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Effective strength parameters of Danish clay tills - revisited

Résistance effective paramètres des sols argileux Danois - revisité

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ABSTRACT: Effective strength parameters φ' and c' cannot be measured in tests. They are derived from the linear equation defining the tangent to a point on the failure envelope and cannot be addressed individually. For years Danish Practice has been to assess the effective stress failure envelope from the linear stress path in undrained triaxial compression tests (CAU$_c$), or alternatively assuming that the effective friction angle is a function of Plasticity Index (using $\varphi' = A - B \cdot \log I_P$) in combination with effective cohesion as function of the undrained shear strength ($c' = C \cdot c_u$). The paper discusses the shortcomings of these practices for Danish clay tills and presents a model that ensures that paired values of φ' and c' can be consistently addressed from curved failure envelopes that are uniquely defined by void ratio and Plasticity Index.

RÉSUMÉ: Les paramètres de résistance effective φ' et c' ne peuvent pas être mesurés lors d'essais. Ils sont dérivés de l'équation linéaire définissant la tangente à un point sur l'enveloppe de rupture et ne peuvent pas être traités individuellement. Pendant des années, la pratique danoise a consisté à évaluer l'enveloppe de rupture sous contrainte effective à partir du chemin de contrainte linéaire dans les essais de compression triaxiale non drainée (CAU$_c$), ou en supposant que l'angle de frottement effectif est une fonction de l'indice de plasticité (en utilisant $\varphi' = A - B \cdot \log I_P$) en combinaison avec la cohésion effective en fonction de la résistance au cisaillement non drainé ($c' = C \cdot c_u$). L'article discute des lacunes de ces pratiques pour les sols argileux danois et présente un modèle qui garantit que les valeurs appariées de φ' et c' peuvent être traitées de manière cohérente à partir d'enveloppes de rupture courbes qui sont définies de manière unique par le taux de vide et l'indice de plasticité.

Keywords: Clay till; effective strength parameters; curved failure envelope; triaxial tests; plate load tests.

1 INTRODUCTION

Effective strength parameters, φ' and c' cannot be measured in tests. These parameters are substitutes for the effective shear strength as modelled by Mohr-Coulomb. Their derivation comes from an evaluation of tests using the slope and interception point of the tangent to each point on the effective strength failure envelope.

Since the pioneering works by Jacobsen (1970), the most common test in Denmark for adressing the effective strength parameters for cohesive soils (including overconsolidated clay tills) is the anisotropically consolidated triaxial compression test CAU$_c$ with reconsolidation to preconsolidation stress and unloading typically to the in situ stress, see Figure 1.

The effective friction angle φ'_{tr} (Eq.1) and the effective cohesion c'_{tr} (Eq.2) in a single CAU$_c$ test is derived from the tangent fitting the linear part of the stress path using the slope $\tan\beta$ and interception point a, i.e., assuming a linear failure envelope for the stress interval covered by the tangent.

$$\varphi'_{tr} = \arcsin\left(\frac{1}{1+2\tan\beta}\right) \tag{1}$$

$$c'_{tr} = a \cdot \tan\beta \cdot \tan\varphi'_{tr} \tag{2}$$

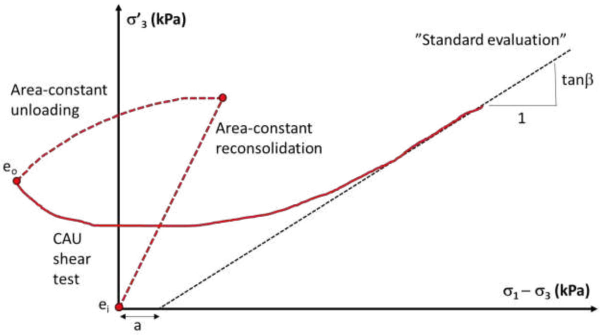

Figure 1. Danish "standard evaluation" of CAU$_c$ triaxial tests. Reconsolidation and unloading changes specimen void ratio from its initial value e_i to e_o

It was suggested by Jacobsen (1970) that the effective strength parameters in triaxial compression tests could be described solely as a function of the initial void ratio, e_i:

$$\varphi' = 35.3^0 - 9e_i \qquad (3)$$

$$c' = 422kPa \cdot exp(-7.3e_i) \qquad (4)$$

Jacobsen (1970) found that the use of φ'_{tr} and c'_{tr} from directly derived from triaxial tests (CAU$_c$ and CAD$_c$) in many cases overestimated the bearing capacity observed in drained plate load tests in clay tills. The same was observed using Eqs.(3)-(4) and Jacobsen (1970) suggested Eq.(5) as an alternative to Eq.(4) to be used in combination with φ'_{tr}.

