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A hybrid reliability methodology for rock tunnel stability analysis coupled with polymorphic uncertainty modelling of rock properties

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ABSTRACT: A hybrid reliability methodology is proposed for the rock tunnel stability analysis considering the polymorphic nature of uncertainties in input properties. This method combines a fuzzy approach to model epistemic uncertainties in Geological Strength Index, considers transformational uncertainty of empirical models, and stochastic methods to model aleatory uncertain properties. Further, detailed guidelines are proposed for the characterization and fuzzification of transformational uncertainties by using the data collected through an extensive literature review. A modified Convergence-Confinement method is proposed and demonstrated by performing the hybrid analysis of a railway tunnel in Jammu and Kashmir, India. Hybrid Monte-Carlo simulations were performed to estimate p-boxes of response parameters of the tunnel, i.e., plastic zone radius and radial deformation. Further, the results of the hybrid analysis evaluated from the developed methodology were compared with the traditional methods-based results and it was concluded that the proposed methodology is appropriate for the tunnel stability analysis with input parameters having different uncertainty types.

Keywords: Hybrid Monte-Carlo simulations; fuzzy approach; P-box; polymorphic uncertainty

1 INTRODUCTION

Modelling of uncertainties related to rock properties and model parameters have always been challenging and demanding in rock mechanics. The probabilistic analysis has found widespread use in this field when combined with reliability methods like the First/Second-Order Reliability Method, Point Estimate Methods, Monte-Carlo Simulation (MCS), etc. (Hoek, 1998; Langford and Diedrich's, 2013; Tiwari and Gali, 2017; Rasmussen et al., 2019).

The primary need for using probabilistic methods is the availability of precisely measured, sufficient data for accurate estimation of probability density function (PDF). However, rock properties data are never sufficient in quantity or adequately precise for design purposes in rock mechanics. This is because sampling of rock masses is usually inadequate due to the costly, time-consuming, and often limited access to the work site.

Moreover, the majority of parameters in rock mechanics are empirical and obtained from expert opinion/subjective approach, like GSI (Geological Strength Index), RMR (Rock Mass Rating), RSR (Rock Structure Rating), etc. This makes probabilistic methods incapable of accurately modeling the uncertainties arising from different sources.

To model the uncertainties arising from quantitative and qualitative lack of information, many researchers have used non-stochastic methods like fuzzy set theory and intervals (Alefeld and Mayer, 2000; Li and Mei, 2004; Park et al, 2008; Zhang et al, 2009; Park et al, 2012).

However, total uncertainty estimation in structural response parameters requires the total uncertainty quantification in rock mass properties. In rock mechanics, rock mass properties are generally predicted using GSI-based empirical relations instead of lab/in-situ testing of rock masses due to the aforementioned complexities. Due to the presence of natural variability in intact rock characteristics, imprecise uncertainty in GSI, and transformation model uncertainties resulting from experimentally calculated relations, the overall uncertainties in rock mass properties are polymorphic.

Polymorphic uncertainty is defined as the combined uncertainty due to aleatory (natural variability of materials) and epistemic (limited or/and inaccurate information) uncertainties (Moller and Beer, 2004).

According to the best of the author's knowledge, very few attempts have been made to provide a thorough and unified uncertainty framework to quantify polymorphic uncertainty in rock mass properties that could apply to general sites as well as their application for real-world rock engineering problems (Aladejare and Wang, 2017).

In this spirit, this paper proposes a hybrid reliability methodology for rock tunnel stability analysis coupled with polymorphic uncertainty modeling of rock properties. Major objectives of this paper are - i) Estimation of transformational correction factors to deal with the empirical nature of GSI-based models ii) Mathematical improvement of Convergence - Confinement method (CCM) to introduce transformational correction factors iii) Fuzzy characterization of imprecise uncertain variables iv) Conversion of fuzzy numbers to p-boxes to enable the propagation of combined (aleatory and epistemic) uncertainties. v) Providing easy-to-use steps to carry out hybrid MCS to estimate the total uncertainty in final outputs. The applicability of the method is demonstrated for a tunnel case study along a proposed railway line in Jammu and Kashmir State of India.

