Proceedings of the XVIII ECSMGE 2024
GEOTECHNICAL ENGINEERING CHALLENGES
TO MEET CURRENT AND EMERGING NEEDS OF SOCIETY

ISBN 978-1-032-54816-6
DOI 10.1201/9781003431749-343

An energetic approach to the soil mechanical behaviour description in a zero-dimensional formulation

Une approche énergétique de la description du comportement mécanique du sol dans une formulation à dimension zéro

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ABSTRACT: Geotechnical risks assessment in limit state design involves the use of mathematical models to predict the stress-strain state of the soil under the external loads. Most of these models are based on continuum mechanics solutions and rely on the concepts of "moduli" as parameters of stress-strain curve. For dispersed media, these concepts are devoid of physical meaning, since the mechanism of resistance to volumetric compression, shear strain and plastic yielding are fundamentally different, and are not caused by the rigidity of the bonds between the crystal lattice elements. The proposed energy approach allows to estimate the energy spent on each process occurring in a dispersed soil under mechanical action. The change in porosity, internal friction, hardening due to dilatancy and the increase in pore pressure are expressed by separate energy parameters, each of which in turn is described by a nonlinear stress state function. User defined threshold strain levels allow to turn on and off various mechanisms. The proposed approach better corresponds to the physical meaning of the processes. The method can be easily applied to adapt the theory of plasticity solutions. The development of a mathematical model based on the proposed approach will improve the quality of forecasting the occurrence of limit states in the soil bases.

RÉSUMÉ: L'évaluation des risques géotechniques dans la conception à l'état limite implique l'utilisation de modèles mathématiques pour prédire l'état de contrainte-déformation du sol sous les charges externes. La plupart de ces modèles sont basés sur des solutions de mécanique des milieux continus et reposent sur les concepts de "modules" comme paramètres de la courbe contrainte-déformation. Pour les milieux dispersés, ces concepts sont dépourvus de signification physique, car le mécanisme de résistance à la compression volumétrique, à la déformation par cisaillement et à la déformation plastique est fondamentalement différent et n'est pas causé par la rigidité des liaisons entre les éléments du réseau cristallin. L'approche énergétique proposée permet d'estimer l'énergie dépensée pour chaque processus se produisant dans un sol dispersé sous action mécanique. Le changement de porosité, le frottement interne, le durcissement dû à la dilatance et l'augmentation de la pression interstitielle sont exprimés par des paramètres énergétiques distincts, chacun étant à son tour décrit par une fonction d'état de contrainte non linéaire. Les niveaux de contrainte seuil définis par l'utilisateur permettent d'activer et de désactiver divers mécanismes. L'approche proposée correspond mieux à la signification physique des processus. La méthode peut être facilement appliquée pour adapter la théorie des solutions de plasticité. Le développement d'un modèle mathématique basé sur l'approche proposée améliorera la qualité de la prévision de l'occurrence des états limites dans les bases du sol.

Keywords: Energy approach; triaxial compression; dispersed soil; mathematical model.

1 INTRODUCTION

Dispersed soils mechanical properties study is largely complicated by the parallel flow of various processes. During volumetric strain compaction occurs, which, in turn, leads to the formation of additional contacts, changes in shear resistance conditions. In plasticity theory this process is described by hardening – an increase in the yield strength as plastic straining occurs.

At the same time, during the shear, phenomena due to the discreteness of the structure are observed – dilatancy and contraction – which cause a change in the volume of the pore space. As a consequence, the direct determination of the stress-strain state parameters mutual dependence is difficult. Various researchers have obtained dependence data for certain values of third-party factors, for example, the degree of density, water saturation, drainage regime, etc. A comprehensive analysis of these data, in fact, has not yet been carried out: most of the laws of soil mechanics are phenomenological in nature, and do not explain the physical nature of the processes taking place.

In addition to the multifactorial nature of the processes under study, the fundamental description of the dispersed soils mechanical behaviour laws is complicated by the limited capabilities of the test equipment. In no other field of materials science or, in a broader sense, solid mechanics, such a large number of devices of various designs are used. This is due to practical difficulties when trying to implement a homogeneous stress-strain state in the sample, but since most of the stress-strain state components are vector quantities, the resulting deviations lead to an increase in the discrepancy between the exact analytical solution and the actual experiment. Together with the change in the sample structure (and, as a consequence, the physical and mechanical properties) during the experiment, this does not allow for unambiguous interpretation of the test results.