$$c' = \frac{N_{co}c_u}{N_c} = \frac{5.14c_u}{(N_q-1)cot\varphi'_{tr}} \qquad (5)$$

However, the use of Eqs. (3)-(4) or φ'_{tr} and Eq.(5) does not remove the mismatch between predicted and recorded bearing capacity in drained plate load tests. This questions the use of φ'_{tr} and c'_{tr} based on a linear failure envelope for a selected stress range (say σ'_3 = 100-150 kPa).

If no site specific tests are available, the obsolete Danish Codes (DS415:1977, 1984) suggest the use of $c' = 0.1c_u \leq 20\ kPa$ in combination with $\varphi' = 30^o$ for preliminary analyses. Unfortunately a common misuse of $c' = 0.1c_u$ (without the cap on c') in combination with $\varphi' > 30^o$ has sneaked into practice, although it clearly disregards that $\varphi'$and c' cannot be assessed separately.

The ratio $c'/c_u = 0.1$ has its origin in triaxial tests by Jacobsen (1970) showing $c'/c_v \geq 0.1$ (see Table 3) in which c$_v$ is the vane shear strength. The 20 kPa cap on the effective cohesion c' is likely to have its origin from Jacobsen's (1970) observation that by using φ'_{tr} in back calculation of c' from the plate load tests resulted in values $c' < c'_{tr}$ and $c' \leq 20$ kPa.

The relationships Eqs.(3)-(4) by Jacobsen (1970) are still in use, but has been surpassed in Danish practice by the approach where φ' taken as a function of Plasticity Index I_p, (Eq.6), and c' assessed using Eq.(5) in combination with φ':

$$\varphi' = 45^o - (14^o\ a\ 15^o) \cdot log\left(\frac{I_p}{1\%}\right) \qquad (6)$$

Equation (5) indicates a coupling between c' and φ' as $N_c = f\{\varphi'\}$, However, most often this coupling is neglected by simply applying $c' = 0.1c_u$.

The arbitrariness of Eqs.(3)-(6) is exposed by comparing their assessment to the values of φ'_{tr} and c'_{tr} from triaxial tests. Figures 2 and 3 show that classification parameters like void ratio and Plasticity Index individually cannot properly describe the effective strength parameters φ'_{tr} and c'_{tr}.

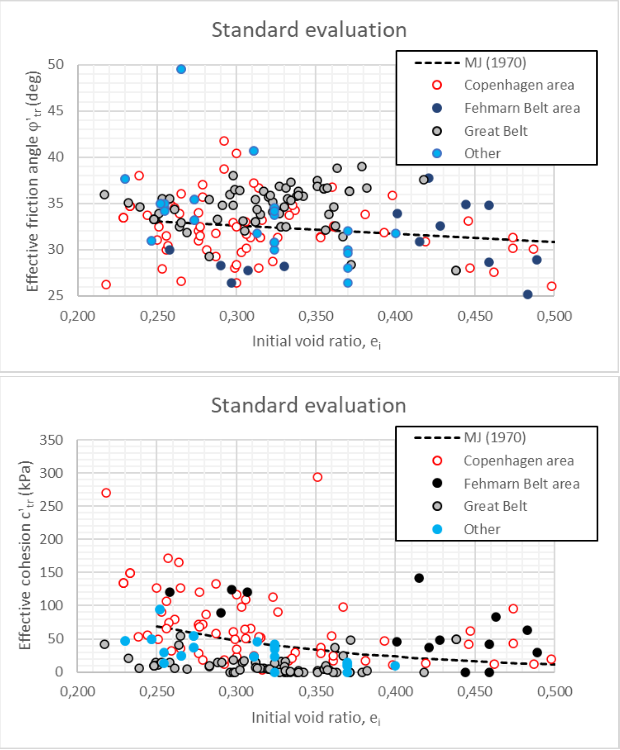

Figure 2. Effective strength parameters φ' and c' assessed using Eqs.(3)-(4) compared to φ'_{tr} and c'_{tr} from CAUc tests (H/D=1) using standard interpretation (Figure 1).