2 DETAILS OF HYBRID RELIABILITY METHODOLOGY

2.1. Transformation model uncertainty

Transformational uncertainty is the uncertainty that accounts for the discrepancy between the parameter's actual value and its estimated value from the empirical model (Zhang et al. 2018). The correction factors (i.e., t_m) can be used to compute transformational error, as illustrated below.

$$t_m = \frac{True\ value\ of the\ property\ (X)_T}{Predicted\ value\ of\ the\ property\ (X)_P} \tag{1}$$

The viability of a genuine field measurement of rock mass raises concerns about the accuracy of the offered correction factors. In this consideration, non-stochastic techniques such as fuzzy numbers, and intervals should be utilised for modelling quantified correction factors, i.e., t_m.

For the proposed tunnel analysis, two different types of transformational uncertainty were needed.:–

2.1.1. Associated with the Hoek-Brown criterion (Hoek-Brown, 1980), t_{m1}

$$t_{m1} = \frac{(\sigma_1-\sigma_3)_T}{(\sigma_1-\sigma_3)_P} = \frac{(\sigma_1-\sigma_3)_T}{\sigma_{ci}\left(m_b\frac{\sigma_3}{\sigma_{ci}}+s_b\right)^{0.5}} \tag{2}$$

where $(\sigma_1 - \sigma_3)$ is deviatoric stress obtained from triaxial tests, σ_{ci} is the uniaxial compressive strength of intact rock, m_b and s_b are Hoek-Brown strength parameters of the rock mass.

2.1.2 Associated with the deformation modulus relation (Hoek and Diederichs, 2006), t_{m2}

$$t_{m2} = \frac{(E_m)_T}{(E_m)_P} = \frac{(E_m)_T}{E_i\left(0.02\frac{1}{1+e^{\frac{60+15D-GSI}{11}}}\right)} \tag{3}$$

where E_i is the elastic modulus of intact rock, D is the disturbance factor (varies from 0 for undisturbed in situ rock masses to 1 for very disturbed rock masses)

2.2 Modified mathematical expression of Convergence-Confinement Method (CCM)

CCM is a common analytical approach for the analysis of tunnels using the Ground Reaction Curve (GRC). In this study, the plastic zone radius $'r_p'$ and the radial deformation,$'\ d_i'$, which are the response parameters for the tunnel, are computed under the presumption that the support pressure is zero ($p_i = 0$). GRC was derived by considering the rock mass as Hoek-Brown material (Hoek-Brown, 1980). To account for transformational uncertainty in the Hoek-Brown strength criterion (Equation 2), the mathematical formulas of CCM were adjusted as follows:

Assuming the tunnel to be a plane-strain axisymmetric problem, and rock mass to be an elastic-perfectly plastic material, expressions for response parameters of the tunnel were derived (similar to Hoek-Brown, 1980) using the modified Hoek-Brown strength model (Equation 2). Response parameters of the tunnel can be calculated in the following sequence:

$$m_b = m_i exp\left(\frac{GSI-100}{28-14D}\right) \tag{4}$$

$$s_b = exp\left(\frac{GSI-100}{9-3D}\right) \tag{5}$$

$$M = \frac{1}{2}\left\{\left(\frac{m_b t_{m1}}{4}\right)^2 + \frac{m_b p_0}{\sigma_{ci}} + s_b\right\}^{0.5} - \frac{m_b t_{m1}}{8} \tag{6}$$

$$G = \left\{\frac{-m_b}{m_b+4\left\{\frac{m_b}{\sigma_{ci}}(p_0-Mt_{m1}\sigma_{ci})+s_b\right\}^{0.5}}\right\} \tag{7}$$