2 THEORETICAL ANALYSIS

The solutions to the mechanics of granular media appear to be the direct method of describing dispersed soils mechanical behaviour, taking into account the discreteness of their structure (Kandaurov, 1966). Based on the characteristic particle size, a calculation scheme can be constructed that allows us to consider the equilibrium of individual particles, taking into account their mutual location. Unfortunately, as the number of particles increases, the number of equations in the solution also increases, and the introduction of different sized particles or deviations from the idealized packaging into the scheme leads to a stochastic solution, the result of which coincides in the limit with the solutions of solid mechanics.

The most well-known generalizing theory linking shear resistance parameters with physical characteristics is the critical state theory (Roscoe et. al, 1958). This theory assumes that each value of the average stress corresponds to its own value of the porosity coefficient, and the porosity coefficient, in turn, determines the amount of resistance to plastic yielding. Despite the active development of this theory in literary sources, in most implementations it is impossible to describe hardening or softening, as well as shear straining with its help.

In most cases, physical characteristics are completely excluded from the problems of soil mechanics, and the solution is carried out for an idealized solid body, the parameters of which are moduli, linking stresses and strains with linear or nonlinear laws. These parameters are obtained by laboratory tests and comparison of experimental values of stress-strain state components. At the same time, a significant number of studies have been carried out linking the values of these parameters with physical characteristics, in particular density, consistency, and granulometric composition (Goldin and Rasskazov, 2001; Kryzhanovsky et. al., 1985). The correlations obtained in these studies do not allow us to describe the causes of dependence, but only quantify it.

An alternative method of interpreting the results of laboratory tests can be the energy method, which is widely used in the mechanics of deformable solids and, in particular, in the theory of plasticity (Drucker and Prager, 1952). A change in the stress-strain state of some medium at a point is accompanied by energy transformations. On the one hand, external forces acting on this medium do the work. On the other hand, the energy expended is partially stored by elastic forces, and partially spent on overcoming internal friction and irreversible structural changes. Taking into account that in the conditions of a laboratory experiment the system can be considered as closed, the energy approach can be confidently applied as an additional method of interpreting the results.

The pioneers in the application of the energy method were specialists in the field of the plasticity theory of metals. The hypothesis of internal energy constancy during plastic yielding allowed to formulate the von Mises failure criterion. Most theories of plastic yielding (first of all, Prandtl–Reissner, Saint-Venant–Mises, Hill) take into account the work of plastic strain or its power. For example, the analysis of the work of strains allows us to describe the hardening during plastic yielding on the basis of Prandtl diagram.

In relation to soil mechanics, this approach was first adapted in the law of unassociated flow and a specialized flow condition. Also known are the works of a group of researchers in the field of critical state theory (Roscoe et. al., 1958), the main provisions of which were derived from energy prerequisites. A group of researchers considered a method for determining the transition point to plastic yielding based on the work expended (Crooks and Graham, 1976). Currently, the energy approach is used in the study of soil properties under dynamic loading, which makes it possible to assess the degree of energy absorption by comparing the work of elastic strain and the energy dissipated during one loading cycle (Ishihara, 2006). Based on the work of strain, it is proposed to evaluate the so-called "strain instability" - the strain levels at which structural changes occur in dispersed soils (Voznesensky, 2018; Usov and Voznesensky, 2016). Also, energy methods are used to determine the history of loading (re-compaction parameters), among them the methods of preconsolidation pressure determination (Becker, 1987; Wang and Frost, 2004).

3 RESEARCH METHODOLOGY

In this paper, we propose a method for interpreting the test results of axisymmetric triaxial compression from the standpoint of the energy and work of strain. This test method was chosen as, on the one hand, quite widespread, and on the other – allowing measurements of all stress-strain components during the experiment.

Triaxial compression tests can be carried out in various modes that determine both the initial state of the sample and the drainage conditions during the experiment. If we do not consider the unconsolidated regime as highly specialized, then at the initial moment of the experiment, filtration consolidation in the sample is considered completed (there is no excess pore pressure, the density of the skeleton is constant and can be unambiguously determined). In the case of full water saturation in the undrained mode during loading, the volume of the pore space remains constant due to the incompressibility of the liquid. This means that only shear strain will occur in the soil skeleton, and the increment of the average stress will cause an increase in pore fluid pressure instead of volumetric strain. The pore pressure change will also be caused by an additional change in the pore volume during the shearing – the pressure will fall in dilating soils and grow in contracting ones. Consequently, in the course of undrained tests, energy will be spent on elastic shear strain, plastic shear strain and pore pressure changes.

