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Numerical studies on the use of zero thickness interfaces in cyclic soil structure interaction analysis

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ABSTRACT: In FE analyses interfaces are crucial for the adequate modelling of the soil structure interaction (SSI). In this paper zero thickness interfaces modelled with varying constitutive models and interface properties are evaluated for cyclic SSI problems with primarily lateral loading. Therefore, a simplified 2D FE model is used that simulates the cyclic seasonal temperature deformation of an integral bridge abutment. Linear elastic and elastoplastic constitutive models are applied for the interface and soil. The study reveals that zero thickness interfaces subjected to large lateral loading can cause interpenetration effects that significantly influence the mobilised earth pressures in the adjacent soil. When modelling interfaces by means of constitutive models with stress-dependent stiffnesses these effects can be amplified with increasing cyclic loading. The underlying causes for these effects are explained and different approaches are investigated to reduce the obtained effects. Comparisons are drawn to calculations without interfaces and with thin layer continuum interfaces.

Keywords: cyclic soil structure interaction; zero thickness interfaces; Plaxis; Hardening Soil model; interpenetration

1 PROBLEM DEFINITION

An ongoing research project at the University of Technology Graz focuses on the numerical analysis of the cyclic soil structure interaction (SSI) of integral railway bridges with its granular backfill. Especially due to temperature deformation of the horizontal bridge superstructure a seasonal interaction of the abutment and the backfill occurs, as illustrated in Figure 1(a). Experiments as well as numerical analyses (e.g. Lehane, 2013; Banks and Bloodworth, 2018) have shown that this SSI can lead to a continuous cyclic increase of earth pressure behind the abutment in the summer position over a bridge's lifetime. In (Stastny and Tschuchnigg, 2022) 2D FE analyses (FEA) of the SSI of a realistic 40 m long integral bridge on spread footing (Figure 1(b)) were performed employing both elastoplastic and hypoplastic soil models. The investigation confirmed the trend of continuing cyclic stress increase. In this study interfaces were applied behind the abutment to describe the frictional contact zone between structure and soil, namely to simulate the reduced strength as well as relative displacements, either slipping or gapping, in the local interaction zone. Therefore, interfaces are essential for a realistic simulation of the earth pressure mobilisation in the active winter and passive summer position as well as the cyclic settlement accumulation. The loading in this case acts primarily orthogonal to the abutment i.e. in normal direction to its interface. During the FEA different numerical issues were detected concerning the used zero thickness interfaces. Among others, severe interpenetration effects were observed that increase with each cycle. The effects will be investigated in this paper.

Interpenetration or overlapping, is a known and intrinsic shortcoming of zero thickness interfaces, several authors (e.g. Sharma and Desai, 1992; Stutz, 2016; Bentley, 2022) have reported. At high normal stresses "too low" normal stiffnesses of zero thickness interfaces can cause numerical elastic deformations within the interface that result in an unrealistic overlap of structure and soil. Figure 1(c) exemplarily displays the profound impact of interpenetration effects on K_{mob}. These results have been obtained in preliminary studies using default interface and calculation settings and the Hardening Soil (HS) model (Schanz et al., 1999). With interpenetration effects the horizontal earth pressure (coefficient $K_{mob} = 2E_{mob} / (\gamma \cdot h^2)$) was found to be considerably smaller in the summer positions (with increasing cycles) compared to FEA without interpenetration. Thus, without proper interface and calculation settings the (cyclic) interpenetration effects can significantly compromise such numerical analyses in practice. Therefore, the zero thickness interface behaviour and the interpenetration in cyclic SSI simulations are studied in detail in the following. A simplified numerical model as well as simple soil models are used to isolate and highlight the problem. Strategies to eliminate interpenetration are investigated. Calculations with thin layer continuum interfaces as well as without interfaces serves as comparison.

