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Numerical simulation and optimization of dike geometry using multi-objective evolutionary algorithm NSGA-II

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ABSTRACT: In geotechnical engineering, the implementation of optimization tools into the designing process has been addressed in literature in the past. Since design optimization is undoubtedly one of the most time-consuming parts of civil engineer's responsibilities, an understanding of how an optimization tool works is crucial for obtaining feasible results. This paper deals with the application of a nondominated sorting genetic algorithm (NSGA-II) for optimizing a multi-objective geotechnical problem by means of using the strength reduction finite element analysis for proving the stability of a dike. The inclinations of the waterside and landside slopes of a dike are parametrised and serve as variables in the algorithm. The objectives of the problem are minimum material demand and minimum structure exploitation. The study explains the purpose of parameters as well as shows how selection of input parameters, that control the algorithm, influence the efficiency of the algorithm in finding a set of Pareto-optimal solutions.

Keywords: Dikes; SRFEA; structure optimization; Multi-objective evolutionary algorithm; NSGA-II

1 INTRODUCTION

Design optimization is considered to be one of the most time-consuming parts of civil engineering. Once a feasible design of a construction is found, it undergoes an optimization process to reduce costs, increase sustainability, or improve construction workflow. During this process, different parameters of a structure are varied with the aim of identifying the most efficient solution in relation to previously set objectives. The optimization process can be conducted in various manners, but it has been proven that evolutionary algorithms have a significant advantage in terms of efficiency.

Over the last decades, the application of various optimization tools in civil and geotechnical engineering has been improving, as stated by Dede et al. (2019). Different types of Evolutionary Algorithms (EA) and ways of their implementation have been proposed for solving geotechnical problems, e.g. by Andrab et al. (2017), Benayoun et al. (2020), Cui and Sheng (2006), Jesswein and Liu (2022), Kinzler (2011), and Meier et al. (2008).

Optimization algorithms like EA or its subgroup of Genetic Algorithms (GA) can be a substantial contribution to the decision-making process, as stated by Okonwo et al. (2022). It can be said that a decision in relation to the final design of a structure is an inevitable responsibility of most engineers.

This study focuses on the implementation of an elitist nondominated sorting genetic algorithm (NSGA-II) for solving a multi-objective optimization problem by means of finite element strength reduction techniques (SRFEA). Corresponding studies, which did not implement numerical methods for verification of structure stability, can be found e.g. in Das et al. (2016).

With the aim of investigating the influence of single parameters on the algorithm outcomes, a set of input parameters is modified to conduct various optimization runs. The obtained results are compared to one another with the intent of identifying the relevance of parameter selection.

2 THEORY

2.1 *Multi-objective problems*

2.1.1 *General information*

Multi-objective problems do not have a single best solution, but rather a set of optimal solutions known as Pareto-optimal or non-dominated solutions. These solutions form a Pareto front, which is a visual representation of the trade-offs between the defined objectives for a multi-objective optimization problem. A solution is considered to dominate another solution if it is superior in one or more objectives, but not worse in any other objective. In other words, a feasible solution is considered Pareto optimal if any other set of input variables would simultaneously reduce one objective while increasing at least one other objective function, as stated by Nayak (2020).

2.1.2 *Nondominated sorting genetic algorithm II (NSGA-II)*

The Nondominated Sorting Genetic Algorithm (NSGA-II) is a Multi-Objective Evolutionary Algorithm (MOEA) that is based on crowding and can be used to solve multi-objective problems. The algorithm offers several advantages, such as an elitist-preserving approach and a fast non-dominated sorting procedure. It was first proposed by Deb et al. (2002).

The NSGA-II combines a random parent population P_{t} with an offspring population Q_{t}, to create a combined population R_{t} of size $2N$. The combined population is then sorted using a fast nondominated sorting procedure, which assigns a nondomination level (or rank) to each member. In the next optimization run, where mutation takes place, solutions from the best set (e.g., F_1, F_2) are chosen for the new population $P_{\mathrm{t+1}}$ in order to preserve the elitism of the next generation. The remaining non-dominated fronts are then chosen based on their rank until no more members can be accommodated in the new population. To select exactly N population members, the last front is sorted in descending order and the best solutions are transferred to the new population (Deb et al., 2002). The procedure which drives the NSGA-II is presented in Figure 1.

