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Performance-based interaction domains for pile groups

Domaines d'interaction basés sur les performances pour les groupes de pieux

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ABSTRACT: In many engineering problems the design of piles is ruled by a threshold acceptable value of absolute and differential settlements as well as rotations under working loads. To verify that settlements and rotations of a piled foundation, under some combined axial load-moment action, do not exceed acceptable values, the construction of 'performance-based' interaction domain would be a useful design tool. To this end, the response of pile groups subjected to combined actions including vertical load, Q, and moment components along x and y axes, M_x and M_y, is first investigated by means of a simple ad-hoc numerical code, fixing the settlement and/or rotation to a given threshold value. Closed-form expressions for the performance-based interaction domains in the Q-M_x-M_y space are then obtained by matching the numerical results. Due to the limited number of parameters involved, which are already required in routine design, the proposed domains can be easily employed for the design of pile groups.

RÉSUMÉ: Dans de nombreux problèmes d'ingénierie, la conception des pieux est déterminée par une valeur seuil acceptable des tassements absolus et différentiels, ainsi que des rotations sous des charges de travail. Pour vérifier que les tassements et les rotations d'une fondation sur pieux, soumise à une action combinée de charge axiale et de moment, ne dépassent pas les valeurs acceptables, la construction d'un domaine d'interaction "basé sur la performance" serait un outil de conception utile. À cette fin, la réponse de groupes de pieux soumis à des actions combinées, comprenant une charge verticale, Q, et des composantes de moment le long des axes x et y, M_x et M_y, est d'abord étudiée au moyen d'un code numérique simple ad hoc, en fixant le tassement ou la rotation à une valeur seuil donnée. Des expressions analytiques pour les domaines d'interaction basés sur la performance dans l'espace Q-M_x-M_y sont ensuite obtenues en mettant en correspondance les résultats numériques. En raison du nombre limité de paramètres impliqués, qui sont déjà nécessaires dans la conception courante, les domaines proposés peuvent être facilement utilisés dans la conception courante de groupes de pieux.

Keywords: Pile groups; eccentric loading; performance-based design; analytical solution.

1 INTRODUCTION

Piled foundations are designed to ensure an adequate safety margin against failure (Ultimate Limit State, ULS) and a satisfactory behaviour in terms of settlements and rotations under working loads (Serviceability Limit State, SLS). However, in many engineering problems the design is ruled by SLS checks, i.e., by some threshold values of settlements and/or rotation which must not be exceeded. The example of a wind turbine is paradigmatic in this sense, given that the foundation is generally designed not to exceed a rotation of 0.5° to guarantee that it is tolerated by the structure without loss of serviceability during the entire lifespan. In light of the above, employing interaction domains fixing acceptable values of settlements and/or rotations would facilitate the assessment of SLS requirements. While expressions for failure envelopes of pile groups under combined loads have been proposed by recent works (Di Laora et al., 2019 and 2022; Iovino et al., 2021; Sakellariadis and Anastasopoulos 2022; Cesaro et al., 2024), there are still no solutions available for performance-based interaction diagrams. To fill this lack, a simple numerical approach is first proposed to investigate the response of pile groups subjected to a vertical load, Q, and to an external moment vector inclined by the angle α_M with respect to the y axis (with M_x and M_y moment components along x and y axes, Figure 1), fixing settlements or rotations to a given value. Using the results obtained with the code as benchmarks, closed-form solutions for performance-based interaction diagrams are thereby provided which allow to assess all the combinations of loads leading to a specific value of displacement of the pile group. Note that the horizontal load component has been disregarded; this simplification is consistent with the design of tall structures (such as wind turbines or bridge piers) since, from an engineering viewpoint, in this case the horizontal load is usually negligible compared to the vertical eccentric component. The proposed simple formulae are found to be in remarkable agreement with the more rigorous solution, and therefore represent a useful practical tool for the design of pile groups.