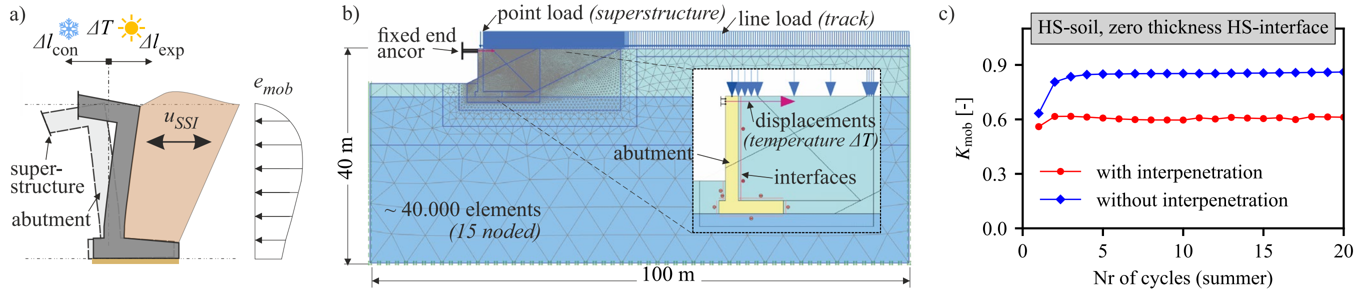

Figure 1. (a) Seasonal soil structure interaction mechanism of an integral abutment and its granular backfill, (b) FE model from (Stastny and Tschuchnigg, 2022), (c) Example for the influence of interpenetration on the earth pressure coefficient K_{mob}

2 ZERO THICKNESS INTERFACES

In FE analyses interfaces are typically modelled with special interface elements, so-called zero thickness elements (e.g. Goodman et al., 1968; Day and Potts, 1994), or by means of a thin layer of continuums elements with adjusted strength (and stiffness) properties (e.g. Desai et al., 1984). The advantages and limitations of these elements have been broadly discussed in literature (e.g. Sharma and Desai, 1992; Potts and Zdravković, 2001). Zero thickness interfaces consist of node pairs, one at the structure and one at the soil. The two nodes are connected by two elastic – perfectly plastic springs, which separately control relative gap / interpenetration (in normal direction) or slip deformations (parallel to the interface) between structure and soil. In Plaxis (Bentley, 2022) the elastic and plastic state of the interface is distinguished by means of the Coulomb criterion, where failure associated with permanent slip is reached when:

$$|\tau| = \sigma_n \cdot \tan \varphi_i + c_i \quad (1)$$

with the effective normal stress σ_n (positive for compression), the shear stress τ, the effective interface friction angle φ_i and cohesion c_i. A reduction factor $R_i \leq 1$ is used to adjust the strength (and stiffness) properties of the interface in regard to the properties of the adjacent soil (default) or the user input (data set):

$$c_i = R_i \cdot c_{soil} \quad (2)$$

$$\tan \varphi_i = R_i \cdot \tan \varphi_{soil} \quad (3)$$

Gapping between structure and soil occurs when the tension $\sigma_{t,i}$ in the interface reaches a cut-off criterion. In most cases no tension is allowed, i.e. gapping starts at $\sigma_n = \sigma_{t,i} = 0$ kPa. A “virtual thickness” t_i (= $VTF \cdot ES_{avg}$) is assigned to every zero thickness interface. In Plaxis this virtual thickness is calculated internally, based on the virtual thickness factor VTF (default value: 0.1) and the average element size ES_{avg}, during the meshing process. Together with the input (spring) stiffness this artificial thickness controls the elastic relative deformation between the interface node pair. The interface stiffness must be high enough (and the virtual thickness small enough) to reduce these relative deformations to a minimum. However, too high stiffnesses (and too small virtual thicknesses) will yield numerically ill conditions (e.g. Day and Potts, 1994; Potts and Zdravković, 2001). In any case small (insignificant) deformations will occur. Both relative slip $\Delta v_{i,rel}$ and interpenetration $\Delta u_{i,rel}$ deformation in the elastic interface can be calculated by:

$$\Delta v_{i,rel} = \frac{\tau}{K_s} = \frac{\tau}{(t_i \cdot G_i)} \quad (4)$$

$$\Delta u_{i,rel} = \frac{\sigma_n}{K_n} = \frac{(\sigma_n \cdot t_i)}{E_{oed,i}} \quad (5)$$

The elastic interface normal ($K_n = E_{oed,i} / t_i$) and shear ($K_s = G_i / t_i$) stiffness are determined by the interface one-dimensional compression ($E_{oed,i}$) and shear (G_i) modulus where $E_{oed,i} = 11G_i$ holds for a fixed Poisson's ratio of $\nu_i = 0.45$. The interface stiffnesses are connected to the soil stiffnesses by Equation (6) and (7):