Case 1: If support pressure (p_i) > critical stress ($p_i^{cr} = p_0 - \sigma_{ci}t_{m1}M$) then deformation around the tunnel is elastic:

$$\frac{d_i}{r_i} = \frac{(1+\nu)}{(E_m)_T}(p_0 - p_i) \tag{8}$$

Case 2: If $p_i < p_i^{cr}$ then plastic failure occurs around the tunnel

$$r_p = r_i exp\left[\frac{2}{t_{m1}}\left(\frac{p_0-M\sigma_{ci}t_{m1}}{m_b\sigma_{ci}} + \frac{s_b}{(m_b)^2}\right)^{0.5} - \frac{2}{t_{m1}}\left(\frac{p_i}{m_b\sigma_{ci}} + \frac{s_b}{(m_b)^2}\right)^{0.5}\right] \tag{9}$$

$$\text{For } \frac{r_p}{r_i} < \sqrt{3}, \quad R = 2\ ln\left(\frac{r_p}{r_i}\right)\ G \tag{10}$$

$$\text{For } \frac{r_p}{r_i} > \sqrt{3}, \quad R = 1.1\ G \tag{11}$$

$$\mathrm{A}= \left\{ 2 \times \left(\frac{(1+\nu)}{(E_m)_T} \sigma_{ci} t_{m1} M \right) - \left(\frac{2\left(\frac{(1+\nu)}{(E_m)_T}\sigma_{ci} t_{m1} M\right)\left(\frac{r_p}{r_i}\right)^2}{\left\{\left(\frac{r_p}{r_i}\right)^2 - 1\right\}\left(1+\frac{1}{R}\right)} \right) \right\} \left(\frac{r_p}{r_i}\right)^2 \quad (12)$$

$$\frac{d_i}{r_i} = 1 - \left[\frac{1 - \left(\frac{2\left(\frac{(1+\nu)}{(E_m)_T}\sigma_{ci} t_{m1} M\right)\left(\frac{r_p}{r_i}\right)^2}{\left\{\left(\frac{r_p}{r_i}\right)^2 - 1\right\}\left(1+\frac{1}{R}\right)} \right)}{1+A} \right]^{0.5} \quad (13)$$

where, p_0 is the magnitude of hydrostatic in-situ stress; ν is Poission's ratio of rock mass; r_i is the radius of the tunnel; $(E_m)_T$ is the true elastic modulus of rock mass, which can be calculated by using Equation (3).

2.3 Fuzzification of uncertain data

The data uncertainty can be modelled using fuzzy numbers if more than one sample element is provided. Mathematical procedures can be used to provide the membership function ($\mu_A(x)$) of the fuzzy number containing the given data (Moller and Beer, 2004). In this approach, the data are represented as a histogram to create an early draught of the membership function. The bin range with the most sample items is chosen. While the right portion of the histogram is used to compute the right-hand branch of $\mu_A(x)$, the left portion is utilised to compute the left-hand branch. Both branches take into account the bin range with the highest frequency.

The method of least squares is used to find the left- and right-hand branches of $\mu_A(x)$, making sure that the total of the squared discrepancies between the actual number of sample elements in each bin range and the functional value $\mu_A(x)$ in the center of the bin is as low as possible.

This fuzzification technique will be demonstrated for the tunnel problem in section 3.1.3.

2.4 Fuzzy number to P-box conversion

According to Bedi (2014), fuzzy numbers are a series of nested intervals that are given a level of possibility based on membership values i.e., $\mu_A(x)$ between 0 and 1. The most likely (mean) value of the fuzzy number is said to exist at precisely one of the x values for that $\mu_A(x) = 1$.

If x is an interval at $\mu_A(x) = 1$ then the fuzzy number is referred to as a fuzzy interval. A fuzzy number/ fuzzy interval can also be represented by a non-parametric p-box consisting of the same level of information (Bedi, 2014) (Figure 1).

The techniques below can be used to acquire the lower and upper-bound CDFs of the P-box:

a) Generate n random probability values $p_i : (p_1, p_2, p_3, \dots, p_n)$ where, $p_i \in [0, 1]$; $i = 1,2, \dots, n$
b) For lower bound CDF ($\underline{P}_X$), estimate $x_i = U^{-1}(p_i)$ where, U^{-1} denotes inverse uniform distribution and $x_i \in [p, q]$.
c) Similarly, for upper-bound CDF ($\overline{P}_X$), estimate $x_i = U^{-1}(p_i)$ where, $x_i \in [r, s]$.
d) For fuzzy numbers, take $q = r$ to estimate P-box.

