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An advanced method to predict umbrella arch behaviour in soft ground tunnelling

Une méthode avancée pour prédire le comportement de voûte parapluie dans la réalisation de tunnels en sol meuble

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ABSTRACT: When tunnelling in soft ground, umbrella arch is often being used to improve the conditions around tunnel face. The paper describes an advanced method to predict the umbrella arch behaviour based on the displacement measurements. An established empirical method initially proposed by Barlow is used as the foundation. The displacement functions for sections ahead of the face and for the supported part of the tunnel are retained while in the unsupported area and unexcavated area close to the face the functions are modified by introducing a set of analytical equations. For the arch element the elastic beam theory is adopted in combination with the theory of the beam on elastic foundation (Winkler soil model). The proposed method is kept relatively simple by adopting analytical solution for semi-infinite beam in front of the tunnel face. The analytical solution is compared to the results of 3D numerical analysis. Despite the simplification in the approach the analytical solution gives very similar results both in terms of beam displacement and bending moment.

RÉSUMÉ: Lors du creusement de tunnels dans un sol meuble, la voûte parapluie est souvent utilisée pour améliorer les conditions autour du front du tunnel. L'article décrit une méthode avancée basée sur des mesures de déplacement pour prédire le comportement de la voûte parapluie. Une méthode empirique établie, initialement proposée par Barlow, est utilisée comme base de cette méthode. Les fonctions de déplacement pour les sections en avant du front et pour la partie supportée du tunnel sont conservées tandis que dans la zone non supportée et la zone non excavée proche du front, les fonctions sont modifiées en introduisant un ensemble d'équations analytiques. Pour la voûte, la théorie des poutres élastiques est utilisée en combinaison avec la théorie des poutres sur fondation élastique (modèle de Winkler). La méthode proposée reste relativement simple en adoptant une solution analytique pour une poutre semi-infinie devant le front du tunnel. La solution analytique est comparée aux résultats de l'analyse numérique 3D. Bien que l'approche soit simplifiée, cette méthode donne des résultats similaires à ceux de l'analyse numérique, tant en termes de déplacement de la poutre que de moment fléchissant.

Keywords: Tunnel; umbrella arch; displacement function; 3D analysis.

1 INTRODUCTION

Umbrella arch also known as pipe roof is one of the pre-support methods in tunnelling used to improve stability and safety conditions around the unsupported section of the tunnel. For safe but also cost-efficient design of umbrella arch it is necessary to understand the behaviour of the pipe elements during the excavation. To design umbrella arch several analytical methods exist, but none has achieved widespread acceptance (Oke et al., 2016).

An empirical method proposed by Barlow (1986) and further modified by Sellner (2000) describes the tunnel displacements as a function of time and excavation advancement. The method has proven to give very good results describing the tunnel displacements, however it has some limitations with regard to describing the umbrella arch displacements. The paper proposes a modification of the method to overcome these shortcomings.

2 TUNNEL DISPLACEMENT METHOD

The displacement function defines vertical displacements of tunnel crown at a given chainage (monitoring section) as a function of time and the distance between tunnel face position and the observed chainage. Barlow proposed three separate functions to describe tunnel crown displacement along a tunnel section: displacement in front of the face (Eq. 1), displacement in unsupported section (Eq. 2) and displacement in supported section (Eq. 3):

$$C_{pf}(x,t) = \left[Q_1 \cdot C_{pf}(x) - Q_k \cdot P_k^+(x)\right] \cdot \left[C_{x\infty} + A \cdot C_2(t)\right] \tag{1}$$

$$C(x,t) = [Q_1 + Q_2 \cdot C_1(x) - Q_k \cdot P_k^+(x)] \cdot [C_{x\infty} + A \cdot C_2(t)] \quad (2)$$

$$C(x,t) = \frac{[Q_1 + Q_2 \cdot C_1(x) + K \cdot C_S - Q_k \cdot P_k^-(x)]}{[1 + K \cdot (C_{x\infty} + A \cdot C_2(t))]} \cdot [C_{x\infty} + A \cdot C_2(t)] \quad (3)$$