$$G_i = R_i^2 \cdot G_{soil} \quad (6)$$

$$G_{soil} = \frac{E_{soil,input}}{(2 - 2\,\nu_{soil})} \quad (7)$$

The stiffness parameter $E_{soil,input}$ and the Poisson's ratio ν_{soil} allow to define the elastic interface behaviour with different constitutive models (by default Plaxis chooses the parameters corresponding to the adjacent soil). In many cases it is essential to consider a stress-dependent interface stiffness. Tschuchnigg (2012) for example showed that the mobilisation of skin friction for piles is strongly affected by the stiffness definition of the interface. According to Tschuchnigg (2012) the stiffness of an interface modelled with the Hardening Soil model (Schanz et al., 1999) in Plaxis is defined as:

$$G_i = R_i^2 \cdot G_{ur,ref} \left(\frac{\sigma_n \cdot \sin \varphi_{soil} + c \cdot \cos \varphi_{soil}}{p_{ref} \cdot \sin \varphi_{soil} + c \cdot \cos \varphi_{soil}} \right)^m \cdot \Lambda \quad (8)$$

with the un- and reloading shear modulus $G_{ur,ref}$ of the soil at reference pressure p_{ref} and the power index m as well as the internal defined factor Λ (considers additional cap-plasticity). From Equation (5) follows, that too low normal stiffnesses will lead to unrealistic large interpenetration $\Delta u_{i,rel}$. Especially the factor $R_i \leq 1$ should be treated with caution. While it reduces the interface strength linearly, it reduces the stiffness properties to the square (Equation (6)). Common recommendations to limit interpenetration effects are:

- create a separate interface data set with $R_{i,E} = 1$ (and reduce strength properties $R_{i,\varphi}$ manually)
- reduce the *VTF* parameter (min. value: 0.01)
- increase interface (normal) stiffness manually.

3 2D FE ANALYSIS

The simplified FE model of an integral bridge in Figure 2 was used to study the interface and interpenetration behaviour. A plane strain model was set up in Plaxis CONNECT V22 (Bentley, 2022) with a size of 80(l) x 8(h) m. The FE mesh was discretized with 15 noded elements (shape function of 4th order) and gradually refined towards the interaction zone. The 8 m high abutment is idealized by a very stiff plate element. It is fixed at the model bottom to allow a free foot point rotation of the plate. The cyclic temperature deformation of the horizontal superstructure was simulated by prescribed displacements $u_{x,top} = \pm 5$ or ± 10 mm at the top of the plate, corresponding to bridge lengths of ca. 40 – 80 m. After the initial phase (K_0 procedure with $K_0 = 1 - \sin \varphi$) 20 seasonal cycles were calculated in 40 alternating winter and summer phases. Drained conditions were assumed. The granular backfill was modelled either with a linear elastic model (LE: $E = 45$ MPa, $\nu = 0.2$) or the elastoplastic Hardening Soil (HS) model (Schanz et al., 1999) with parameters given in Table 1. The HS model captures the nonlinear stress strain behaviour of soils by means of separate deviatoric and volumetric yield surfaces. In the interaction zone between plate and soil either zero thickness or thin layer interfaces were defined. Further calculations were performed without an interface. For the zero thickness interfaces separate data sets with the elastic – perfectly plastic Mohr-Coulomb model (MC: $\varphi = 40°$, $c = 0$ kPa, $E = 45$ MPa, $\nu = 0.2$) or the HS model with parameters from Table 1 were selected. The interface stiffnesses were adjusted with the factor $R_{i,E} = 0.5$ or 1 while the strength properties in all calculations were equivalently reduced with a factor of $R_{i,\varphi} = 0.5$ ($\varphi_i = 22.8°$). In a separate calculation with $R_{i,E} = 1$ the interface stiffness $E_{soil,input}$ was further increased times 100 (in relation to the soil stiffness E_{soil}). The thin layer interface was modelled with 15 noded elements and a thickness of $t_{i,thin}$ = 4 cm ≈ 5 – 10 d_{50}, where d_{50} is the median grain size (Saberi et al., 2018). Preliminary sensitivity analyses related to the layer thickness showed no significant changes and verified this choice. For the thin layer interface the HS parameters from Table 1 in combination with reduced strength properties ($\varphi_i = 22.8°$, equivalent to $R_{i,\varphi} = 0.5$) were defined. The calculations were performed with the default value of max load fraction *l.f.* = 0.5 per step. This parameter controls the maximum size of a load step within a calculation phase. With *l.f.* = 0.5 the applied load (unbalance) will be solved in at least 2 steps. However, the program will automatically use more steps if convergence is slow (Bentley, 2022). The influence of this parameter was analyzed by a variation of *l.f.* = 0.5 – 0.001.