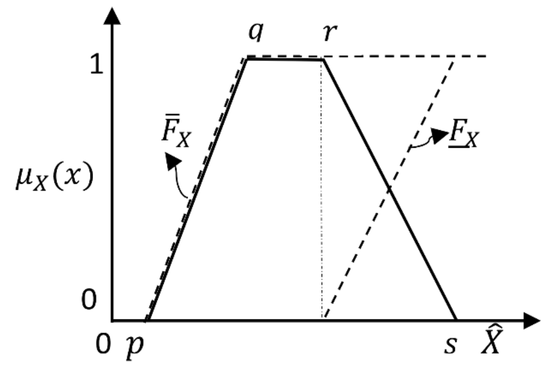

Figure 1. Fuzzy interval as a P-box.

2.5 Hybrid Monte-Carlo simulations (HMCS)

In a typical MCS, the statistical qualities of the input properties 'X' (mean, standard deviation, and PDF 'p_X') are used to produce random realisations (x_i) of the input properties, and the corresponding responses ($g(x_i)$) are evaluated. The random realisations of X are obtained as intervals $[\underline{x}_i, \overline{x}_i]$ when the input property is a p-box that is limited by the CDFs $\underline{P}_X$ and $\overline{P}_X$. In the case of combined uncertain properties, such as aleatory uncertain variable 'A' (modelled using PDF (p_A)) and epistemic uncertain variable 'E' (modelled as p-box ($\underline{P}_E$, $\overline{P}_E$)), the HMCS method (Zhang et al., 2010) can be used to estimate the response parameters i.e., $G(\mathrm{A}, \mathrm{E})$ by following the steps below:

a) Generate n standard uniform random numbers $u_i: (u_1, u_2, \dots, u_n)$; $i = 1, 2, \dots, n$
b) Estimate the n interval realisations of E i.e., $[\underline{e}_i, \overline{e}_i]$; as :$\underline{e}_i = \underline{P}_E^{-1}(u_i)$ and $\overline{e}_i = \overline{P}_E^{-1}(u_i)$
c) Generate n random realizations of 'A' according to PDF p_A i.e., $a_i: (a_1, a_2, \dots, a_n)$
d) Perform optimization on response functions to estimate response interval [$\underline{g}(a_i, e_i), \overline{g}(a_i, e_i)$]:
   $\underline{g}(a_i, e_i) = \min G(\mathrm{A}, \mathrm{E})$
   $\overline{g}(a_i, e_i) = \max G(\mathrm{A}, \mathrm{E})$
   for $a = a_i$, and $e \in [\underline{e}_i, \overline{e}_i]$
e) Plot the frequency distributions of the response's minimum and maximum values to obtain the probability boundaries (P-box) for the response parameters.

3 APPLICATION CASE STUDY

This section explains how the suggested methodology can be used to analyse a planned railway tunnel in Jammu and Kashmir, India. With a radius (r_i) of 4 meters and an in-situ hydrostatic pressure (p_0) of 10 MPa, the tunnel was considered to be circular. The surrounding rock mass had a Poisson's ratio (ν) of 0.23. The surrounding rock was assumed to be undisturbed (D =0) for this case study. Two performance functions (PFs) (Lü and Low, 2011) were used to assess the tunnel's stability.:

$$M_1 = 2 \times r_i - r_p \tag{14}$$

$$M_2 = 0.02 \times r_i - d_i \tag{15}$$

where, r_p is the plastic zone radius, r_i is the radius of the tunnel, d_i is the tunnel convergence.

3.1 Analysis of tunnel stability

3.1.1 Estimation of rock properties

Table 1 lists the statistics and best-fit PDFs for m_i, E_i and σ_{ci} based on 31 values for E_i and σ_{ci} and 22 values for m_i that were determined by laboratory testing.

Table 1. Statistical parameters and PDFs of intact rock properties

Property	Mean	Std	PDF
$\boldsymbol{m_i}$	12.8	4.08	Weibull
$\boldsymbol{E_i}$ (GPa)	65.7	18.03	Weibull
$\boldsymbol{\sigma_{ci}}$(MPa)	114.5	50.13	Weibull

Data for GSI was collected from Tiwari and Latha, 2019. The histogram of the GSI is shown in Figure 2(c) for the rock joint feature.