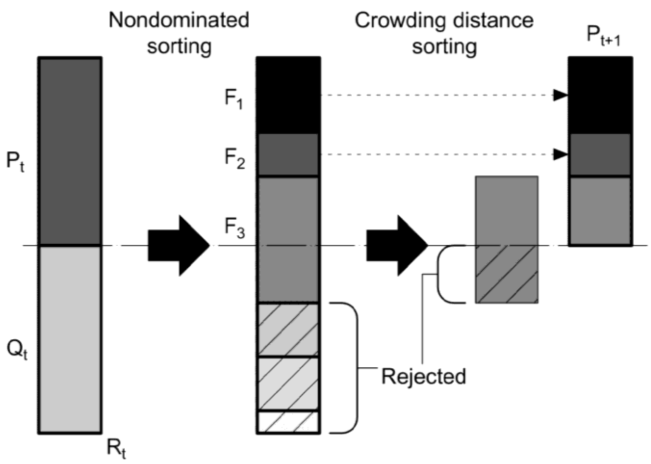

Figure 1. NSGA-II procedure

2.2 *Strength reduction finite element analysis (SRFEA)*

During this study a finite element code Plaxis (Plaxis, 2020) is used for all displacement-based analyses, as it has been widely applied for investigations of slope stability (e.g. Obernhollenzer et al., 2018; Tschuchnigg et al., 2015a, 2015b).

The displacement finite element method can be used to obtain the factor of safety (FoS) by means of SRFEA, which involves decreasing the friction angle φ' and the cohesion c' simultaneously until the investigated structure collapses. Equation (1) presents the formulation of the FoS, where 'mobilized' refers to mobilized strength parameters for a failure stage. It is important to note that all calculations in this study were conducted for characteristic values, therefore no safety factors are applied for SRFEA.

$$FoS = \frac{\tan \varphi'}{\tan \varphi'_{mobilised}} = \frac{c'}{c'_{mobilised}} \qquad (1)$$

2.3 *Scripting interface*

The Plaxis finite element code has a scripting interface that allows a user to automate the calculation process by using a code written with the Python programming language (Van Rossum & Drake, 2009). Python allows for the use of predefined modules that provide definitions and statements that can be applied in a programmed code. In this paper, we implement the multi-objective optimization framework, *pymoo* (Blank & Deb, 2020), in order to use the NSGA-II optimization algorithm and find a Pareto front for a given geotechnical multi-objective problem. By using Python scripting, it is possible to link the NSGA-II optimization algorithm with a finite element analysis conducted in the commercial Plaxis software. Therefore, the results obtained from the numerical simulation can be used to define the problem within the optimization algorithm.

3 COMPUTATIONAL MODEL

3.1 *Finite element model*

The SRFEA employs a plain strain model that is composed of 15-noded triangular elements. In order to achieve a reliable Factor of Safety (FoS), a relative element size of 0.015 was selected for the model discretization, which corresponds to a very fine mesh. The total number of finite elements varies depending on the different model configurations, as the geometry of the domain also changes across different simulations.

3.2 *Material properties*

In this study, a linear elastic perfectly-plastic material model with a Mohr-Coulomb failure criterion was utilized for all soil polygons within the modeled domain. Four soil layers were defined within the model, based on a recent geotechnical survey for a dike reconstruction project. The soil properties are listed in Table 1, and the model geometry including the soil layers is shown in Figure 2. The soil parameters were not altered in any of the simulations presented in this paper.

3.3 Geometrical boundaries

The size of the modeled domain was determined based on the guidelines for earth structure models included in Mestat et al. (2004). Figure 2 illustrates the relationships between dike width B and height of the dike crest H and the total height and width of the model domain.