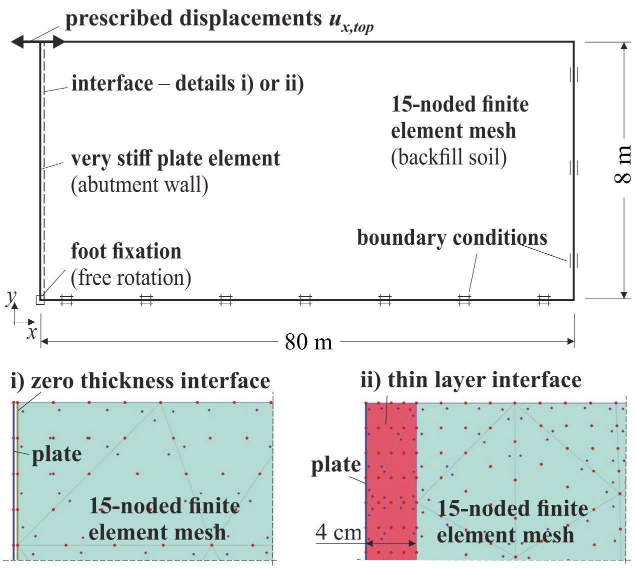

Figure 2. Scheme of the simplified 2D FE model

Table 1. HS parameters from Stastny&Tschuchnigg (2022)

φ [°]	E_{50}^{ref} [MPa]	E_{oed}^{ref} [MPa]	E_{ur}^{ref} [MPa]	m [-]	c_{ref} [-]	ψ [°]	ν_{ur} [-]
40	45	45	135	0.55	0	0	0.2

4 RESULTS

At first the cyclic interface and interpenetration behaviour is analyzed with a linear elastic (LE) backfill and zero thickness interfaces with MC and HS models and varying interface parameter ($R_{i,E} = 0.5 - 1$, $m_i = 0 - 0.55$). The results *I) – V)* are summarized in Figure 3. Figure 3(a-d) displays the horizontal interface deformation $u_{x,i}$ in cycle 1, 2, 10 and 20 in relation to the plate's winter (W) and summer (S) position ($u_{x,top}$ = ±10 mm). All calculations show a similar gapping effect between the plate and the interface / soil in the winter positions, because the tension cut off criterion $\sigma_n = \sigma_{t,i} =$ 0 kPa is reached over large parts of the interface, compare Figure 3(e-h). In the summer position the interface does also not coincide with the plate position. The reason for this are the relative deformations (interpenetration) $\Delta u_{i,x,rel}$ in the interface. While this interpenetration is small and constant for the calculation *IV)* with MC-interface (Figure 3(d)), it is significantly larger for the calculations *I)* and *II)* with HS-interfaces and seems to increase (strongly) with every cycle – compare Figure 3(a) and (b). As illustrated later in more detail, the