where Q_1 is a displacement proportion ahead of the face, C_{pf} is a displacement function ahead of the face, Q_k is a displacement proportion to account for the effect of support, P_k^+ is a support function ahead of support installation point, $C_{x\infty}$ is time independent displacement, A is a displacement proportion to account for the effect of time, C_2 is a displacement function to account for time effect, Q_2 is a displacement proportion behind the face, C_1 is a displacement function behind the face, P_k^- is a support function behind the support installation point, K is a support stiffness and C_S is the convergence at the point of support installation.

The advantage of the method is relatively small number of parameters. Four main parameters ($C_{x\infty}$, Q_1, A, K) govern the shape of three functions, which makes the process of fitting calculated to measured displacements relatively straightforward. Since the displacements of umbrella arch don't differ significantly from the ground displacement, the displacement function can be used to describe the umbrella arch behaviour. Once the deformed shape of umbrella arch is known, internal forces can be easily calculated.

However, as it can be seen from Figure 1, the analytical displacement functions describe continuous displacements, whereas their derivatives are not continuous at the connection points between individual displacement functions. Such displacement functions do not represent the actual behaviour of tunnel lining well enough to be used for the assessment of umbrella arch internal forces due to ground displacements. Although the arch elements aren't very stiff compared to the ground, they can still locally redistribute stresses from the unsupported section to the tunnel support on one side and to unexcavated ground ahead of the face. This results in a more gradual displacement curve around two crucial points (Figure 1, red line). In the next chapter, a modification of displacement functions is proposed.

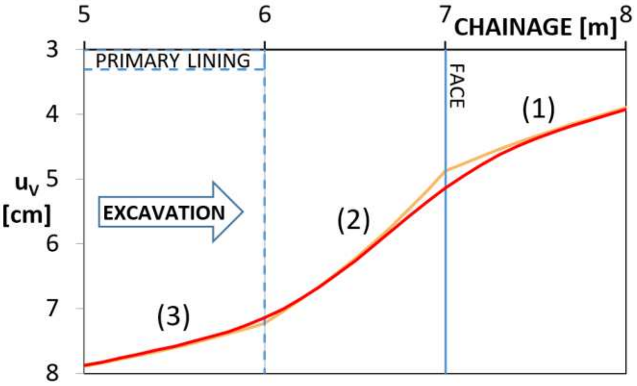

Figure 1. Three displacement functions (brown curve) and measured/numerically obtained displacement curve (red).

3 MODIFICATION OF EMPIRICAL METHOD

As shown in Figure 1, redefinition of the intermediate displacement function (Equation 2) is particularly required. Similarly, the function ahead of the face at close proximity to the face also deviates considerably from measured curve (Figure 1) and needs an adjustment having in mind that we are interested in obtaining good estimates of derivatives, which will yield bending moment and shear force distribution in arch elements. Therefore, two new intermediate functions are proposed.

Sequential excavation method is considered in the study and this results in an oscillating pattern of bending moment and shear force distribution along the arch element (Figure 2). The original displacement function formulation doesn't allow to simulate such behaviour. The proposed modification of displacement functions is therefore written in terms of displacement increments between two consecutive excavation stages instead of total displacements.

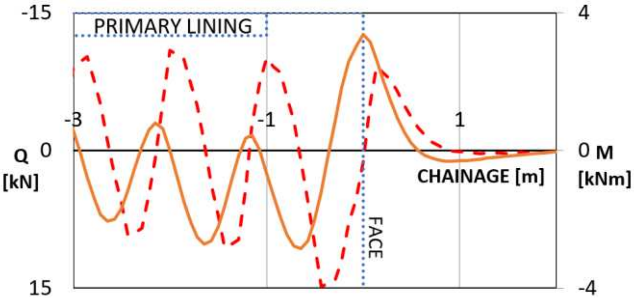

Figure 2. Bending moment curve (solid line/secondary axis) and shear force curve (dashed line) of the arch element.