In the drained mode, the pore space is open during the experiment, therefore, the pore pressure does not change, and an increase in the average stress p will cause volumetric strain. This usually happens when using the standard stress trajectory (CTC) where an increase in the maximum main stress also causes an increment in the average stress. The processes of dilatancy and contraction proceed freely. Energy is spent on elastic and plastic shear and volumetric strain, and the plastic volume strain consists of the action of medium stress and dilatancy/contraction.

The implementation of the loading trajectory with a constant average stress (TC) leads to the fact that any volumetric strain in the experiment occurs solely due to dilatancy/contraction. Energy is spent on elastic and plastic shear strains, and volume strains are exclusively plastic and are caused by dilatation/contraction. Thus, in the presence of a series of experiments in different modes, with different loading trajectories and at different average stresses, parallel processes can be divided and the work expended on them can be compared.

The total work can be determined through the vertical force F and the movement of the rod Δh (1). The resulting value should be attributed to the volume of the sample in order to proceed to the specific work of strain.

$$\Delta W_{tot} = \Delta F \cdot \Delta h \qquad (1)$$

where ΔW_{tot} (J/m^3) is the specific work of strain, ΔF (N) is the vertical force increment, Δh (m) is the rod displacement.

4 EXPERIMENTAL STUDIES

As an experimental basis, this paper considers the results of triaxial compression tests of model soils (quartz grains of 2-5 mm fraction), performed in a stabilometer with a sample diameter of 300 mm, manufactured by NPP Geotek LLC for the Moscow State University of Civil Engineering. The tests were performed at an average stress of 200, 400 and 800 kPa. All tests were performed in kinematic mode with a constant strain rate. As a result, the dependences of vertical strain on the stress deviator, as well as volumetric strain on the vertical, were obtained, taking into account all the necessary corrections due to the design of the device. The resulting data set allows you to build any dependencies for other stress-strain state components.

The simplest analysis is performed for tests along the TC trajectory. If we select the last section corresponding to the constant slope of the dependence graph $W(\gamma)$, then we can assume that only plastic strain develops in this section (Fig. 1). At the same time, both shear strains and volume strains develop due to dilatancy. Considering that the energy costs in this case are caused solely by overcoming friction, the ratio of linear approximation equations can be considered as a function of internal friction, which is confirmed by the linear dependence of this ratio on the cell pressure. This ratio corresponds in meaning to the plastic ratio λ between the increment of plastic strain and the intensity of tangential stresses in the theory of plastic flow (2):

$$d\lambda = \frac{dA_p}{2\tau^2} \qquad (2)$$

Where $\Delta\lambda$ is the specific work of strain, ΔA_p (J) is the work increment, τ (Pa) is the shear stress.

It is noteworthy that these lines intersect the ordinate axis at almost the same point corresponding to the shear strain $\gamma = 0.0108$-0.0129. Such shear strain corresponds to axial strain $\varepsilon = 0.007$, which in absolute terms is 4.2 mm – approximately the average particle size in the samples studied.

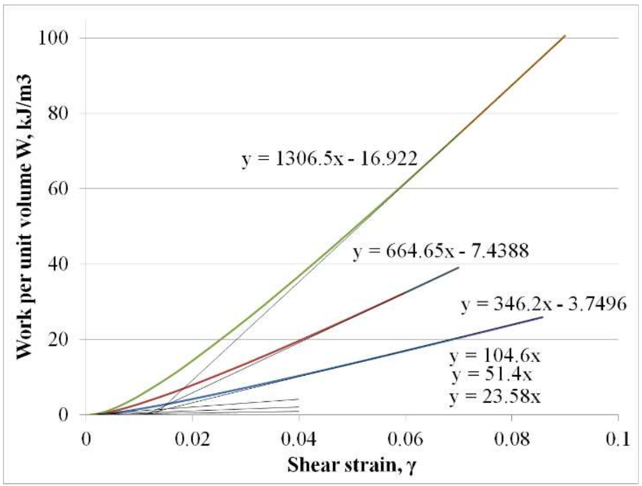

Figure 1. Dependences of the specific work of strain on the shear strain for tests at a pressure in the chamber of 200, 400 and 800 kPa with the correlation equations of the initial and final linear sections.

For tests performed at different average stresses, these ratios are obtained differently, since the resistance to plastic flow is determined mainly by internal friction. If we plot the obtained values of $d\lambda$ on a graph depending on the average stress, then these values are well approximated by a linear function (3) reflecting the dependence of the resistance to plastic flow on the average stress:

$$d\lambda(p) = 1.6p + 25 \quad (3)$$

where $\Delta\lambda$ is the specific work of strain, p (Pa) is the mean stress.

The threshold shear strain at which the transition to plastic flow occurs is γ = 0.03-0.05 for all tests.