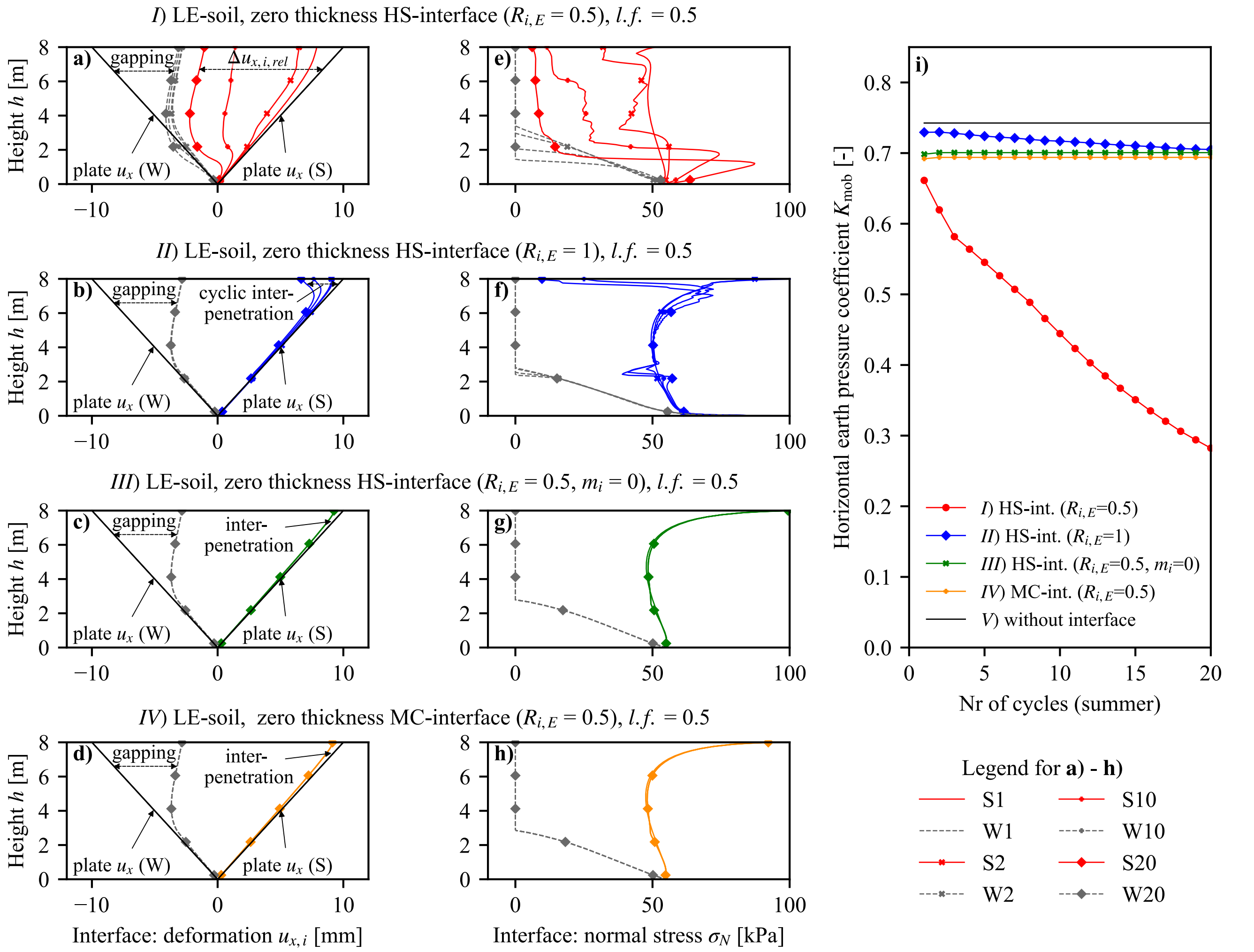

Figure 3. Calculation I – V) with linear elastic (LE) soil and different zero thickness interface soil models (MC, HS) and properties ($R_{i,E}$, m_i) after 20 cycles (S = summer; W = winter) with l.f. = 0.5: (a-d) Interface deformation $u_{x,i}$ compared to the plate deformation at $u_{x,top}$ = ±10 mm; (e-h) Normal stresses σ_n; (i) Earth pressure coefficient K_{mob} with increasing cycles

stress-dependent stiffness of the HS model is causing this accumulation of relative deformation in the interface (in combination with a too low number of load steps per calculation phase). This is verified by the results *III)* in Figure 3(c). With the power exponent $m_i = 0$ no stress-dependency of stiffness in the HS-interface is taken into account (see Equation (8)) and a constant stiffness, similar to the MC-interface, is adopted. As a consequence (analog to Figure 3(d) – MC-interface), the interpenetration effects are reduced significantly and no cyclic increase occurs.

The cyclic interpenetration obtained with the HS-interfaces has a severe impact on the normal stresses σ_n, compare Figure 3(e) and (f). With every cycle the normal stresses decrease (at the interface where the interpenetration increases). The reason for this is that the external loading (prescribed displacement) u_x at the plate in the summer phase is causing significant relative deformations $\Delta u_{i,x,rel}$ within the interface and thus the adjacent soil experiences less and less loading. Correlatively, also the mobilised horizontal earth pressure K_{mob} decreases extremely with every cycle, see Figure 3(i).

The much higher stiffness in calculation *II)* with $R_{i,E} = 1$ strongly mitigates the interpenetration effects, but still shows a cyclic decrease of earth pressure. For the calculation with a constant interface stiffness (*III, IV*) no cyclic impact on σ_n is detected. However, the small interpenetration in these calculations still causes a decrease of earth pressure in the summer position compared to the calculation *V)* without an interface (Figure 3(i)). Additional calculations with no strength reduction in the interface ($\varphi_i = \varphi_{soil} = 40°$) confirmed that the main cause for this ΔK_{mob} is indeed the interpenetration and thus the interface stiffness.

