Proceedings of the XVIII ECSMGE 2024
GEOTECHNICAL ENGINEERING CHALLENGES
TO MEET CURRENT AND EMERGING NEEDS OF SOCIETY

ISBN 978-1-032-54816-6
DOI 10.1201/9781003431749-161

Application of the isotache law and the power creep law on prediction of long-term consolidation behaviour

Application de la loi isotache et de la loi de fluage à la prédiction du comportement de consolidation à long terme

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ABSTRACT: In this study, it was shown that the isotache law is essentially the same as the power creep law, but the formulations of these two are different. The model equations are applied to the results of the multi-stepwise constant rate of strain (CRS) consolidation test. It was clarified that the isotache law (or power creep law) is applicable for the test result. Strain rate dependency of two Osaka Bay clays (marine clays collected from the Kansai International Airport and the Yumeshima Island) was discussed through comparison of isotache parameters.

RÉSUMÉ: Dans cette étude, il a été démontré que la loi d'isotache est essentiellement la même que la loi de fluage, mais que les formulations de ces deux lois sont différentes. Les équations du modèle sont appliquées aux résultats de l'essai de consolidation à vitesse de déformation constante (CRS) en plusieurs étapes. Il a été précisé que la loi d'isotache (ou loi de fluage) peut s'appliquer aux résultats de l'essai. La dépendance de la vitesse de déformation de deux argiles de la baie d'Osaka (argiles marines prélevées à l'aéroport international du Kansai et sur l'île de Yumeshima) a été examinée en comparant les paramètres d'isotache.

Keywords: Long-term consolidation; strain rate dependency; isotache; creep.

1 INTRODUCTION

Over the past 70 years, many researchers have attempted to apply the isotache law, which expresses a unique relationship among stress-strain-strain rate, i.e., the strain rate dependency of compression curve, focusing on the effect of viscosity in long-term consolidation of clay.

In this study, first, it was shown that the isotache law is essentially the same as the power creep law, which is one of the classical mechanical models, but the formulations of these two are different. Based on the isotache law and the power creep law, model equations were derived, respectively. And then, the model equations are applied to the results of the multi-stepwise constant rate of strain (CRS) consolidation test. Finally, strain rate dependency of two Osaka Bay clays (marine clays collected from the Kansai International Airport and the Yumeshima Island) was discussed through comparison of isotache parameters used in the models, and calculated long-term consolidation settlement behaviour as well.

2 MODEL DESCRIPTION

The first author and his colleagues (Watabe et al. 2008; 2012; Watabe and Leroueil 2015) have conducted long-term consolidation tests and multi-stepwise CRS consolidation tests for various worldwide clays, including the marine clays at the Kansai International Airport, and have modelled the strain rate dependency in long-term consolidation behaviours. The strain rate dependency has been rheologically modelled as "isotache law", which is essentially the same approach as "power creep law in conjunction with overstress concept" in the classical mechanics, even though these are expressed by different equations. An outline of these models is described as follows.

The isotache law of compression curve in consolidation behaviour was first found by Šuklje (1957). In previous studies by the first author and his colleagues, the "isostache law" is rheologically modelled by finding the strain rate dependency of the consolidation yield stress by fitting the "shape" of the compression curve which is normalized by the consolidation yield stress σ'_p based on the method proposed by Leroueil et al. (1985)

The equations can be written in the form of visco-plastic strain ε_{vp} and described by Equations (1) – (3):

$$\varepsilon_{vp} = \varepsilon - \varepsilon_e \quad (1)$$

$$\sigma'/\sigma'_p = f(\varepsilon_{vp}) \quad (2)$$

$$\sigma'_p = g(\dot{\varepsilon}_{vp}) \quad (3)$$

Watabe et al. (2008) showed that the consolidation yield stress converges to the lower limit σ'_{pL} as the visco-plastic strain rate $\dot{\varepsilon}_{vp}$ decreases, and proposed Equation (4) for the strain rate dependency of the consolidation yield stress σ'_p:

$$\ln\frac{\sigma'_p-\sigma'_{pL}}{\sigma'_{pL}} = c_1 + c_2\ln\dot{\varepsilon}_{vp} \quad (4)$$

where c_1 and c_2 are constants, and together with the lower limit of consolidation yield stress, σ'_{pL}, are referred to as isotache parameters. Watabe et al. (2012) conducted experiments for a variety of clays and found that the values of the isotache parameter are almost common regardless of the clay type or condition. The parameters commonly applicable to many clays are $\sigma'_{pL}/\sigma'_{p0}$ = 0.70 and c_1 = 0.935, where σ'_{p0} is the consolidation yield stress corresponding to $\dot{\varepsilon}_{vp}$ = 1.0 × 10^{-7} s^{-1} that is almost equivalent to the strain rate at elapsed time of 24 hours in incremental loading oedometer test. Using these values, the parameter c_2 is automatically determined as in Equation (5):

$$c_2 = \frac{\ln\frac{\sigma'_p-\sigma'_{pL}}{\sigma'_{pL}}-c_1}{\ln\dot{\varepsilon}_{vp}} = \frac{\ln\frac{1-0.7}{0.7}-0.935}{\ln(1.0\times10^{-7})} \quad (5)$$

