki, C.H. (2020) *Protection of Buried Pipelines Using Geosynthetics Under Different Loading Conditions*. In Prashant, A., Sachan, A. and Desai, C.S. (eds.). Springer Singapore.
- DiscoverWater (2022) *Find out how water companies in England & Wales are performing*. Available at: [/discoverwater.co.uk](http://discoverwater.co.uk) (Accessed: 8 October 2022).
- Fang, Q., Zhang, D. and Wong, L.N.Y. (2011) Environmental risk management for a cross interchange subway station construction in China. *Tunnelling and Underground Space Technology*, 26 (6): 750–763.
- Hecke, B. Van (2020) *Lose the leaks*. Available at: <https://nickelinstitute.org> (Accessed: 9 October 2022).
- Karoui, T., Jeong, S.Y., Jeong, Y.H. and Kim, D.S. (2018) Experimental study of ground subsidence mechanism caused by sewer pipe cracks. *Applied Sciences*, 679.
- Li, X., Yao, J., Sun, Y. and Wu, Y. (2022) Material point method analysis of fluid-structure interaction in geohazards. *Natural Hazards*, 114, 3425–3443.
- Martinelli, M. (2016) Soil-water interaction with material point method. *Double-Point Formulation. Report on EU-FP7 research project MPM-Dredge*.
- Monzer, A., Faramarzi, A., Yerro, A. and Chapman, D. (2023) MPM Investigation of the Fluidisation Initiation and Post-Fluidisation Mechanism Around a Pressurised Leaking Pipe, *Journal of Geotechnical and Geoenvironmental Engineering*, (under review).
- Monzer, A., Murphy, J., Yerro, A., Faramarzi, A. and Chapman, D. (2022) Simulation of Soil Fluidization Around a Pressurised Leaking Pipe Using the Material Point Method. In *Geo-Congress 2022*, 363-374.
- Richards Jr, R., Elms, D.G. and Budhu, M. (1990) Dynamic fluidization of soils. *Journal of Geotechnical Engineering*, 116 (5): 740–759.
- Rogers, C.D.F., Chapman, D.N. and Royal, A.C.D. (2008) Experimental investigation of the effects of soil properties on leakage: final report [R]. *Report prepared for Thames Water Limited and Three Valleys Water Limited. Department of Civil Engineering, The University of Birmingham, Edgbaston Birmingham*.
- Sulsky, D., Chen, Z. and Schreyer, H.L. (1994) A particle method for history-dependent materials. *Computer methods in applied mechanics and engineering*, 179–196.
- Teeluckdharry, S. (2017) *An experimental investigation of leakage flow paths in soil surrounding leaks in water distribution systems*. University of Cape Town.
- Wang, D., Bienen, B., Nazem, M., Tian, Y., Zheng, J., Pucker, T. and Randolph, M.F. (2015) Large deformation finite element analyses in geotechnical engineering. *Computers and Geotechnics*, 65: 104–114.
- Yerro, A., Alonso, E. and Pinyol, N (2015) The material point method for unsaturated soils. *Geotechnique*.
- Zhao, X., Bolognin, M., Liang, D., Rohe, A. and Vardon, P.J. (2019) Development of in/outflow boundary conditions for MPM simulation of uniform and non-uniform open channel flows. *Computers and Fluids*, 179: 27–33.
- Zhu, H., Zhang, L., Chen, C. and Chan, K. (2018) Three-dimensional modelling of water flow due to leakage from pressurized buried pipe. *Geomechanics & engineering*, 16 (4): 423–433.