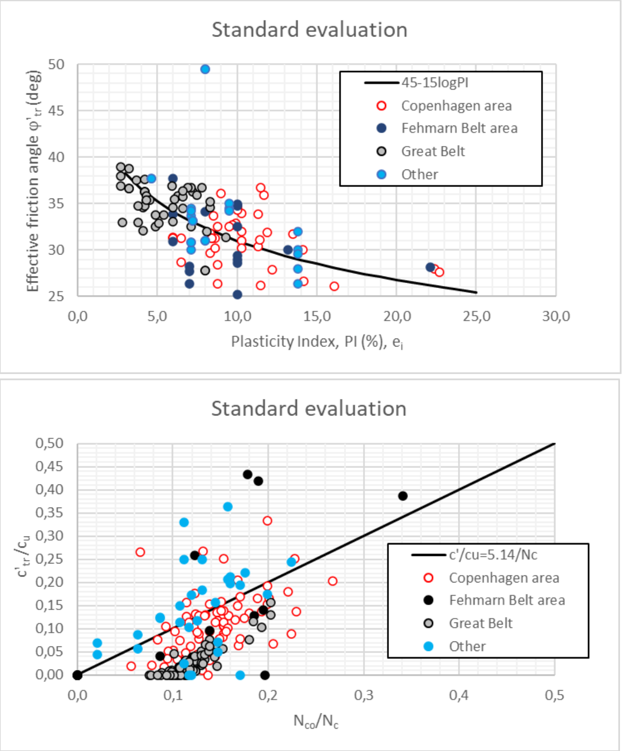

Figure 3. Effective strength parameters φ' and c' assessed using Eqs.(4)-(6) compared to φ'_{tr} and c'_{tr} from standard evaluation of CAUc tests(H/D=1).

2 FAILURE ENVELOPE FOR CLAY TILL

In relation to the København Tunnelbane project in the mid 1970ies, a series of drained triaxial compression tests (MCIDc) was performed with fixed axial stress (σ'_1) and decreasing confining stress (σ'_3) until failure. These showed very high friction angle and low effective cohesion at low effective stress levels at failure, $(\sigma'_3)_f$, indicating a curved failure envelope.

Therefore, a curved failure envelope is adopted in the proposed "*CAU-PI-e_o model*" (Steensen-Bach, 2023) being a function of void ratio and Plasticity Index. The failure envelope is assumed to follow a power law of $(\sigma'_3)_f$ (Eqs. (7)-(13) with coefficients as functions of the void ratio e_o, and Plasticity Index I_p).

$$(\sigma_1-\sigma_3)_f = k_1\cdot\left(\frac{\sigma'_3}{1kPa}\right)^{k_2} \tag{7}$$

$$k_1 = a_1\cdot(e_o)^{-b_1} \tag{8}$$

$$k_2 = a_2\cdot(e_o)^{b_2} \tag{9}$$

$$a_1 = 2.76675 + 0.053924\cdot\frac{I_p}{1\%} \tag{10}$$

$$b_1 = 0.907 + 0.001620\cdot\frac{I_p}{1\%} \tag{11}$$

$$a_2 = 0.8327 - 0.004004\cdot\frac{I_p}{1\%} \tag{12}$$

$$b_2 = 0.02515 + 0.000816\cdot\frac{I_p}{1\%} \tag{13}$$

The effective strength parameters φ' and c' are assessed from the tangent to each point on the curved failure envelope using Eqs.(14)-(15) as input in Eqs. (1)-(2).

$$tan\beta = \frac{1}{k_1 k_2}\cdot\left(\frac{\sigma'_3}{1kPa}\right)^{1-k_2} \tag{14}$$

$$a = k_1\cdot\left(\frac{\sigma'_3}{1kPa}\right)^{k_2} - \frac{\sigma'_3}{tan\beta} \tag{15}$$

An example of the curved failure envelopes is shown in Figure 4 for the case of $I_p = 10\%$ and various void ratios in the range $e_o = 0.25$-0.40.

Figure 5 shows the curved failure envelope for the case $e_o = 0.3$ and $I_p = 5$-20%, and the related effective strength parameters φ' and c' as function of σ'_3. Similar curves can be obtained for other e_o and I_p with the validity ranges $e_o = 0.25$-0.45 and $I_p = 3$-20%.