Equation (4) models the fact that the consolidation yield stress σ'_p converges to the lower limit σ'_{pL} when the strain rate becomes infinitesimal, but it can be easily transformed into an equation similar to the power creep law in conjunction with overstress concept expressed as Equation (6) (Watabe and Leroueil, 2015):

$$\dot{\varepsilon}_{vp} = c_3\left(\frac{\sigma'_p-\sigma'_{pL}}{\sigma'_{pL}}\right)^{c_4} \quad (6)$$

where c_3 = exp($-c_1/c_2$) and c_4 = $1/c_2$. Hereafter, calculations using this method will be referred to as "calculation with isotache law" evaluated by rheological approach.

On the other hand, the description of the power creep law in conjunction with overstress concept in classical mechanics is expressed as follows: Visco-plastic strain occurs when the present effective stress σ' is larger than the present yield stress σ'_y (which, as described later, corresponds to the lower limit of consolidation yield stress σ'_{pL} in the isotache law in the initial state). The model is based on the concept that creep deformation in consolidation will eventually terminate as the structure gradually becomes denser (harder), and the strain rate is expressed as Equation (7):

$$\dot{\varepsilon}_{vp} = a\left(\frac{\sigma'-\sigma'_y}{\sigma'_y}\right)^b \quad (7)$$

where *a* and *b* are constants.

Hereafter, calculations using this method will be referred to as "power creep law" based on the classical mechanical model. The isotache law and the power creep law, although formulations of these methods are different as described above, these two laws are essentially the same.

In the model equations above, the compression curves obtained from the tests can be used directly. However, as with constitutive models of soil in general, a compression curve expressed in strain is assumed as a bi-linear relationship expressed by the compression index λ_ε and the swelling index κ_ε. The parameters for the Pleistocene clay Ma9 collected at the construction site of the Kansai International Airport in Osaka Bay are shown in Table 1 as the representative values obtained in the previous studies, and used in the following calculations.

Table 1. Input parameters for calculation (c_3 and c_4 are calculated from c_1 and c_2, respectively).

	Kansai Airport	**Yumeshima Island**
$\sigma'_{pL}/\sigma'_{p0}$	0.70	0.70
σ'_{p0} (kPa)	1408	242
σ'_y (kPa)	980	169
c_3, a	2.127×10^{-4}	7.889×10^{-4}
c_4, b	9.043	7.873
σ'_{v0} (kPa)	1016	550
e_0	1.420	2.067
C_c (λ_ε)	1.000 (0.1790)	1.204 (0.1705)
C_s (κ_ε)	0.100 (0.0179)	0.126 (0.0179)

The simulation results of the multi-stepwise CRS consolidation test, in which the strain rate was varied stepwise by one order of magnitude sequentially for each 1% increment of strain in the range of 3.3 × 10^{-12} s^{-1} to 3.3 × 10^{-5} s^{-1} after the strain reached 5%, are shown in Figure 1. The results calculated based on the isotache law and the power creep law are superimposed on the same figure, but the results are coincident with each other and cannot be distinguished between them. Here, for the calculation with the power creep law, it was assumed that $\sigma'_y = \sigma'_{pL}$ in the initial state. That means that σ'_y corresponds to the consolidation yield stress of the compression curve with infinitesimal strain rate, considering the relation of $\sigma'_{pL}/\sigma'_{p0}$ (= 0.7). Even in the case of multi-stepwise CRS consolidation test, in which the strain rate is stepwise varied sequentially, isotache properties in strain rate dependency are consistent with the isotache compression curves.

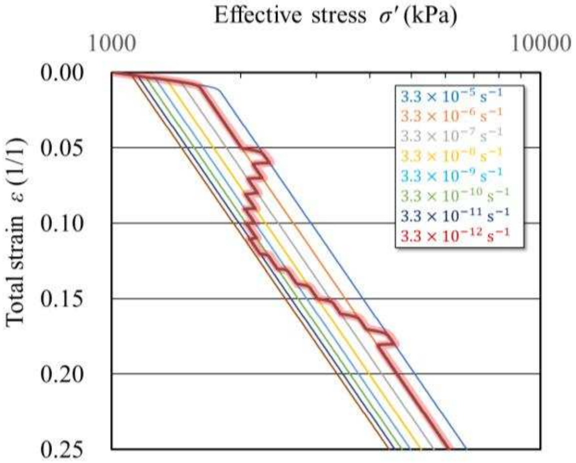

Figure 1. Calculation results for the strain rate dependency (isotache properties) of the compression curve for Ma9 collected from the Kansai International Airport.