3.1 The model assumptions

The modified model for the description of the arch element behaviour is based on the following assumptions:

- the arch element is modelled as linear elastic beam with known relations between displacements and internal forces,
- the model comprises four functions: two original displacement functions at both ends and two modified analytical intermediate functions. The first intermediate function starts at certain distance ahead of the face and ends approximately at the face of the tunnel. From this point onwards the second one describes displacements in the unsupported section and extends slightly into supported section,
- four boundary conditions (displacement, rotation, bending moment, shear force) are fulfilled at each joint between two adjacent functions,
- functions ahead of the face (Eq. 1) and in supported section (Eq. 3) are fitted to measured/calculated displacement increments,
- two intermediate functions are based on analytical solutions for elastic beam and derived at the incremental level (see 3.2 and 3.3). The values a_L, a_M, a_R, a_T and p_L, p_M, p_R, p_T defining load pattern (Figure 4) acting on elastic beam are again obtained by fitting procedure,
- the derivatives and resulting inner forces are obtained analytically.

3.2 Arch load in unsupported section

To model the load in the unsupported section the results of numerical analyses were thoroughly studied. Different distributions were considered (polynomial, trigonometric), but finally the load pattern was simplified to triangular load at the initial part of supported section and two rectangular loads in unsupported section (Figure 3), which significantly reduces the number of parameters but still provides very good agreement with numerical results.

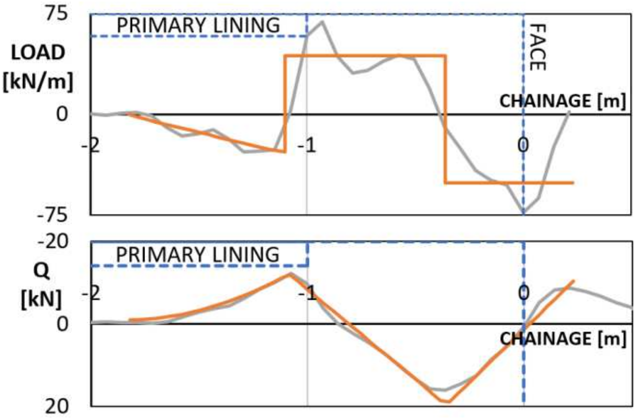

Figure 3. The load distribution in unsupported section (above) and shear force curve (below). Comparison of numerical results (gray) and analytical model (orange).

3.3 Intermediate function ahead of face

The second intermediate function ahead of the face is based on Winkler subgrade model. The analytical solutions for the semi-infinite elastic beam were adopted (Young and Budynas, 2002). This beam type was chosen to keep the solution relatively simple and with low number of parameters. Figure 2 shows that the resulting values of internal forces vanish quickly as one would expect.

The superposition approach was adopted to model the load in both intermediate sections. The ground load was modelled by triangular load p_T (Figure 4) and solutions for concentrated force and moment were included to meet the boundary conditions at the face.

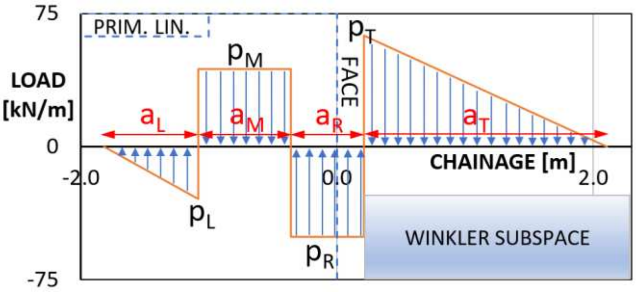

Figure 4. Load and ground model around the tunnel face.