Figure 6 compares measured triaxial test values of $(\sigma_1-\sigma_3)_{max}$ to modelled values $(\sigma_1-\sigma_3)_f$ using the *CAU-PI-e_o model*.

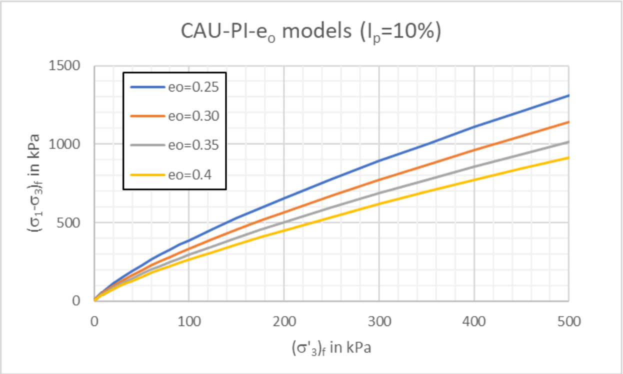

Figure 4. Curved failure envelope for CAU$_c$ tests using Plasticity Index I_p=10% and some selected void ratios.

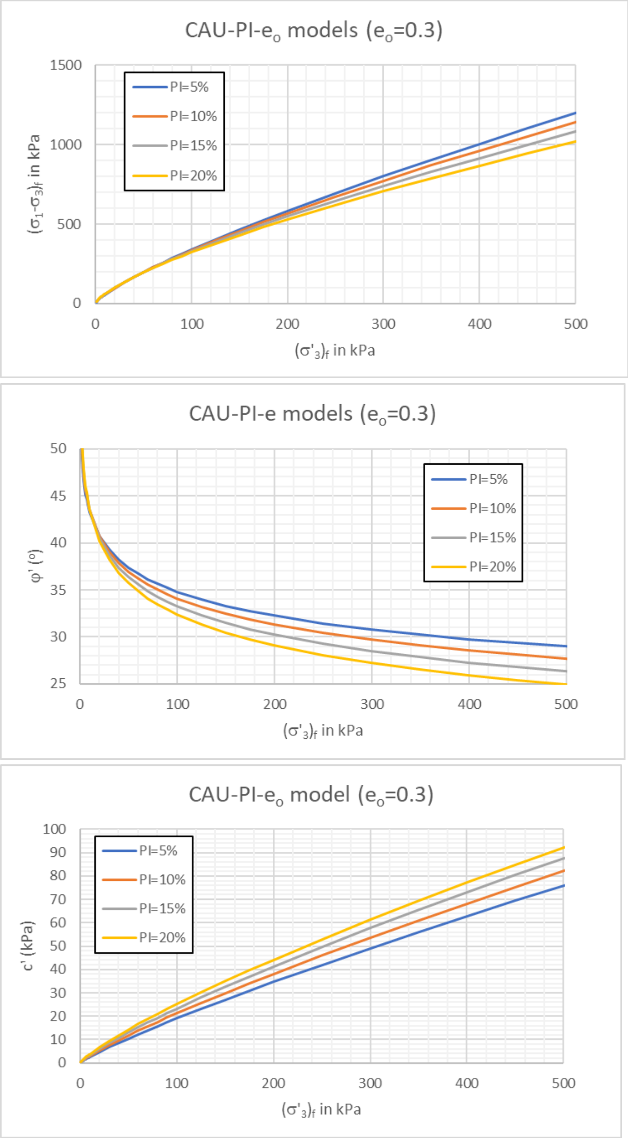

Figure 5. Curved failure envelope and effective strength parameters for CAU-PI-e$_o$ model at selected e_o and I_p.

Table 1 summarizes the ranges of initial void ratio (e_i) and at start of shear phase (e_o), and Plasticity Index (I_p) for CAU$_c$ and CAD$_c$ tests on Danish clay tills shown in Figure 6, and CID$_c$ tests in Figure 7.