The dike crest height H of 6.5 m was selected based on typical dike dimensions along the river Elbe in Germany. The dike crest width B_c of 3.0 m and thickness of dike clay cover were chosen in accordance with German standards, such as Freie und Hansestadt Hamburg (2003). The slope horizontal runs (x_1 and x_2) are variables defining the slope inclination, as described further in section 4.1.2.

3.4 Calculation stages

The model setup consists of four consecutive calculation phases defined in Plaxis. The first phase begins with gravity loading, followed by a plastic nil-step to ensure equilibrium in the model. The third phase is a plastic calculation with a defined high-water level, followed by a SRFEA, where a long-term shear failure is investigated. The plastic calculation does not take the change of pore pressure with time into account. To calculate the initial stresses in the soil body, a water level of 0.5 m above the terrain level was set on the left side of the dike body. The high-water level was defined as 3.5 m above the terrain level. The water level on the right side of the dike body was kept at the same level as the terrain.

Table 1. Soil properties for dike

	Unit	Clay 1	Clay 2	Sand 1	Sand 2
γ_{unsat}	(kN/m³)	16.0	15.0	18.0	19.0
γ_{sat}	(kN/m³)	16.0	15.0	20.0	21.0
E'	(MN/m²)	2.5	2.0	20.0	40.0
ν'	(-)	0.3	0.3	0.2	0.2
c'	(kN/m²)	10.0	10.0	0.1	0.1
φ'	(°)	17.5	17.5	30.0	35.0
ψ'	(°)	0.0	0.0	5.0	5.0
$k_{x/y}$	(m/s)	$1 \cdot 10^{-9}$	$1 \cdot 10^{-10}$	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$