3.1.2 Estimation of transformational uncertainties in GSI-based empirical models

Triaxial test data for the rock was gathered for the quantification of t_{m1} after thorough literature research (Zhang et al., 2018). Data on the deformation modulus for t_{m2} was gathered from Hoek et al., 2019, and Kayabasi et al., 2003. Several t_{m1} and t_{m2} values were calculated by utilising the proposed methodology (Section 2.1). The estimated t_{m1} and t_{m2} histograms are shown in Figure 2 (a,b).

3.1.3 Estimation of fuzzy membership functions of uncertain variables

As indicated in Section 2.1, it was challenging to accurately characterise the mean, standard deviation, and PDF of transformational errors since there was insufficient availability of true/in-situ rock mass properties. Similarly, accurate calculation of the PDF of an uncertain variable (GSI) based on an expert's subjective method was not found a good idea.

This led to the development of fuzzy membership functions to represent the transformational uncertainties (t_{m1} and t_{m2}) and GSI. For t_{m1}, t_{m2}, and GSI, fuzzy membership functions were computed using the approach described in section 2.3. Figure 2 presents the initial draft of the left and right branches of the membership functions along with confidence bounds. These confidence bounds represent the range in which the true membership value lies at a certain level of confidence. These confidence bounds were found tighter for t_{m1} and t_{m2} than GSI due to the dispersed nature of GSI data.

However, the two branches (Figure 2) typically have a point of intersection. This point of intersection gives the mean value of the fuzzy set. The two nearby zeroes of the membership value mark the interval bounds of the support. The acquired membership function is finally normalized so that the functional value at the mean value point is unity. Figure 3 presents the final normalised fuzzy membership functions of t_{m1}, t_{m2} and GSI i.e., ($\mu(t_{m1})$, $\mu(t_{m2})$, $\mu(GSI)$).

3.1.4 Fuzzy number to P-box conversion

Using the instructions in section 2.4, the fuzzy t_{m1}, t_{m2} and GSI were transformed into P-boxes. Figure 4 consists of the P-boxes (lower and upper bound CDFs) of t_{m1}, t_{m2} and GSI. Figure 4 is just the P-box representation of triangular fuzzy numbers (Figure 3) to enable the propagation of combined (aleatory and epistemic) uncertainties by using HMCS.

3.1.5 Estimation of P-boxes of response functions

According to Equations (4–13), the response parameters of the tunnel—the plastic zone radius (r_p) and the radial deformation (d_i)—are functions of both aleatory (σ_{ci}, m_i, E_i) and epistemic (t_{m1}, t_{m2}, and GSI) uncertain variables. Where, in section 3.1.1, σ_{ci}, m_i, and E_i were probabilistically characterised, and t_{m1}, t_{m2}, and GSI were modelled using P-boxes (Sections 3.1.3, 3.1.4). In light of this, P-boxes of response parameters were determined by HMCS (Section 2.5).

Additionally, based on the provided performance functions (Equations 14-15), the probability of tunnel failures (%Pf) corresponding to P-boxes of response parameters was estimated. In addition to that, a traditional stability study of the rock tunnel was also carried out by disregarding the uncertainties in the transformation model (taking $t_{m1} = 1$, $t_{m2} = 1$) and taking the GSI into account as a probabilistic variable (mean = 41.13; std = 6.45; PDF - lognormal).

The findings of the study utilising both conventional and advanced approaches are shown in Figure 5.