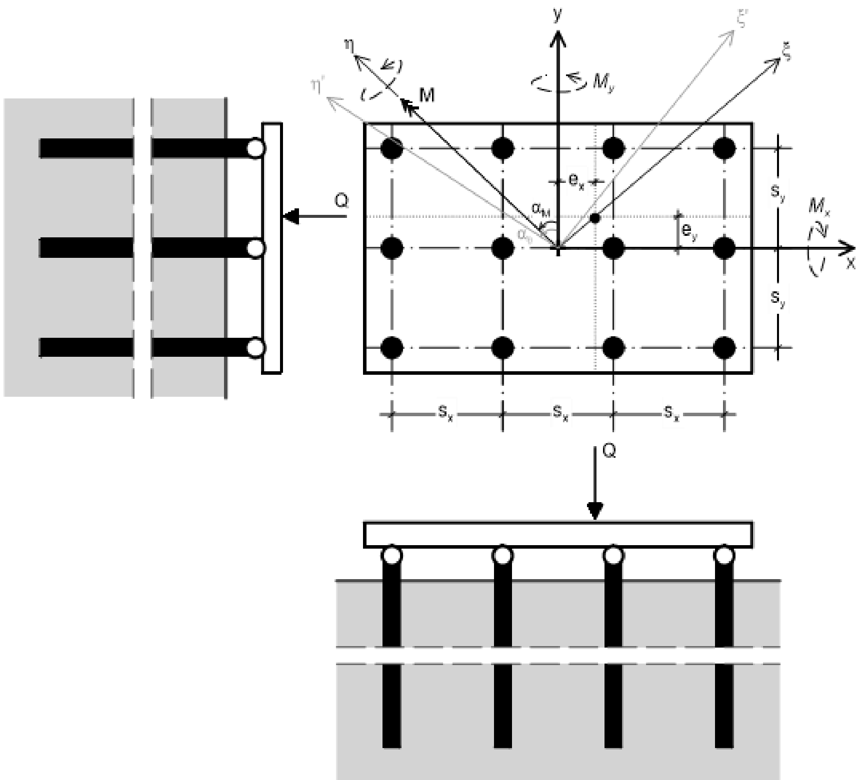

Figure 1. Pile group subjected to a vertical eccentric load (e_x and e_y, s_x and s_y are the eccentricities and the pile spacings along x and y axes, respectively).

2 NUMERICAL CODE

The numerical procedure to compute pile group settlement and rotation under an arbitrary load path and hence obtain the performance-based interaction diagrams, is detailed in the ensuing.

The following simplified assumptions have been made (Mandolini and Viggiani, 1997):

(a) Piles are modelled as uniaxial elements, characterized by a failure load in compression (N_u) and in uplift ($-S_u$), and by an elastic stiffness (k_s). The single pile load-settlement curve is described employing a hyperbolic relationship as suggested by Chin (1972) and the non-linearity is simulated using a stepwise linear incremental procedure;

(b) Piles heads are connected through a rigid free-standing cap having infinite strength;

(c) To account for pile group effects, interaction factors are used superimposing the effect of any pair of piles in the group (Poulos, 1968; Randolph and Wroth, 1979). Pile-to-pile interaction affects only the elastic component of the pile settlement because the non-linearity is essentially concentrated at the pile-soil interface (Randolph, 1994; Kanellopoulos and Gazetas, 2020).