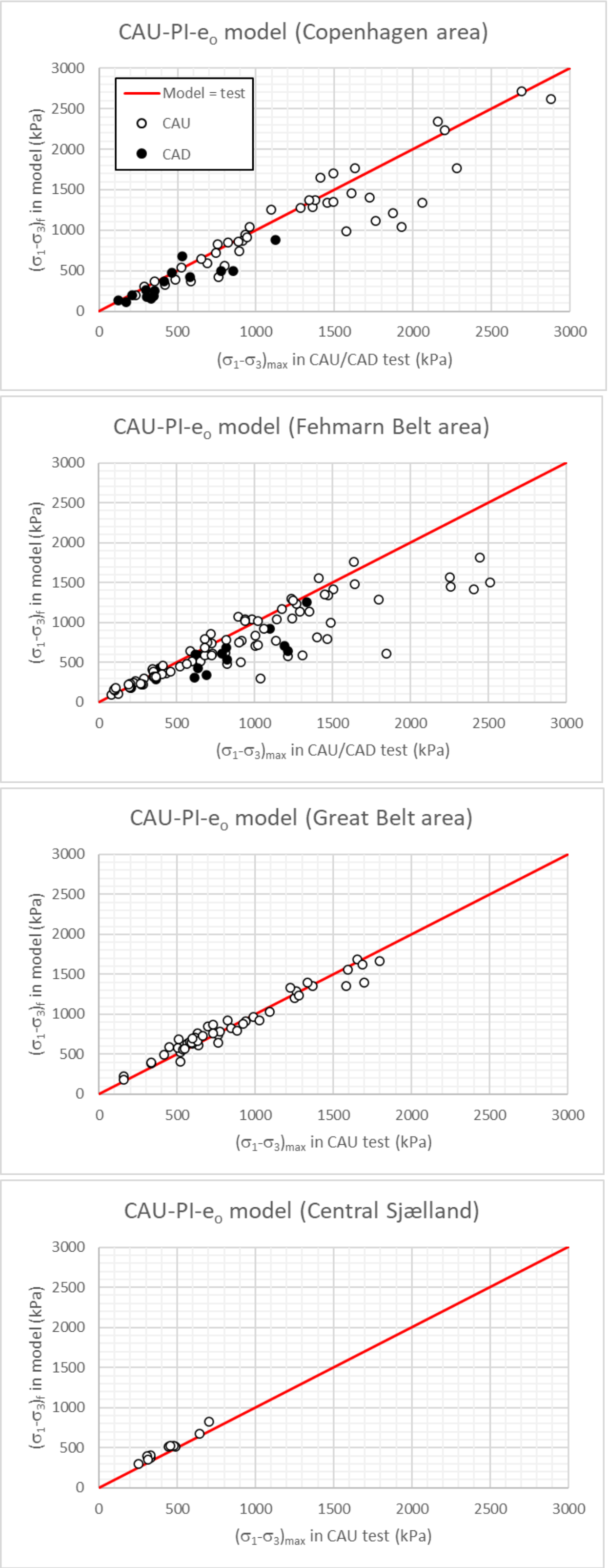

Figure 6. Prediction of $(\sigma_1\text{-}\sigma_3)_{max}$ in 186 CAU$_c$ tests and 33 CAD$_c$ tests on Danish clay tills (Clay content L= 10-25%) using the CAU-PI-e$_o$ model, Eqs.(7)-(13).

The use of the *CAU-PI-e$_o$ model* appears also to be applicable for some clay tills outside Denmark, as illustrated in Figure 7 for a Canadian clay till.

Table 1. Test type and basic classification parameters for clay tills tests analysed in this document.

Test	**H/D**	**I_p (%)**	**e_i (-)**	**e_o (-)**
CAU$_c$	1	2.7-22.7	0.23-0.55	0.16-0.49
CAD$_c$	1	3.0-27.1	0.17-0.47	0.15-0.41
CID$_c$	2	5.0-16.0	0.22-0.72	0.20-0.65

H/D = height to diameter ratio of specimen.

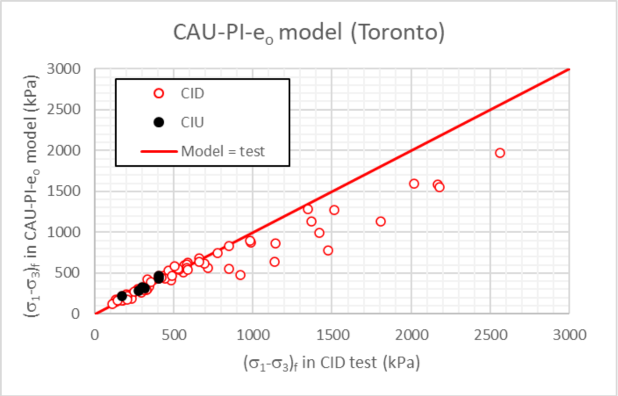

Figure 7. Prediction of $(\sigma_1\text{-}\sigma_3)_{max}$ in 71 CID$_c$ and 6 CIU$_c$ tests on clay till from Toronto (OCR$_{test}$=1-2, Clay content L= 9-42%) using the CAU-PI-e$_o$ model, Eqs.(7)-(13).