The calculation *I)* with HS-interface and $R_{i,E} = 0.5$ is repeated with different max load fractions *l.f.* = 0.5 – 0.001. As illustrated in Figure 4(a), the decrease of horizontal earth pressure with increasing summer cycles can be eliminated if the max load fraction is low enough, i.e. enough load steps per phase are applied. In this case *l.f.* = 0.001 (at least 1000 load steps per phase) was sufficient. This can be exemplarily explained based on the evaluation of relative deformations $\Delta u_{i,x,rel}$ and normal stresses σ_n in the interface node pair at $y =$

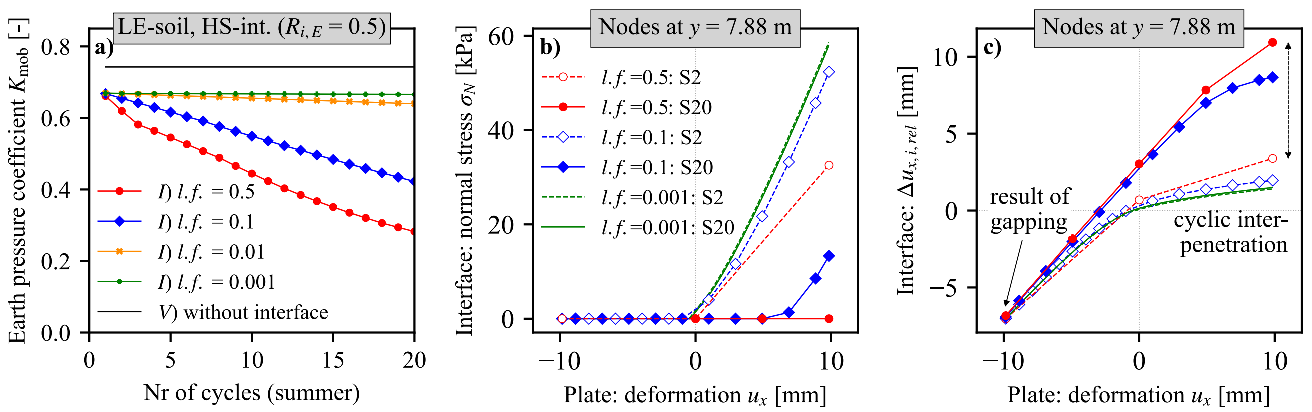

Figure 4. Calculation I) with LE-soil, zero thickness HS-interface ($R_{i,E}$ = 0.5) and varying max load fraction l.f. = 0.5 – 0.001: (a) Earth pressure coefficient K_{mob} in summer 1 – 20; (b-c) Relative deformations $\Delta u_{i,x,rel}$ and normal stresses σ_n of the interface node pair at y = 7.88 m in relation to the corresponding plate deformation u_x in summer phase S2 and 20

7.88 m for all load steps of calculation phase summer 2 and 20, see Figure 4(b) and (c). Due to the fact that the stiffness of the HS-interface is controlled by the normal stress σ_n (compare Equation (8)) the initial (normal) stiffness at reloading in the summer phase is rather low, particularly when gapping ($\sigma_n = \sigma_{t,i} = 0$ kPa) occurs. In the first load step after the gap closure, i.e. at $\sigma_n > 0$ kPa, this leads to a strong increase of relative deformation $\Delta u_{i,rel}$ in the interface when the load step is large (*l.f.* = 0.5). Since the stress-dependent stiffness is updated at the beginning of each calculation step no adjustment of the interface stiffness takes places during the iteration procedure. Consequently, the step size (*l.f.*) has a big impact on the computed results. The relative deformation $\Delta u_{i,rel}$ in the summer phase increases with every cycle, which reduces the actual loading the adjacent soil experiences. This is evident when looking at the decrease of normal stress with increasing cycles in the interface nodes at $y = 7.88$ m (Figure 4(b)) and the decrease of the earth pressure coefficient in Figure 4(a). If (very) small load steps are applied, i.e. 1000, the HS-interface stiffness is sufficiently adjusted with the increasing normal stress σ_n. Hence, no cyclic accumulation of relative deformation $\Delta u_{i,rel}$ in the summer phase occurs and therefore (similar to calculations *III*) and *IV*)) a constant K_{mob} is obtained (Figure 4(a)). Yet, the (normal) interface stiffness in this case is still too low to prevent (non-cyclic) interpenetration effects. Only a significant increase of the interface normal stiffness could eliminate (relevant) interpenetration effects completely. This will be shown in the following examples by using the HS-soil.