As concerns the elastic axial stiffness, the adoption of a unique value for both compression and uplift is acceptable from an engineering standpoint since in many applications the base contribution is a small percentage of the total stiffness (Randolph and Wroth, 1978) and can thus be neglected. It should be pointed out that the numerical procedure can be employed by adopting a different load-settlement law to reproduce the load-settlement response of the single pile; likewise, the interaction coefficients can be evaluated by using one of the expressions and/or approaches available in literature. In this work the interaction coefficients, α_{ij}, are calculated using the approximated expression proposed by Randolph and Wroth (1979):

$$\alpha_{ij} = \frac{\ln(r_m/s_{ij})}{\ln(r_m/r)} \quad (1)$$

where s_{ij} is the distance between the piles i and j, r is the radius of pile j and r_m is the "magic radius", i.e., the distance at which the deformative field associated to pile j displacement, and measured from its axis, is assumed to be negligible:

$$r_m = \left\{0.25 + [2.5\rho(1-\nu_s) - 0.25]\frac{G_L}{G_b}\right\}L \quad (2)$$

In the above equation L is the pile length; G_L and G_b are the soil shear moduli at the depth L and underneath pile base, respectively; ν_s is the Poisson's coefficient of the soil; ρ is a parameter depending on the soil homogeneity (equal to 1 for a homogeneous medium and to 0.5 for a "Gibson-type" soil), defined as $G_{L/2}/G_L$, where $G_{L/2}$ is the soil shear modulus at the depth $L/2$. As shown by Mylonakis and Gazetas (1998), for a floating pile the magic radius can be estimated as:

$$r_m \approx \chi_1\chi_2 L(1-\nu_s)\mathrm{u} \quad (3)$$

where $\chi_1\chi_2$ is equal to 2.5 for an homogeneous halfsapce and 1 for a Gibson soil on rigid bedrock at depth 2.5L.

The elastic settlement, w_i, of a pile in the group of n piles, is given by:

$$w_i = \sum_{j=1}^{n} \frac{1}{k_{sj}} P_j \alpha_{ij} \quad (4)$$

Note that, the hypothesis (a) and (c) assure that the interaction factors do not vary with the load level. To account for pile non-linear behaviour under its own load, assuming Chin's hyperbole as the load-settlement curve for the single pile, the tangent compliance to be employed in the stepwise linear incremental procedure is given by:

$$\frac{dw_i}{dP_i} = \frac{1}{k_{si}\left(1-\frac{P_i}{P_{iU}}\right)^2} \quad (5)$$

where P_i and P_{iU} are respectively the load acting on pile i and its axial capacity in compression (N_u) or in uplift ($-S_u$) depending on the sign of P_i. This means that for i=j the right-hand side of eq. (4) must be multiplied by the following factor:

$$\alpha_\psi = \frac{1}{\left(1-\frac{P_i}{P_{iU}}\right)^2} \quad (6)$$

For any (Q, M) pair applied to the pile group, the distribution of the axial loads at the head of each pile as well as the displacement components of the rigid raft must satisfy the following compatibility and equilibrium equations:

$$w_i = w_0 + \theta_x y_i + \theta_y x_i \quad (7a)$$

$$\sum_{i=1}^{n} P_i = Q \quad (7b)$$

$$\sum_{i=1}^{n} P_i x_i = M_y \quad (7c)$$

$$\sum_{i=1}^{n} P_i y_i = M_x \quad (7d)$$

where w_0 is the settlement of the cap evaluated at the origin of the reference system, θ_x and θ_y, are the rotations of the cap about x and y axis, respectively. Note that Equation (7a) must be written for each of the n piles.

The construction of the performance-based interaction domains corresponding to fixed values of the vertical settlement is shown in Figure 2 for a 3×4 pile group. With reference to loading paths in which a vertical eccentric load increases at constant eccentricity, M/sQ, with α_M = 0, the points of the domain are obtained by deriving Q (and, consequently, M) corresponding to a given w/d (e.g., 2, 5, 10%) in the load-settlement curves (Figure 2a) calculated by the above-described numerical procedure. From Figure 2b it is possible to notice that the perfomance-based domains are superimposed to the theoretical failure surface (Di Laora et al., 2019; Cesaro et al., 2024) until the selected w/d is reached (i.e., $w/d \leq 2\%$ if the performance diagram is calculated for w/d = 2%). It follows that these domains contract only on the side of positive Q, that is in the same direction of the considered w/d.