The *CAU-PI-e$_o$ model* underestimates $(\sigma_1\text{-}\sigma_3)_{max}$ in CAD$_c$ tests (see Figure 8) and thus provides a conservative estimate of the effective strength parameters in drained tests.

It is believed that the main reason for some CAD$_c$ tests being stronger than predicted by the *CAU-PI-e$_o$ model* is related to the strain-hardening due to dilation taking place in drained tests, but not in undrained tests. The difference in strain hardening between CAD$_c$ and CAU$_c$ tests increases with increasing OCR and could explain why the Toronto tests (OCR=1-2) are better predicted by *CAU-PI-e$_o$ model* than CAD$_c$ tests on the more heavily preconsolidated Danish clay tills.

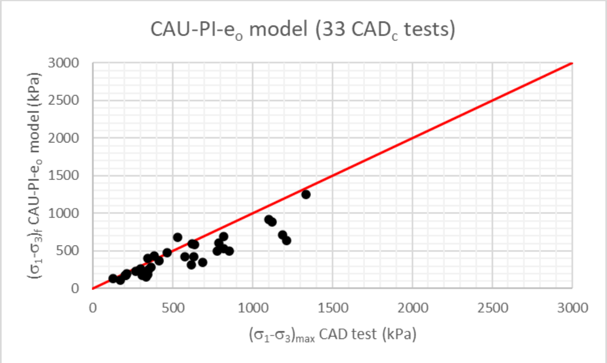

Figure 8. Prediction of $(\sigma_1\text{-}\sigma_3)_{max}$ in 33 CAD$_c$ tests on Danish clay tills using the CAU-PI-e$_o$ model, Eqs.(7)-(13).

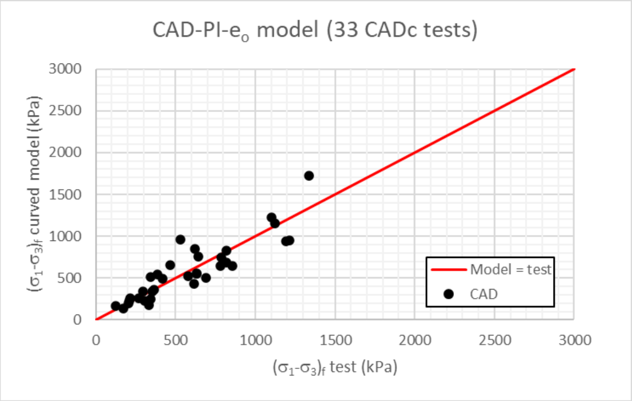

Figure 9. Prediction of $(\sigma_1\text{-}\sigma_3)_{max}$ in 33 CAD$_c$ tests on Danish clay tills using the CAD-PI-e$_o$ model, Eqs.(16)-(22).

To account for the strain-hardening in CAD$_c$ tests and obtaining a better prediction of the drained tests (see Figure 9) the coefficients a_1, b_1, a_2 and b_2 used in the *CAU-PI-e$_o$ model* are modified in the curved failure envelope for CAD$_c$ tests (*CAD-PI-e$_o$ model*). The coefficients a_3, b_3, a_4 and b_4 in the *CAD-PI-e$_o$ model,* Eqs.(16)-(18), are given by Eqs.(19)-(22).

$$(\sigma_1 - \sigma_3)_f = k_3 \cdot \left(\frac{\sigma'_3}{1kPa}\right)^{k_4} \tag{16}$$

$$k_3 = a_3 \cdot (e_o)^{-b_3} \tag{17}$$

$$k_4 = a_4 \cdot (e_o)^{b_4} \tag{18}$$

$$a_3 = 0.8 \cdot a_1 \tag{19}$$

$$b_3 = 1.3 \cdot b_1 \tag{20}$$

$$a_4 = 1.05 \cdot a_2 \tag{21}$$

$$b_4 = 1.4 \cdot b_2 \tag{22}$$

The coefficients a_1,b_1,a_2 and b_2 are given by Eqs. (10)-(13). The effective strength parameters φ' and c' for each point on the curved failure envelope (*CAD-PI-e$_o$*) are derived using Eqs.(23)-(24).