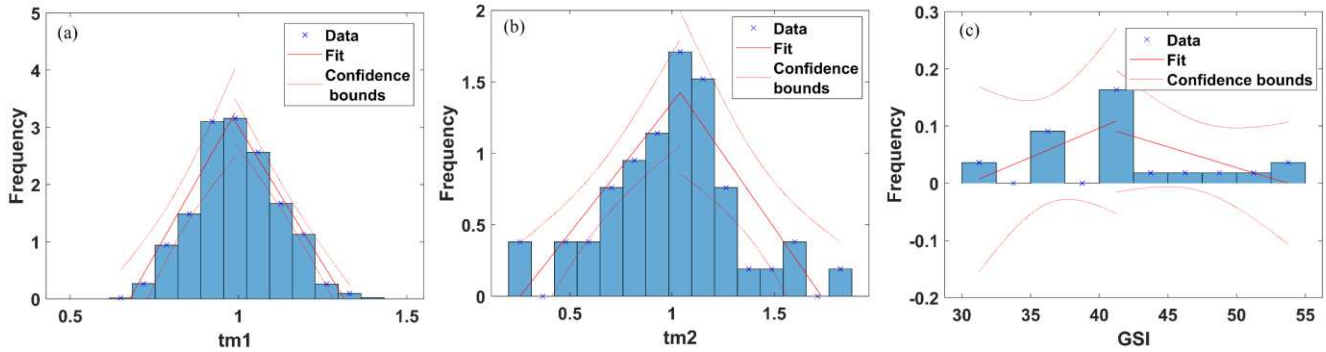

Figure 2. Best fitted linear left and right branches of membership functions on histograms of a) t_{m1} b) t_{m2} c)GSI