4 RESULTS

The intended use of the proposed method is primarily during tunnelling by fitting displacement functions to measured displacements of a tunnel lining. In this paper, instead of using measurements, displacements were obtained by advanced 3D numerical models. A series of numerical analyses with different scenarios were performed (Kuder et al., 2022) by using Plaxis 3D finite element software (Figure 5). Results of one such case are presented in this paper using the properties shown in Table 1.

Hardening soil material model was used for the ground. Primary lining was modelled with special concrete constitutive model.

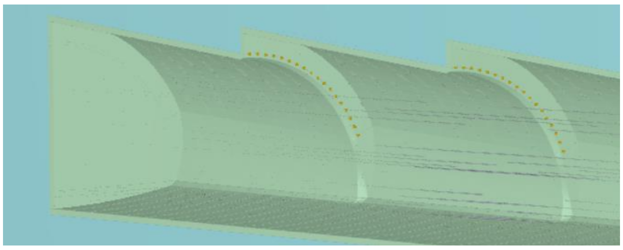

Figure 5. 3D model of top heading with temporary invert.

To acquire the parameters, four functions were fitted to numerically obtained displacement curves by using MS Excel solver. Table 2 shows the fitted displacement function parameters for analysed case. Figure 6 shows very good agreement for both incremental displacement and bending moment.

Table 1. Main properties of numerical model.

Parameter	Value
Average top heading radius	approx. 6 m
Overburden height	55 m
Shotcrete thickness	25 cm
Ground angle of internal friction	28°
Ground cohesion	35 kPa
Ground modulus $E_{50} = E_{\text{oed}}$	100 MPa
Pipe diameter	114 mm
Pipe bending stiffness	655 kNm^2

Table 2. Displacement function properties.

Parameter	Value
$C_{x\infty}$	9.88 cm
Q_1	0.26
K	46.3 m^{-1}
Fitting parameter X	2.46 m
Subgrade modulus	260 MN/m^3
Load p_L	-29.5 kN/m
Load p_M	43.6 kN/m
Load p_R	-50.9 kN/m
Load p_T	62.6 kN/m
Load area a_L	0.73 m
Load area a_M	0.73 m
Load area a_R	0.57 m
Load area a_T	1.90 m

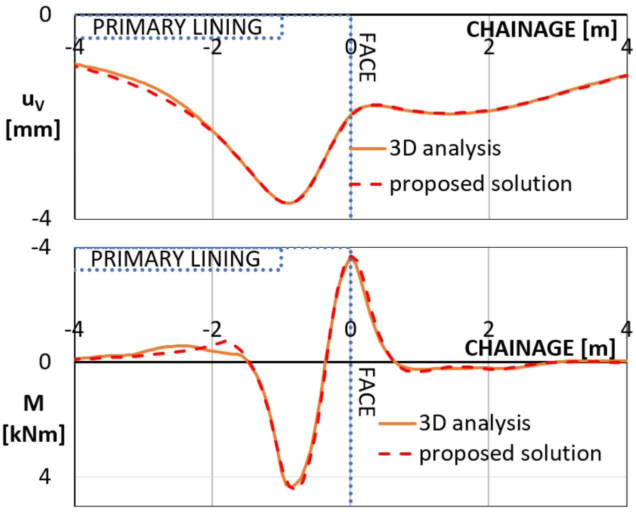

Figure 6. Incremental phase displacement (above) and resulting bending moment (below).

5 CONCLUSION

The paper presents an advanced method for predicting umbrella arch behaviour. For this purpose, the established displacement function has been modified. The proposed analytical solution for intermediate unsupported section behind the excavation face efficiently connects two original empirical displacement functions and provides such derivatives that perfectly fit the numerically obtained bending moments and shear forces within umbrella arch pipe.

Complex 3D numerical analyses are usually required to design an umbrella arch efficiently. The proposed solution is useful as complementary method during tunnel construction to make use of usually reliable displacement measurements and when calibration of sophisticated numerical models may not be a proper choice. The results of the analyses on a test case show that the proposed method can perform this task very well.

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