$$tan\beta = \frac{1}{k_3 k_4} \cdot \left(\frac{\sigma'_3}{1kPa}\right)^{1-k_4} \tag{23}$$

$$a = k_3 \cdot \left(\frac{\sigma'_3}{1kPa}\right)^{k_4} - \frac{\sigma'_3}{tan\beta} \tag{24}$$

There is no significant effect of Plasticity Index I_P or void ratio e_o on the ratio between $(\sigma_1\text{-}\sigma_3)_{max}$ in the CAD$_c$ tests and $(\sigma_1\text{-}\sigma_3)_f$ modelled using the *CAD-PI-e$_o$ model.* This is valid within the ranges of void ratio (e_o=0.15-0.40, e_{mean} = 0.27) and Plasticity Index (I_P=3-27%, $I_{p,mean}$ =11%) covered by the CAD$_c$ tests.

3 DRAINED PLATE LOAD TESTS

For curved failure envelopes the question is how to assess the relevant stress level $(\sigma'_3)_f$. Meyerhof (1950) suggested that stress level (drained bearing capacity of footings, p_f) is obtained through an iterative process solving Eqs.(25)-(26):

$$(\sigma'_3)_f = f(p_f) \tag{25}$$

$$p_f = \frac{1}{2}\gamma' B s_\gamma N_\gamma + q' s_q N_q + c' s_c N_c \tag{26}$$

The function $f(p_f)$ for clay till that relates to the *CAU-PI-e$_o$ model*, is established using the 28 drained plate load tests on four clay tills (Table 2), using circular plates of diameter *D* tested at effective overburden pressure *q',* as reported by Jacobsen, (1970).

The width B in Eq.(26) has been taken for the circular plates as for an equivalent square footing $B \approx 0.866D$. Bearing capacity factors N_γ, N_c, N_q and shape factors for square footing (s_γ=0.6, s_q=s_c=1.2) as given in DS415:1984. For clays, it is Danish practice to use the triaxial friction angle in N_γ, N_c and N_q.

For each plate load test the bearing capacity is modelled using the *CAU-PI-e$_o$ model*. Figure 10 shows that on the relevant failure envelope (e,I_p) there is a unique σ'_3 and thus (φ',c') for each plate test, where modelled capacity $p_{f,model}$ equals $p_{f,test}$.

Table 2. Drained plate load tests on clay tills (L=15-25%).

	Carls. I	**Carls. II**	**Kratbj.**	**Sabro**
Nos.	1	6	17	4
D (cm)	7.5	7.5-25	5-15	15-30
q' (kPa)	23	0-30.5	0-20.5	1-7.8
e	0.24	0.30	0.32	0.52
I_p (%)	10.3	8.8	7.1	13.8
c_v (kPa)	700	250	200	58
φ'_{tr} (°)	33.0	30.6	32.6	30.6
c'_{tr} (kPa)	70.6	27.9	39.2	10.8

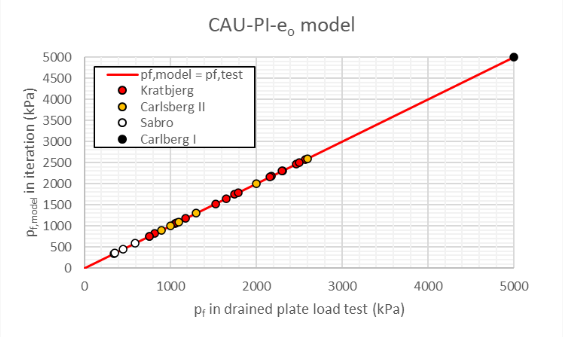

Figure 10. Bearing capacity in test (Jacobsen, 1970) and for selected σ'_3 in CAU-PI-e$_o$ model giving $p_{f,model}$=$p_{f,test}$.

Figure 11 shows that for three of the clay tills investigated there is a distinct relationship between bearing capacity p_f in test and σ'_3. The coefficients a and b (Eq.19) are modelled as functions of in situ void ratio e and I_p (Steensen-Bach, 2023):

$$f\{p_f\} = b \cdot \left(\frac{p_f}{1kPa}\right)^a \qquad (27)$$

$$a = 1.5724 + 0.05974\left(\frac{I_p}{1\%}\right) - 0.004098\left(\frac{I_p}{1\%}\right)^2$$

$$b = 0.002731 - 0.0232(e) + 0.004968(e)^2 \geq 0.000242 \qquad (28)$$

The coefficient a (ranging 1.62-1.79) decreases with increasing I_p and coefficient b (ranging 0.000242-0.0041) increases with increasing in situ void ratio e. The lower bound of b = 0.000242 (e = 0.3) is applied to Carlsberg I clay till.