3 MULTI-STEPWISE CRS CONSOLIDATION TEST

Constant rate of strain (CRS) consolidation test was conducted in the manner proposed by Watabe et al. (2019) for Pleistocene clay Ma9 collected from the Kansai International Airport in Osaka Bay under back pressure of 200 kPa. Consolidation pressure was increased to the effective overburden stress σ'_{v0} by applying a constant strain rate of 3.3×10^{-6} s^{-1}, then switched to constant stress control and preliminary consolidation under the effective overburden stress was conducted for 7 days, and then, a constant strain rate (3.3×10^{-6} s^{-1}) loading was applied. After the specimens were sufficiently yielded, the strain rate was stepwise varied sequentially from 3.3×10^{-5} s^{-1} to 3.3×10^{-8} s^{-1}, and the loading pattern was set so that the specimens would experience twice at each strain rate. Figure 2 shows the compression curve obtained by multi-stepwise CRS consolidation test after preliminary consolidation at the effective overburden stress. The vertical axis of the figure is the total strain, however, the value shown in the figure were obtained by offsetting at the completion of 7 days of preliminary consolidation at the effective overburden stress.

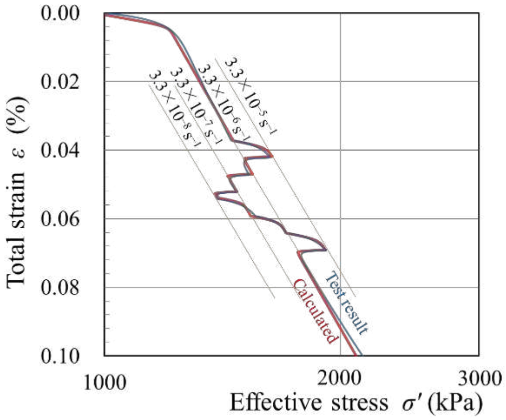

Figure 2. Comparison of test result and calculation result of the muti-stepwise strain-rate loading consolidation test for Ma9 collected from the Kansai International Airport.

To obtain a better fitting to the test results, the compression index C_c (or λ_ε) and the swelling index C_s (or κ_ε) were modified by a factor of 0.97 from the value shown in Table 1 and the value of parameter c_1 by a factor of 0.877. The strain rate dependency of the compression curves obtained as a test result can be well expressed by the isotache model.

Similar tests were conducted for Pleistocene clay Ma12 collected from the Yumeshima Island in Osaka Bay, and an example of the results is shown in Figure 3. The strain on the vertical axis is calculated from the initial specimen height, but is shown offset by the amount of strain at the elapsed time of 7 days in the preliminary consolidation under the effective overburden stress σ'_{v0}.

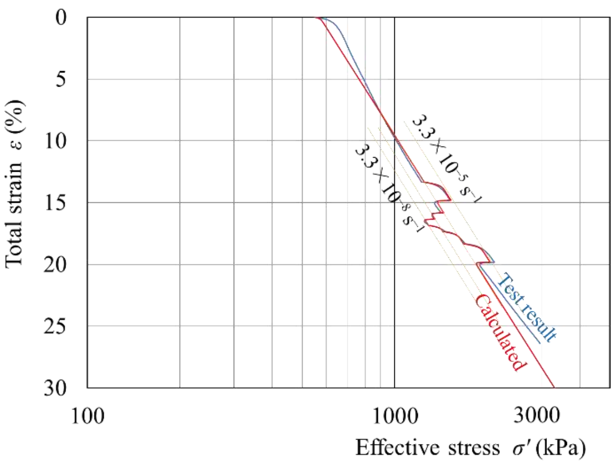

Figure 3. Comparison of test result and calculation result of the muti-stepwise strain-rate loading consolidation test for Ma12 collected from the Yumeshima Island.

The strain rate dependency of the compression curve is modelled and expressed by the isotache law using Equation (4) or Equation (5) (note here that it is the same as the power creep law using Equation (6)). The parameters used in the calculation are listed in Table 1. The isotache parameters were set to $\sigma'_{pL}/\sigma'_{p0}$ =

0.7 (from previous studies) and c_1 = 1.20 (by fitting) and c_2 = 0.127 (automatically calculated from both $\sigma'_{pL}/\sigma'_{p0}$ and c_1). Calculated curve corresponding to multi-stepwise CRS loading is well expressed the test results.