Likewise, it is possible to derive the domains limiting the rotation, θ, to a given value. The procedure can be applied by simultaneously setting θ and w/d (Figure 5). It should be observed that, contrary to the case of fixing w/d, choosing θ as performance parameter corresponds to a symmetric contraction of the domain along the M axis - a consequence of adopting the same curve for both positive and negative values of the rotation.

3 SIMPLIFIED FORMULATION

To provide an easy-to-use tool to estimate the performance-based interaction domains, simplified formulae are proposed.

If the performance parameter is the vertical displacement, the interaction domain can be approximated with the following expression:

$$\sqrt{\left(\frac{M_x}{M_{xu}}\right)^2 + \left(\frac{M_y}{M_{yu}}\right)^2} = \frac{4[\lambda_w Q(Q_c+Q_t) - Q^2 - \lambda_w^2 Q_c Q_t]}{\lambda_w^2 (Q_c - Q_t)^2} \quad (8)$$

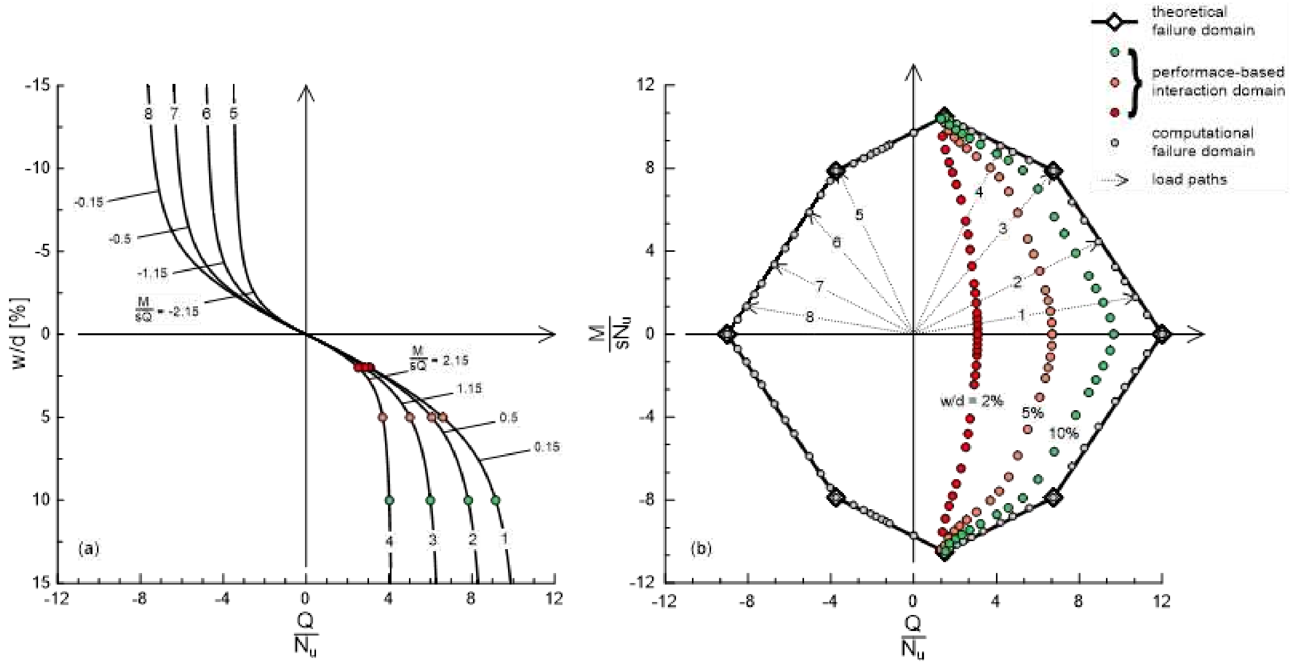

Figure 2. Load-settlement curves for a 3×4 pile group with applied vertical eccentric loads (Su/Nu = 3/4, s/d = 3, r_m/d=40, $k_s d/Nu$=80, α_M = 0, where d is the pile diameter) (a) and dimensionless interaction domains in the Q-M plane (b).