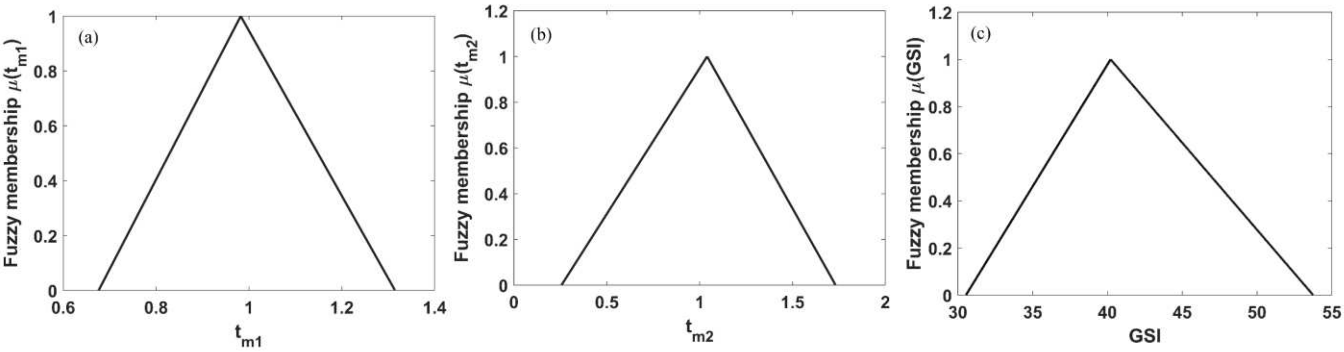

Figure 3. Normalised fuzzy membership functions of a) t_{m1} b) t_{m2} c)GSI

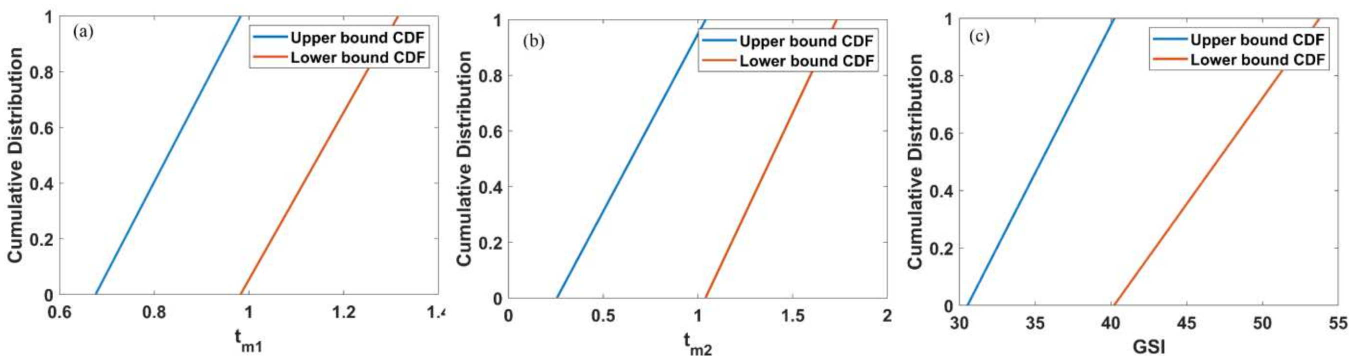

Figure 4. P-boxes of a) t_{m1} b) t_{m2} c)GSI

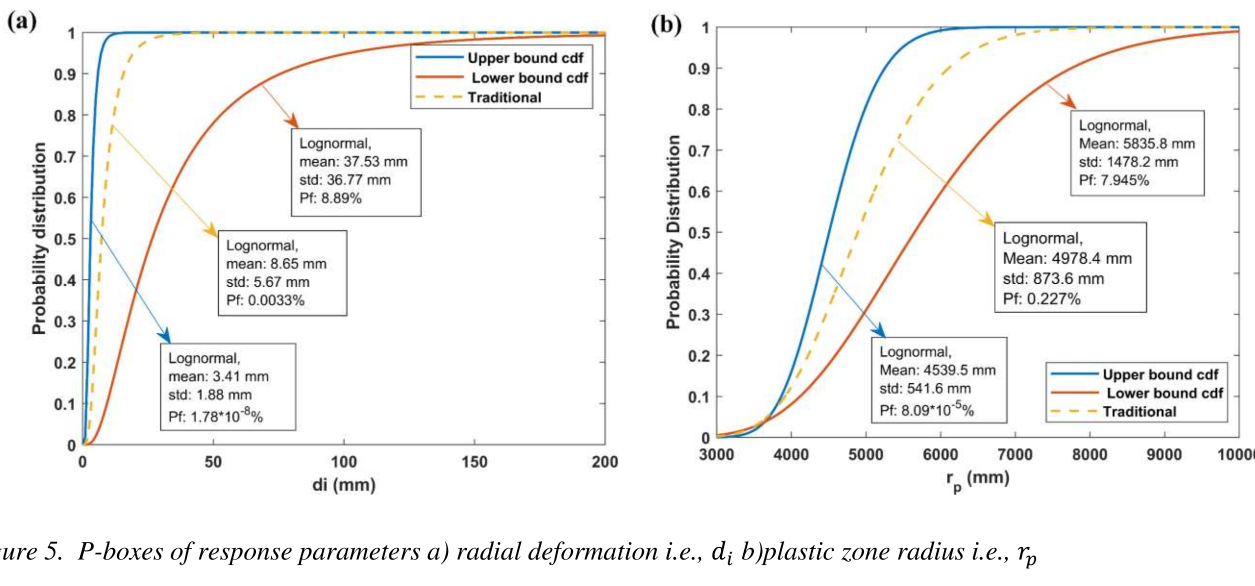

Figure 5. P-boxes of response parameters a) radial deformation i.e., d_i b)plastic zone radius i.e., r_p

4 COMPARATIVE ANALYSIS

The outcomes of conventional and proposed approaches are compared in this section. The tunnel in this case study was expected to be safe (Pf $\leq$ 1%) using conventional approaches, but it was discovered to be failing (Pf > 1%) when applying advanced methods (Figure 5). For the proposed technique, the range of total uncertainties (coefficient of variations (COVs)) in the radius of the plastic zone (r_p) and the radial deformation (d_i) were estimated to be (11.9-25.3)% and (55.13-98)%, respectively. The magnitude of the range of COV (the gap between the upper bound and lower bound values) represents the epistemic uncertainty, whereas the value of COV represents the aleatory uncertainty in response parameters. Whereas traditionally estimated COV of $\mathrm{r_p}$ and $\mathrm{d_i}$ i.e., 17.55% and 65.5% reflect the effect of only randomness/aleatory uncertainty in response parameters respectively.

5 CONCLUSION

An efficient hybrid reliability methodology is proposed for rock tunnel stability analysis coupled with polymorphic uncertainty modelling of rock properties. Methodology computes the total uncertainty in response parameters based on the combined sources (i.e., aleatory and epistemic). Proposed methodology is demonstrated for a rock tunnel case study located in Jammu and Kashmir State of India. Analysis was also conducted using traditional methodology which neglects the contribution of epistemic uncertainties in the total uncertainty estimation. For the present case study, the underestimation of epistemic uncertainty led to the underestimation in the probability of tunnel failure w.r.t. r_p and d_i by 97% and 99% respectively.

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