4 DISCUSSION

Because naturally deposited intact clay sample was examined in this study, the compression curve in the normal consolidation domain is slightly convex downward. In the calculation, the compression curve was represented by a bi-linear relationship with compression index λ_ε and swelling index κ_ε expressed in terms of strain, therefore, there are some deviations in the low and high consolidation pressure domains. However, the fitting was successful, especially for the portion of the multi-stepwise CRS loading (Figures 2 and 3).

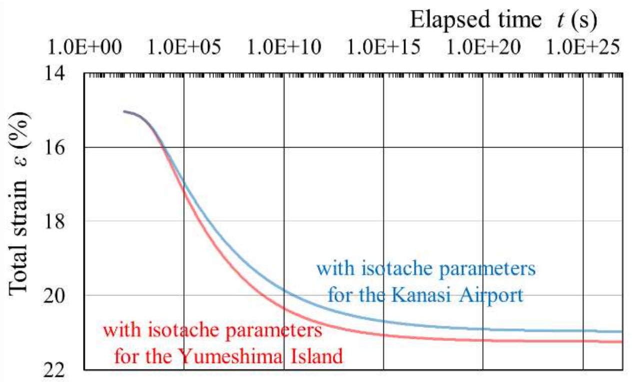

Figure 4. Calculated results of long-term consolidation settlement for the clay collected from the Yumeshma Island as a relationship between total strain and elapsed creep time.

Aiming to express all test results at each site using the same isotache parameters, $\sigma'_{pL}/\sigma'_{p0}$ = 0.7 (obtained from previous studies) was fixed, and c_1 = 1.20 (by fitting) and c_2 = 0.127 (automatically calculated) were obtained. According to a previous studies, the isotache parameters for marine clays at the Kansai International Airport are commonly determined with $\sigma'_{pL}/\sigma'_{p0}$ = 0.7, c_1 = 0.935, and c_2 = 0.111. The value of c_1 for marine clays at the Yumeshima Island is 28% larger than that of marine clays at the Kansai International Airport.

Figure 4 shows an example of calculated results showing how this difference affects the long-term consolidation behaviour. In these calculations, soil parameters obtained for the clay at the Yumeshima Island were used, but only isotache parameters were set respectively for each clay. In the figure, the data is plotted only in creep stage starting at a total strain of 15% against the elapsed creep time. It can be seen that secondary consolidation proceeds slightly faster in the Yumeshima clay than in the Kansai Airport clay, and that the final settlement is slightly larger.

5 SUMMARY

In this study, isotache properties in consolidation behaviour were evaluated based on isotache law, and power creep law as well. Even though the two laws are based on different approaches to express strain rate dependency in consolidation behaviour, these laws are essentially the same, which expresses strain rate dependency using a power law with overstress concept.

The strain rate dependency of consolidation behaviour was experimentally investigated by multi-stepwise CRS test for seabed clays in Osaka Bay. The strain rate dependency of the clay collected from the Yumeshima island showed isotache parameter c_1 of 1.20, which is slightly larger than c_1 of 0.935 for the clays collected from the Kansai International Airport.

REFERENCES

Leroueil, S., Kabbaj, M., Tavenas, F. and Bouchard, R. (1985). Stress-strain-strain rate relation for the compressibility of sensitive natural clays. *Géotechnique*, 35(2), 159–180. https://doi.org/10.1680/geot.1985.35.2.159.

Šuklje, L. (1957). The analysis of the consolidation process by the isotache method. *Proceedings of the 4th International Conference on Soil Mechanics and Foundation Engineering*, Vol. 1, Butterworths, London, pp. 200–206.

Watabe, Y. and Leroueil, S. (2015). Modelling and implementation of isotache concept for long-term consolidation behavior. *International Journal of Geomechanics*, ASCE, 15(5), A4014006. http://dx.doi.org/10.1061/(ASCE)GM.1943-5622.0000270.

Watabe, Y., Udaka, K. and Morikawa, Y. (2008). Strain rate effect on long-term consolidation of Osaka bay clay, *Soils and Foundations*, 48(4), 495–509. http://dx.doi.org/10.3208/sandf.48.495.

Watabe, Y., Udaka, K., Nakatani, Y. and Leroueil, S. (2012). Long-term consolidation behavior interpreted with isotache concept for worldwide clays. *Soils and Foundations*, 52(3), 449–464. http://dx.doi.org/10.1016/j.sandf.2012.05.005.

Watabe, Y., Yoneda, A., Hashizume, H. and Ono, Y. (2019): Multi-stepwise strain-rate loading consolidation test to evaluate strain rate dependency, *Proceedings of Geotec Hanoi 2019*, *Geotechnics for Sustainable Infrastructure Development*, Springer Singapore, pp.1253–1258.