where Q_c and Q_t are the axial capacities of the group in compression and in uplift, M_{xu} and M_{yu} are the maximum ultimate moment capacities of the group around x and y axes and λ_w is the ratio between the vertical load corresponding to the fixed value of the vertical displacement and Q_c (or Q_t). M_{xu} and M_{yu} can be calculated from the failure domain, and for a value of α_M not corresponding to a vertex of the domain, a linear interpolation between two consecutive vertexes of the M_x-M_y failure locus is needed (Cesaro et al. 2024). The group vertical settlement taking into account the pile-to-pile interaction and the non-linearity of the single pile behaviour can be estimated as:

$$w = \frac{\lambda_w Q_u}{n k_s}\left[R_s + \frac{1}{(1-\lambda_w)} - 1\right] \tag{9}$$

where R_s is the amplification factor for group settlement:

$$R_s = \sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij}/n \tag{10}$$

Thus λ_w may be easily calculated as:

$$\lambda_w = \frac{1+\frac{K_v w}{Q_u} - \sqrt{1+\frac{K_v w}{Q_u}\left(\frac{4}{R_S}+\frac{K_v w}{Q_u}-2\right)}}{2\left(1-\frac{1}{R_S}\right)} \tag{11}$$

where K_v (= nk_s/R_s) is the elastic vertical stiffness of the group. Taking r_m = 2L(1-v_s) and v_s = 0.3, it is possible to investigate the influence of the pile slenderness ratio L/d and normalized pile spacing s/d, on the parameter R_S (Figure 3). It follows that R_S can be estimated as:

$$\frac{R_S}{n} = 0.9 exp\left(-0.5\sqrt{\frac{ns}{L}}\right) \tag{12}$$

where $R_G = R_S/n$ is the group reduction factor.

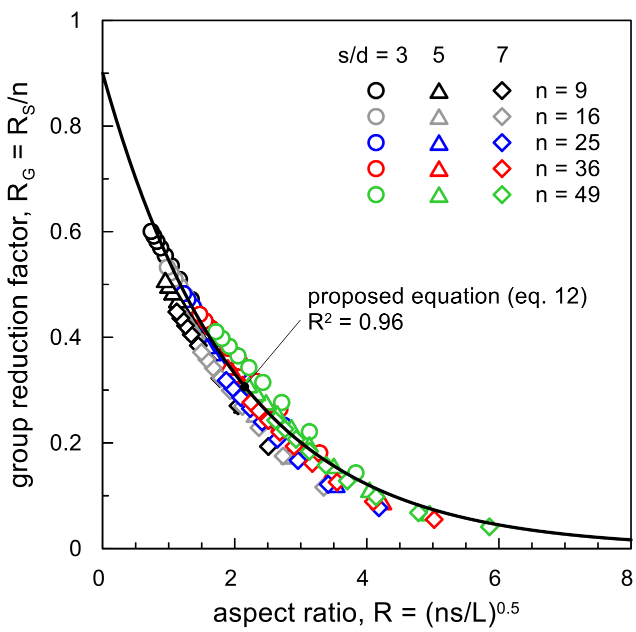

Figure 3. Group reduction factors.

Note that, Equations 9 and 12 allow to simply derive the pile group load-settlement curve under vertical centered load.

On the other hand, if the maximum cap rotation is the chosen performance parameter, the interaction domain can be estimated with the following expression:

$$\sqrt{\left(\frac{M_x}{\lambda_{\theta x} M_{xu}}\right)^2 + \left(\frac{M_y}{\lambda_{\theta y} M_{yu}}\right)^2} = \frac{4\left[\lambda_\theta^{0.5} Q(Q_c+Q_t) - Q^2 - \lambda_\theta Q_c Q_t\right]}{\lambda_\theta (Q_c - Q_t)^2} \qquad (13)$$

where λ_θ is the ratio between the resultant moment corresponding to the fixed value of the rotation and M_u.