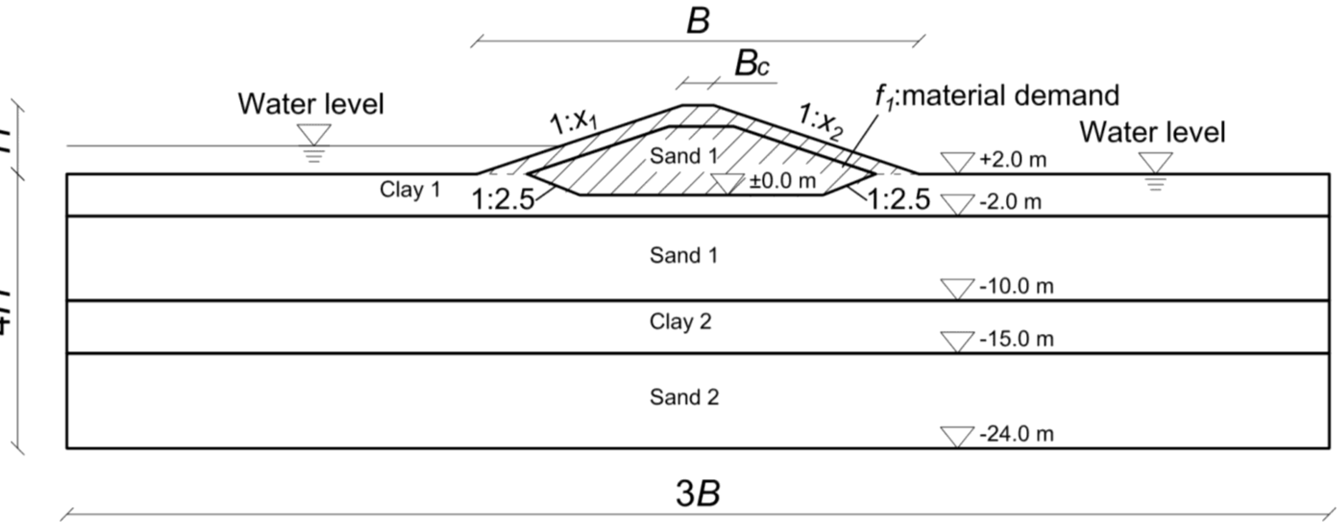

Figure 2. Model geometry and soil layers used in FEM-simulations

4 OPTIMIZATION PROBLEM

4.1 Definition of a problem

4.1.1 Objective functions

For this study two, objective functions f_1 and f_2 were defined. The first objective function f_1 represents the required material for dike construction in each simulation run, based on the computed areas of dike sand core and dike clay cover, as illustrated in Figure 2. The areas of polygons are calculated by incorporating the shoelace algorithm into the Python code, as outlined in Equation (2), where x_i and y_i are coordinates of vertices. The second objective function f_2 relates to the exploitation of the structure and is defined in Equation (3) as a reciprocal of FoS.

$$f_1 = \frac{1}{2} \left| \sum_{i=1}^{n} x_i y_{i+1} - x_{i+1} y_i \right| \quad (2)$$

$$f_2 = \frac{1}{FoS} \quad (3)$$

4.1.2 Variables

As previously mentionied in section 3.3, the slope horizontal runs on both sides of the dike body (x_1 and x_2) are set as variables in the optimization problem. For both variables a range between 0.5 and 5.0 is specified, i.e. the minimum possible value of horizontal run is 0.5 and maximum possible values of variable is 5.0. The lower boundaries x_l and upper boundaries x_u for variables are given by Equations (4-5).

$$x_l = \{0.5; 0.5\} = \{x_{1,min}; x_{2,min}\} \tag{4}$$
$$x_u = \{5.0; 5.0\} = \{x_{1,max}; x_{2,max}\} \tag{5}$$

4.1.3 Constraints

Within the *pymoo* optimization framework, either inequality constraint or equality contraint may be defined. For this study one inequality constraint g_1 was specified that includes the calculated FoS from the SRFEA conducted in Plaxis. If the FEM-simulation gives a FoS greater than 1.0, the solution is accepted.

4.2 Definition of algorithm parameters

4.2.1 Seed number

The NSGA-II algorithm generates a set of random numbers for the initial parent and offspring populations, which are then mutated in subsequent generations. The seed number is used to select the starting point for the random number generator to ensure the reproducibility of the results obtained by the algorithm. The effect of the seed number on the outcomes is discussed further in this paper.

4.2.2 Initial Population size

The size of the initial population, which refers to the number of members, needs to be selected. The initial population is generated before the first reproduction takes place. It can be assumed that the selected size of the first population affects the quality of the results.

4.2.3 Rounding of numbers

The cycle that drives the genetic operations of NSGA-II includes procedures such as selection, crossover, and mutation, during which new random variables are selected for the next generation. With regards to geotechnical structures, which are designed and constructed with maximum accuracy to centimeters, rounding to two decimal places is sufficient. Therefore, the default parameters controlling the rounding of numbers to more than two decimal places may be seen as unnecessary considering the computational effort.

For this study, the original code managing the selection of population members was altered. The initial population and the subsequent members, selected during the crossover and mutation processes, are rounded to a predefined number of decimal places in order to speed up the selection of the optimal solutions with respect to expected accuracy. Additionally, any duplicated values that may occur in the population are eliminated to diversify the obtained solutions and maintain the efficiency of the optimization algorithm.

4.3 Termination criteria

In this study, a specific termination criterion is discussed with the aim of examining its effects on the outcomes.

4.3.1 Maximal number of generations

The maximum number of generations is a user-defined parameter that limits the number of iterations of the algorithm. Once the algorithm has reached the specified number of generations, the optimization process is halted, regardless of whether or not an optimal or near-optimal solution has been found. It simply prevents the algorithm from running indefinitely.

5 STUDIES

An initial population size of 10, maximum generation number of 10, rounding of numbers to two decimal places, and a seed number of 1993 are assigned as parameters for a reference run in the following study. If a parameter is changed for a simulation, it will be noted. The results obtained from parameter variations are always compared to the reference run.

5.1 Varation of rounding

Two parameter sets regarding number rounding were investigated and compared to the reference run. By using the code described in section 4.2.3, a simulation with rounding to five decimal places for initial and subsequent generations, as well as a simulation where only the intial population members were rounded to two decimal places, were conducted.