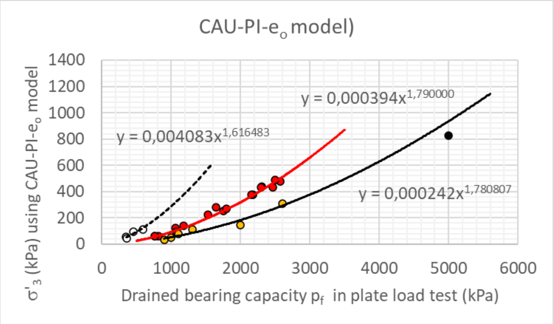

Figure 11. Required stress level σ'_3 in CAU-PI-e_o model to model drained bearing capacity in plate load tests.

Figure 12 shows a comparison of the vertical bearing capacity in Jacobsen's drained plate load tests to the bearing capacity predicted using the *CAU-PI-e_o model* with $(\sigma'_3)_f$ assessed using Eqs.(25)-(28).

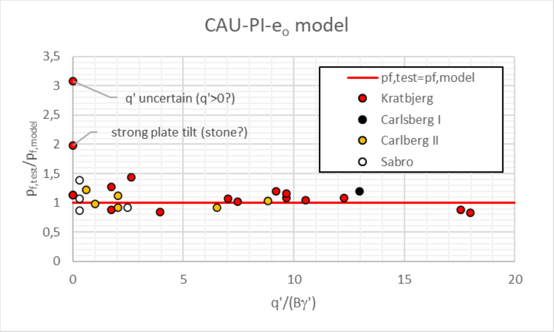

Figure 12. Comparison of measured and predicted values of drained vertical bearing capacity in plate load tests on clay tills using CAU-PI-e_o model and assessing stress level from the iteration model, (Eqs.25-28).

Figure 13 shows the comparison of drained bearing capacity in plate load tests using the commonly used approaches i.e., Eqs.(5)-(6) using I_p and field vane shear strength c_v, or the effective strength parameters from the linear failure envelope parameters (Table 2).

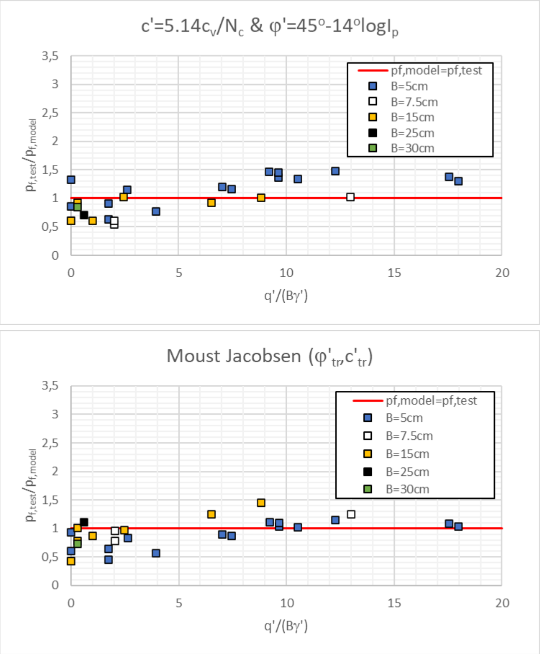

Figure 13. Comparison of measured values and predicted drained vertical bearing capacity in plate load tests.

4 CONCLUSIONS

No amount of modelling can beat parameters established through site specific testing. However, analyses demonstrate the inadequacy in predicting the drained bearing capacity using constant values of φ' and c' from CAU$_c$ or CAD$_c$ tests.

The *CAU-PI-e_o model* offers a rational alternative to the models currently used in practice and can be used to support sites where low number of tests prevents establishing local (curved) failure envelope models.

Instead of the arbitrary selection of constant values of *φ' and c'*, the models support the Danish Practice using undrained triaxial tests to derive effective strength parameters, provided the undrained tests are analysed with due respect to the impact of I_p, in situ e (replaces e_o in models) and $(\sigma'_3)_f$.

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