The cap rotation, taking into account the pile-to-pile interaction and the non-linearity of the single pile behaviour, can be estimated as:

$$\theta = \frac{\lambda_\theta M_u}{K_\theta (1-\lambda_\theta)} \qquad (14)$$

then λ_θ is given by:

$$\lambda_\theta = \frac{\theta K_\theta}{M_u + \theta K_\theta} \qquad (15)$$

where K_θ is the elastic rotational stiffness of the group, depending on the axis orientation about which the group capacity assessment is required (Figure 1):

$$K_\theta = \frac{n}{\sum_{i=1}^{n} \frac{\sum_{j=1}^{n} \xi'_j \alpha_{ij}}{\xi'_i k_s \sum_{z=1}^{n} \xi'^2_z}} \qquad (16)$$

in which

$$\xi'_i = x_i \cos(\alpha_\theta) - y_i sin(\alpha_\theta) \qquad (17)$$

It should be pointed out that for a given resultant moment acting on pile group around a generic axis η, the maximum cap rotation could occurs about a different axis η', i.e., $\alpha_M \neq \alpha_\theta$ (Figure 1). However, it can be observed that for a square group of identical piles the difference between α_M and α_θ is always negligible from an engineering viewpoint, thus the evaluation of the group capacity can be carried out in the same direction as the resultant moment vector.

As in the previous case, it is possible to define the group rotational stiffness as the ratio between the elastic rotational stiffness of the group in absence of interaction ($k_s \Sigma \xi'^2_i$) and the group rotational amplification factor $R_{S\theta}$, that can be estimate through the following expression (Figure 4):

$$R_{S\theta} = 0.4\left[n^{-1.45}\left(\frac{L}{s}\right)^{0.4}\right]^{-0.45} \qquad (18)$$

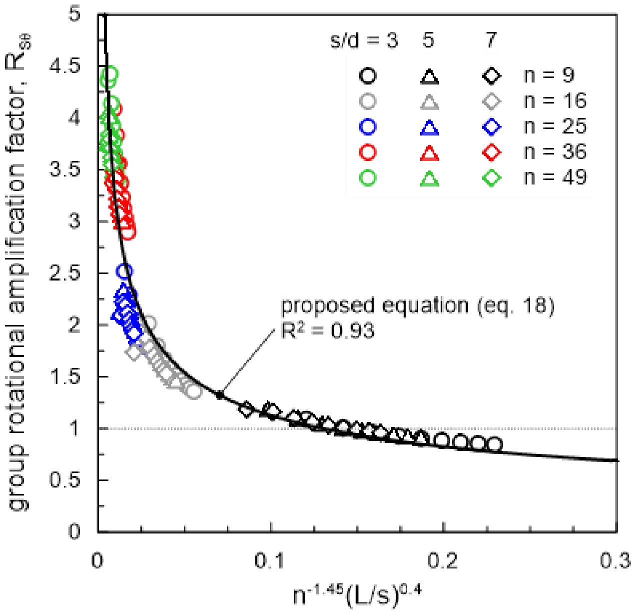

Figure 4. Group rotational amplification factor.

The domain derived by fixing both the performance parameters can be easily computed intersecting Equations (8) and (13) obtaining a very satisfactory match with the numerical results (Figure 5).