The results shown in Figure 3 indicate that the three conducted optimization runs may differ significantly from one another. A partial agreement of solutions obtained from all simulations is observed. It can be stated that in some areas, the rounding to five decimal places and rounding to two decimal places only for the initial population may require more generations to reach a similar level of accuracy as the reference run. However, by adjusting from five to two decimal places, the number of possible population members is reduced by a factor of 1000, yet the accepted tolerance is still maintained.

5.2 Varation of seed number

Similarly, two sets of parameters with different user-defined seed numbers, either divided by a factor of 10 or multiplied by it, are compared. As shown in Figure 4, the respective Pareto fronts are distinguishable from each other.

It should be emphasized that the efficiency of the investigated algorithm, in terms of computational time and the set of obtained optimal solutions, for a limited number of iterations and a relatively small population size, strongly depends on user-defined input parameters. Therefore, a preliminary parameter study is highly recommended.

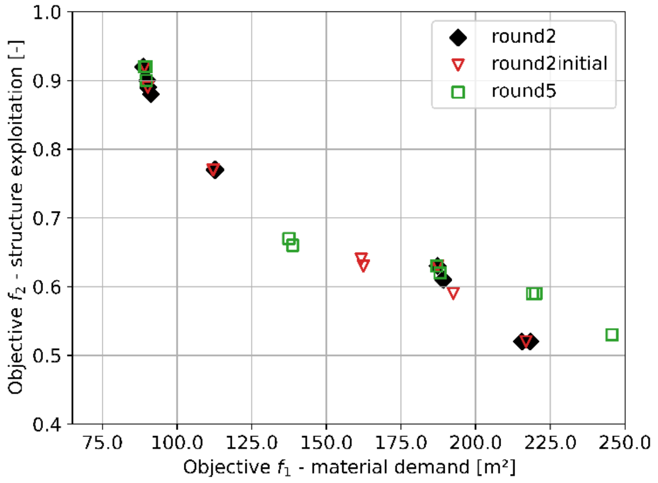

Figure 3. Pareto-optimal solutions of NSGA-II for minimizing material demand and structure exploitation (variation of number rounding: 2 decimal places, 5 places, 2 places for initial population)

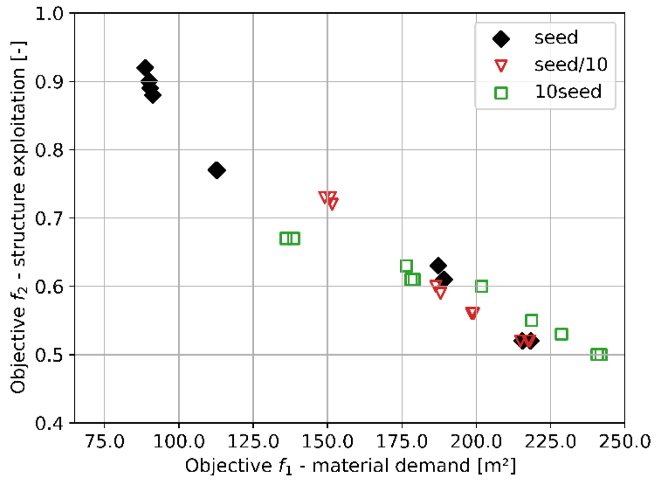

Figure 4. Variation of seed number: original seed number, seed divided by 10, seed multiplied by 10

5.2.1 Variations of population size and maximal number of generations

In this study, four different optimization setups are compared to a reference run. One setup uses an initial population size of 1 with 100 possible generations, while another uses a simulation with 100 members but only 1 possible iteration. Additionally, a case with an initial population of 100 and 10 generations is compared to one with an initial population of 10 and 100 generations.

As shown in Figure 5, the run with only 1 population member and 100 iterations yields a more optimal solution in comparison to the reference run. Additionally, the optimization run with 100 members in the initial population yields 13 Pareto solutions. Although only 1 iteration was conducted, the results are competitive with those of the reference run and the distribution of solutions is better than in the reference run.