The calculations *VI*) – *VIII*) with the HS model for both soil and zero thickness interface are evaluated with varying interface parameters ($R_{i,E}$ = 0.5 – 1, $E_{soil,input}$ = 1 – 100E_{soil}). Calculations *IX*) with a thin layer HS-interface and *X*) without an interface serve as references. Figure 5(a-c) displays the interface deformation $u_{x,i}$ in cycle 1, 2, 10 and 20 in relation to the plate's winter and summer position at $u_{x,top} = \pm 5$ mm. Calculations *VI*) and *VII*) with $R_{i,E}$ = 0.5 – 1 show a cyclic increase of relative deformations $\Delta u_{i,x,rel}$ (cyclic interpenetration), similar to

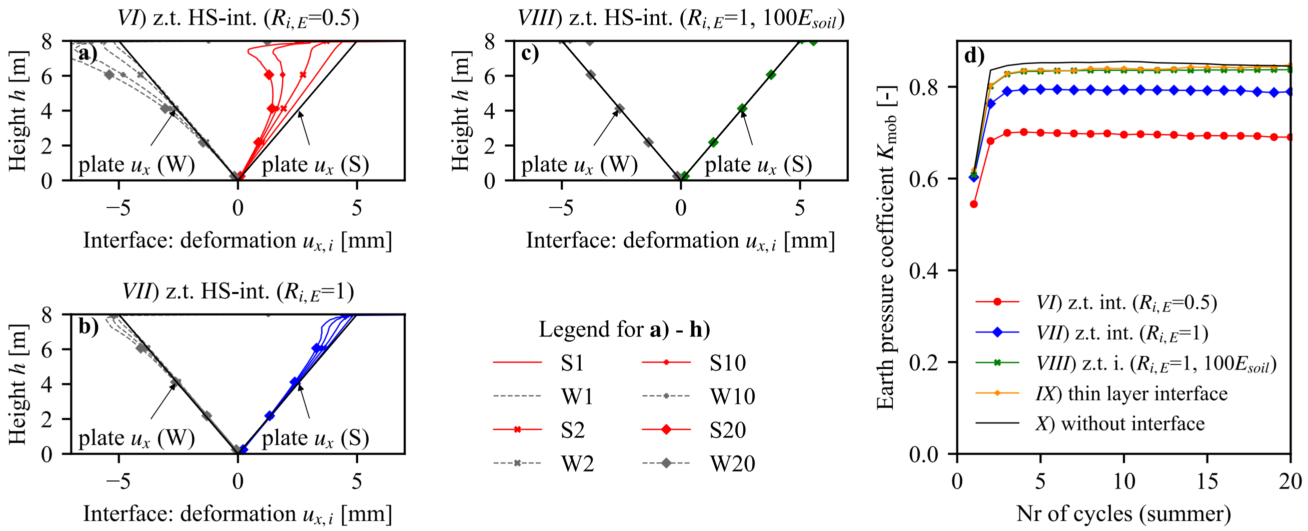

Figure 5. Calculation VI – X) with HS model for soil and interface after 20 cycles (S = summer; W = winter) with varying zero thickness interface properties ($R_{i,E}$, E_i) at a load fraction l.f. = 0.5: (a-c) Interface deformation $u_{x,i}$ compared to the plate deformation at $u_{x,top}$ = ±5 mm: (d) Earth pressure coefficient K_{mob} with increasing summer cycles

calculations *I*) and *II*) with LE-soil. As explained before, this can be traced back to the too low, stress-dependent (normal) stiffness of the HS-interface. This is confirmed by calculation *VIII*) with increased interface stiffness ($R_{i,E} = 1$, $E_{soil,input} = 100E_{soil}$). In this FEA no interpenetration can be seen – the winter and summer position of the interface coincides with the plate position. The increase of the interface stiffness can therefore be recommended as a suitable method to avoid interpenetration. An alternative could be to reduce the virtual interface thickness t_i by means of the *VTF* parameter. Yet, its reduction by the user is limited to the factor 10.