4 CONCLUSIONS

A simple numerical code to derive performance-based interaction domains is proposed based on the procedure presented in an early work by Mandolini and Viggiani (1997). This involved the selection of the load-settlement curve for the single pile and the possibility to account for the pile-to-pile interaction which is considered to affect only the elastic component of the single pile settlement. After calculating the load-displacement curves for different loading paths involving vertical eccentric loads applied to the pile group, the construction of the performance-based interaction domains is carried out by fixing settlement and/or rotation to a threshold value and deriving the corresponding load from the above curves. Simple formulae to employ in routine design are also provided taking into account the non-linear behaviour of the single pile and the pile-to-pile interaction. The results obtained employing the proposed formulae match very satisfactorily those derived with the numerical procedure. It is worth of note that the procedures and formulations presented have general validity since the results are depending on the input data. The proposed domains are straightforward design tool enabling to simultaneously carry out SLS checks for all load combinations necessitating only the capacity and stiffness of the single pile as ingredients, thus avoiding complex finite element modelling of the whole foundation.

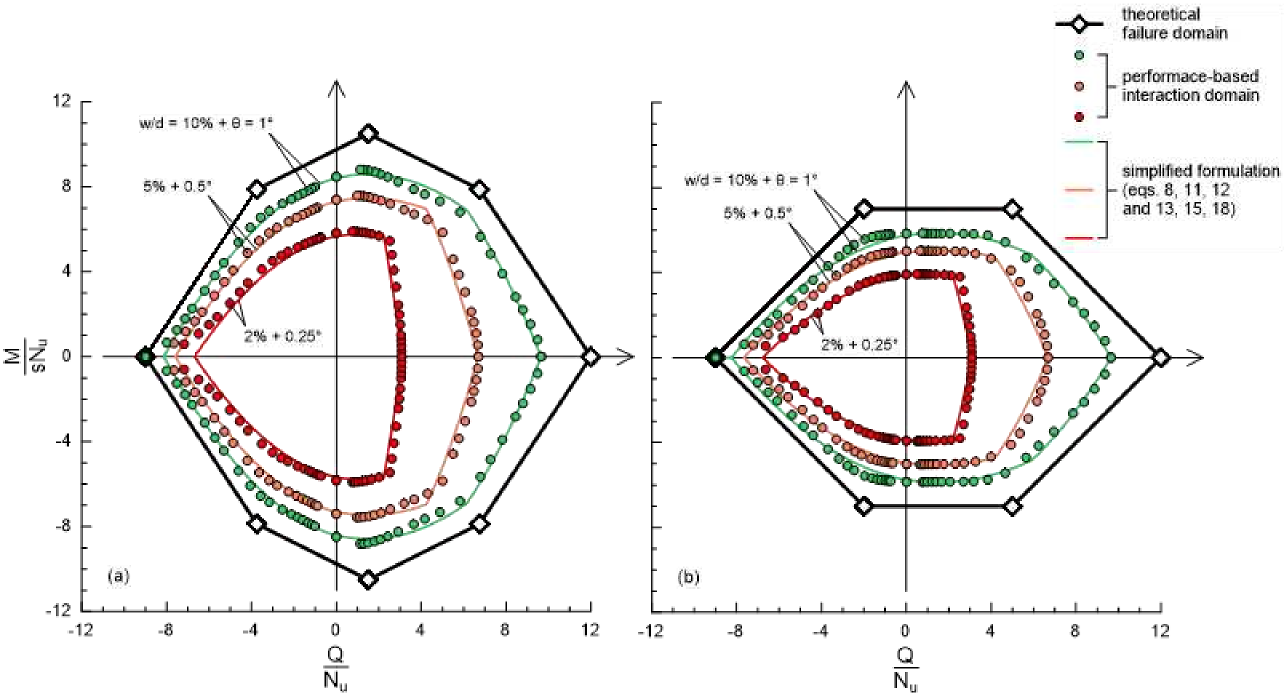

Figure 5. Dimensionless interaction domains in the Q-M plane for a 3×4 piles group (Su/Nu = 3/4, s/d = 3, r_m/d=40, k_sd/Nu=80): α_M = 0° (a); α_M = 90° (b).

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