Figure 6 confirms that increasing the maximum number of generations to 10 for a population of 100 improves both the coverage of the Pareto front and the optimality of solutions. Increasing the number of generations to 100 for a population of 10 resulted in a slight improvement in distribution throughout the Pareto front, but did not affect the accuracy of the solutions.

Overall, it can be concluded that a larger initial population size increases the probability of covering a wider range of Pareto-optimal solutions with an appropriate level of accuracy.

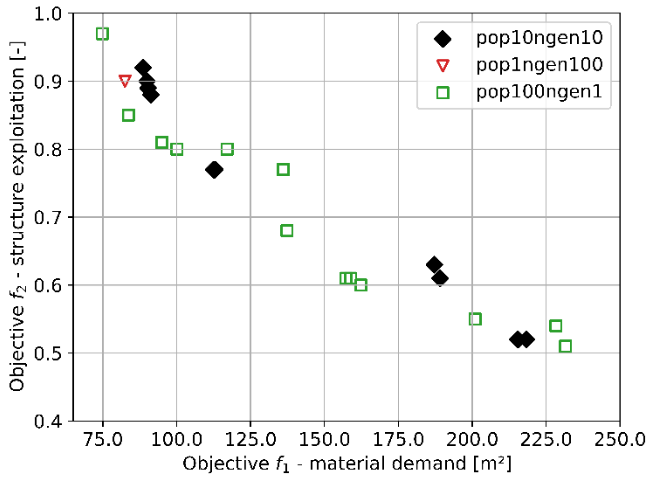

Figure 5. Variation of initial population and generation numbers: initial population 10 number of generations 10, pop. 1 ngen. 100, pop. 100 ngen. 1

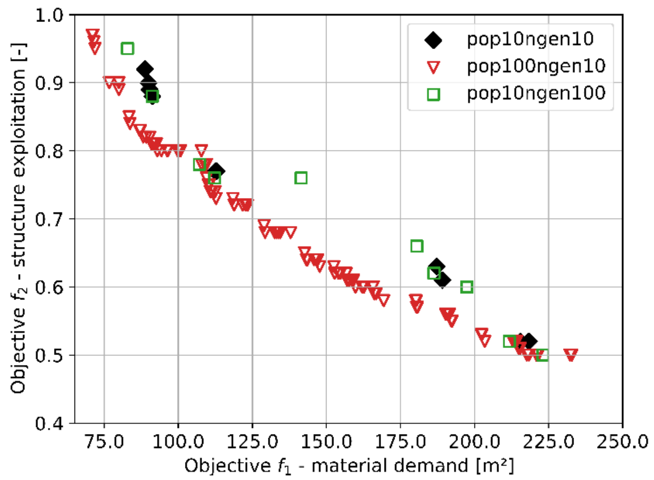

Figure 6. Variation of initial population and generation numbers: initial population 10 number of generations 10, pop. 100 ngen. 10, pop. 10 ngen. 100

6 CONCLUSIONS

The study presented in this paper shows that selecting the appropriate input parameters for the optimization algorithm NSGA-II is crucial for obtaining feasible results with an appropriate level of optimality. With regard to the accepted tolerance for Pareto solutions, e.g. centimeters for geotechnical structures, the number of possible population members to pick can greatly impact the algorithm's efficiency. The choice of seed number, which drives the selection process of the initial population, can lead to differences in the final set of Pareto solutions, therefore different seed numbers should be investigated.

The results of the study show that a larger initial population size ensures a higher quality of the results while also offering a better distribution over the defined ranges. Considering the aim of optimization algorithms, which is to find the Pareto front, it is recommended to prioritize a larger population size over a larger number of generations.

The usefulness of optimization algorithms for solving multi-objective problems is undeniable. However, as highlighted in this paper, the right choice of input parameters can greatly improve the efficiency of the algorithm. Therefore, users and engineers should proceed with caution and use common sense when selecting input parameters for a specific case.

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