Like the calculations *I) – IV*), the results for HS-soil (*VI – VIII*) show a severe reduction of the mobilised horizontal earth pressure coefficient K_{mob} for cases with interpenetration, see Figure 5(d). However, no significant cyclic decrease of earth pressure is visible, even for calculation *V)* with low HS-interface stiffness. Two aspects are decisive for this behaviour: First, contrary to the calculations with LE-soil (*I – IV*), no gapping occurs in the considered example with HS-soil (*V – VIII*) in the winter positions because the tension cut off criterion is not reached ($\sigma_n > 0$ kPa). Thus, the initial (normal) stiffness of the interface upon reloading in the summer phase is higher and consequently less relative deformations $\Delta u_{i,rel}$ take place. Second, plasticity occurs in the adjacent (elastoplastic) soil which leads to stress increases in the summer phase and thus to higher interface stiffnesses. Additionally, due to the soil plasticity, (much) more load steps are required per phase to solve the unbalance. As described before for the results shown in Figure 4, the increase in load steps ensures that the stress-dependent HS stiffness is also adapted. As a result, the decrease of earth pressure with increasing cycles is limited or stopped, depending on the extent of plasticity and the number of load steps. Thus, any cyclic increase of interpenetration can be prevented if the max load fraction *l.f.* is chosen sufficiently small (e.g. 0.001, analog to the results for LE-soil). Finally, the comparison in Figure 6 also highlights that without interpenetration very similar earth pressures are mobilised in calculations using zero thickness interfaces (*VIII*), thin layer interfaces (*IX*) and FEA without interfaces (*X*).

5 CONCLUSIONS

In this paper interpenetration effects of zero thickness interfaces have been studied in cyclic FE analyses with Plaxis 2D. A simplified model of the soil structure interaction of an integral bridge abutment with primarily lateral loading was used. The investigation with linear elastic and elastoplastic soil models showed that interpenetration effects can lead to a significant underestimation of earth pressure behind the abutment. Furthermore, a cyclic increase of the interpenetration and a cyclic decrease of earth pressure were found when interfaces with stress-dependent stiffnesses were applied. The interpenetration is linked to too low normal stiffnesses of the interface. Therefore, for similar analyses, it is advised to carefully choose interface normal stiffnesses high enough. A comparison of interface and structure deformations can help to identify possible interpenetration effects. Additionally, if interfaces with stress-dependent stiffness are required enough load steps per calculation phase should be applied to avoid a cyclic increase of interpenetration. In Plaxis, default calculation and interface settings should be adjusted as recommended in this paper. Without interpenetration similar results have been obtained with zero thickness and thin layer interfaces. Future studies will expand this research to hypoplastic soil models and include other numerical issues of zero thickness interfaces such as increasing stress oscillations under cyclic loading. The investigations are part of ongoing research on the cyclic soil structure interaction behaviour of integral bridges.

6 REFERENCES

Banks, J. R., Bloodworth, A. G. 2018. Lateral stress profiles on integral bridge abutments. *Proceed. of the Institution of Civil Engineers - Bridge Engineering* **17**(3), 155–68.

Bentley 2022. *Plaxis connect edition V22 manual*. Bentley Systems, Exton.

Day, R. A., Potts, D. M. 1994. Zero thickness interface elements—numerical stability and application. *International Journal for Numerical and Analytical Methods in Geomechanics* **18**(10), 689–708.

Desai, C. S., Zaman, M. M., Lightner, J. G., Siriwardane, H. J. 1984. Thin-layer element for interfaces and joints, *International Journal for Numerical and Analytical Methods in Geomechanics* **8**(1), 19-43.

Goodman, R. E., Taylor, R. L., Brekke, T. L. 1968. A model for the mechanics of jointed rock, *Journal of the Soil Mechanics and Foundations Division* **94**(3):637–59.

Lehane, B. M. 2011. Lateral soil stiffness adjacent to deep integral bridge abutments, *Géotechnique* **61**(7), 593–603.

Potts, D. M., Zdravković, L. 2001. *Finite element analysis in geotechnical engineering: Application.* Telford, London.

Saberi, M., Annan, C.-D., Konrad, J.-M. 2018. On the mechanics and modeling of interfaces between granular soils and structural materials, *Archives of Civil and Mechanical Engineering* **18**(4), 1562–79.

Schanz, T., Vermeer, P., Bonier, P., (1999). Formulation and verification of the Hardening Soil model. *Beyond 2000 in Computational Geotechnics*, 281–96, Rotterdam.

Sharma, K. G., Desai, C. S. 1992. Analysis and implementation of thin-layer element for interfaces and joints, *J. of Engineering Mechanics* **118**(12), 2442–62.

Stastny, A., Tschuchnigg, F. 2022. Numerical studies on soil structure interaction of integral railway bridges with different backfills. *Proceed. – IACMAG 2022*, 338–345.

Stutz, H. H. 2016. *Hypoplastic models for soil-structure interfaces – modelling and implementation.* Ph.D. thesis, Kiel University.

Tschuchnigg, F. 2012. *3D finite element modelling of deep foundations employing an embedded pile formulation.* Ph.D. thesis, University of Technology Graz.