If we correlate the obtained rate of work change to the shear strain and the work performed by the loading device, then we can identify a discrepancy – for example, for a sample tested at a pressure in a chamber of 800 kPa, the work of shear strain is 77.1% of the total work of the loading device. The remaining 22.9% of the energy is spent on friction in the elements of the device, deformation of the membrane and – mainly – on plastic volume strain. This observation gives grounds to consider the dependence of volumetric strain on the stress deviator, and similarly to obtain the amount of energy spent on plastic volume strain. Given that the tests were carried out in kinematic mode, the resulting percentage ratio directly allows to determine the dilatancy angle (4):

$$\psi = arcsin\frac{22.9}{77.1} = 17.3°. \quad (4)$$

Where ψ (°) is the dilatancy angle.

On the other hand, the construction of approximating lines from the origin allows us to obtain the energy consumed for elastic strain. Their slope is also proportional to the cell pressure, while these points are approximated by a power function. This correlates well with the power law of the dependence of stiffness on stress, repeatedly described for various conditions (Benz, 2007; Janbu, 1963; Schanz et. al., 1999).

In the test along the trajectory of deviatory crushing (CTC), these processes will overlap with each other, while ceasing to be linear, since during the experiment the average stress increases (which means there is an increase in stiffness and shear resistance). This means that both elastic and plastic strains with increasing stresses will require more energy. But, given that the tests are carried out on the same material, the dependencies obtained earlier will be partially valid.

5 RESULTS AND DISCUSSION

Based on the analysis of the results of the triaxial compression test with a constant average stress, the following patterns were determined:

- specific work of elastic shear strain W_e^s (5):

$$W_e^s = 0.0803p^{1.075}; \quad (5)$$

- specific work of plastic shear strain W_p^s (6):

$$W_p^s = 1.6p + 25; \quad (6)$$

- specific work of plastic volume strain W_p^v (7):

$$W_p^v = 0.297W_p^s; \quad (7)$$

Taking into account that the values of threshold strainss can be determined from the end points of the linear approximation sections, using these parameters it is possible to reproduce the shape of the original experimental curve. The modelling method is quite simple – for a certain step of shear strain (for example, 0.1%), the value of the stress deviator is calculated. In this case, the specific work of strain is calculated by (4-6), depending on the average stress. When the threshold strains are reached, individual terms are turned on or off. The dependencies obtained as a result of modelling are shown in Fig. 2.

It follows from the simulation results that the proposed approach allows reproducing experimental dependencies with high accuracy. The simulation result consists of separate fairly simple mathematical patterns (4-6), each of which acts in the appropriate range of strains: up to the level of 0.007, it is exceptionally elastic, then elastic and plastic, after 0.03 - plastic. At the same time, part of the energy consumed is spent on volumetric plastic strain.

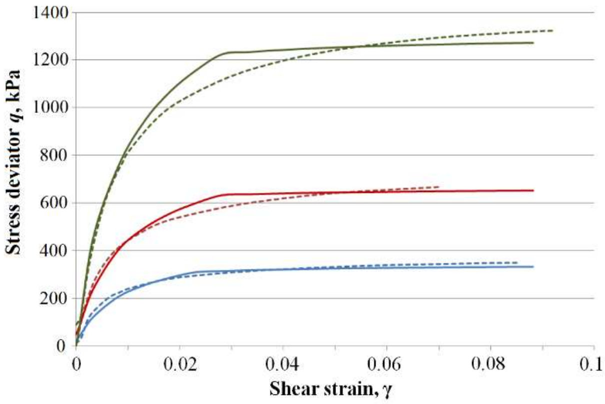

Figure 2. The result of modelling experimental curves. The dotted line shows experimental data, the solid line shows modelling.

The greatest discrepancy is observed at the transition site from elastic-plastic deformation to plastic flow. This is probably due to the smooth development of dilatancy in the real sample, which can be taken into account when processing the test results in more detail.

Based on the joint analysis of tests on two different trajectories, it is possible to separately describe all the ongoing deformation processes.

6 CONCLUSION

The method proposed in this paper for interpreting the results and modelling mechanical behaviour has a number of advantages in relation to solid mechanics.

- the theories of stresses and strains are used separately, the connection between them is established using energy parameters reflecting the physical meaning of the processes occurring during deformation;
- the resulting parameters are scalar quantities, and are represented as simple functions of the average stress;
- threshold strain values can be determined experimentally and correlated with the granulometric composition of samples.
- the proposed method of analysing the results of triaxial compression tests can be applied to other types of laboratory tests (simple shear, oedometric compression), provided that the stress-strain state is constant during the experiment.

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