

Uncovering soft soils with 3D Empirical Bayesian Kriging: A case study on cone penetration test data

Révélation des sols meubles avec la Krigeage Bayésien Empirique en 3D: Une étude de cas sur les données des essais de pénétration au cône

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ABSTRACT: The present work investigates the coexistence of soft and very soft soils in the foundation of a dam and aims to create a 3D map of excess pore pressure using the 3D Empirical Bayesian Kriging geostatistical method. A previous study had revealed that the only distinctive property in these residual soils is the generation of excess pore pressure during shear, which varies significantly depending on the weathering process of the parent rock, and so the pore-pressure parameter is the main feature here. The purpose of the study is not only to demonstrate that zoning the foundation of the dam is possible, but also to clarify that this zoning is more than a simply geometric delimitation of geotechnical units with previously defined deterministic properties. The values of the geotechnical properties of the equivalent soil grouped in a unit, regardless of the zoning method adopted, must represent the partial contributions of each of the soils gathered there, in variable and uncertain proportions, but estimable with some level of confidence. The study concludes that geostatistical methods are useful tools for creating balanced and reliable geotechnical models, particularly when the values of the target properties vary drastically. Overall, the study highlights the importance of accurately representing the properties of soils in geotechnical models to ensure safe and reliable engineering structures.

RÉSUMÉ: Le travail présent étudie la coexistence de sols mous et très mous dans les fondations d'un barrage et vise à créer une carte 3D de la pression de pore excessive à l'aide de la méthode géostatistique de Krigeage Bayésien Empirique en 3D. Une étude précédente avait révélé que la seule propriété distinctive de ces sols résiduels est la génération de pression de pore excessive lors du cisaillement, qui varie significativement en fonction du processus d'altération de la roche mère, entre autres, le paramètre de pression de pore étant ici la caractéristique principale. L'objectif de l'étude n'est pas seulement de démontrer que la zonification des fondations du barrage est possible, mais aussi de préciser que cette zonification va au-delà de la simple délimitation géométrique des unités géotechniques avec des propriétés déterministes préalablement définies. Les valeurs des propriétés géotechniques du sol équivalent regroupées dans une unité, quel que soit la méthode de zonage adoptée, doivent représenter les contributions partielles de chacun des sols regroupés là-bas, dans des proportions variables et incertaines, mais estimables avec un certain niveau de confiance. L'étude conclut que les méthodes géostatistiques sont des outils utiles pour créer des modèles géotechniques équilibrés et fiables, en particulier lorsque les valeurs des propriétés cibles varient considérablement. Globalement, l'étude met en évidence l'importance de représenter avec précision les propriétés des sols dans les modèles géotechniques pour garantir des structures d'ingénierie sûres et fiables.

Keywords: 3D empirical bayesian kriging; cone penetration test; pore pressure parameter; tropical residual soils.

1 INTRODUCTION

The characterization of a material in terms of its strength and deformability is fundamental for any geotechnical project, and equally important is the accurate spatial definition of each material. Incorrectly placing a more resistant material in a potential failure zone could create a false sense of security.

Defining boundaries between materials, such as soil, which exhibit significant spatial variability, is a challenging task fraught with uncertainties. Determining what cannot be seen based on pointwise data requires the interpreter's discernment and some level of confidence (or skepticism) in the models being produced. All models, be they geological or otherwise, are inherently imperfect due to simplifications or

assumptions. It is the responsibility of the technician to determine the degree of error that can be tolerated.

When creating a model, it is of utmost importance to evaluate its implications for the real-world problem. For example, what would happen if a material extended a few meters more than initially assumed or if it turned out to be deeper than anticipated? Could it create a plane of weakness or affect the percolation regime? Whenever a model is presented, it should ideally be accompanied by a map of uncertainties, allowing engineers to account for less favourable scenarios.

This study focuses on a tailings dam located in the Quadrilátero Ferrífero, an iron ore-rich region in Minas Gerais, southwestern Brazil. The dam's foundation comprises two residual soils of dolomitic phyllite, with one significantly softer than the other. This very soft soil exhibits low tip resistance and a high pore pressure ratio, as determined through Cone Penetration Tests (CPTu) detailed in (Viegas et al., 2023). The extent and location of this very soft soil critically affect the dam's factor of safety (FS) since the critical failure surface may intersect this material.

The primary objective of this study is to map this very soft residual soil by interpolating the pore-pressure ratio (B_q) data obtained from CPTu surveys. The aim is to demonstrate how the material's strength, situated on a potential failure surface, can be quantified. For a better understanding, imagine a failure surface passing through a specific area of the foundation and encompassing a certain mass of soil. The goal would be to define, with some confidence and a few parameters, the strength that would be available on average along this surface. Considering a slope stability model discretized into slices, one would be looking for the available resistance at the base of each slice, considering the possibility of stress redistribution between elements (meaning that the material exhibits some ductility).

2 METHODOLOGY

In this section, we describe the methodology employed in this study to map the very soft residual soil within the context of the tailings dam in the Quadrilátero Ferrífero. We utilized the pore-pressure ratio parameter, B_q , which follows a skewed normal distribution and exhibits non-stationarity.

To address the challenges posed by limited data and the absence of stationarity, we applied the geostatistical method known as Empirical Bayesian Kriging (EBK). EBK combines the principles of kriging and Bayesian statistics, and it assumes that the study variable follows a Gaussian distribution. Hence,

since the distribution of the data was skewed, a Gaussian transformation was applied. It generates priors and posteriors distributions, providing more accurate estimates than classical kriging methods and quantifying uncertainty in predictions. EBK represents the stochastic spatial process as a random field, allowing parameters to vary across space, crucial for materials with significant spatial variability, as is often the case with geological materials. For readers interested in a more detailed exploration of the EBK method, we recommend referring to the works of Gribov and Krivoruchko (2020) and Krivoruchko and Gribov (2019).

The model presented here was created using the implementation of EBK 3D in ArcGIS Pro 3.0, and several sensitivity tests were performed to determine the most suitable semivariogram and calibrate model parameters such as the subset size, the number of simulations, the overlap factor and the semivariogram type.

3 RESULTS

The final model is presented in Figure 1, in which the pore-pressure parameter, B_q is displayed with a colour scale, where yellow represents high values of B_q , correlated to the very soft residual soil of dolomitic phyllite and confirmed by the boreholes in that area. The blue colour represents low values of B_q . To evaluate the model, a cross-validation was performed, where the observed data were compared against the calculated data. The Average Continuous Ranked Probability Score (CRPS) is low, with a value of 0.006, the Root-Mean-Square (RMS) and an average standard error are 0.023, reflecting good data reproduction. The Root-Mean-Square Standardized (RMSE) is 0.94, suggesting slight overfitting. The results presented here could be as diverse as possible, depending on the choice of modelling parameters; however, this study serves to illustrate an approach to model soil properties rather than specifically create a terrain model.

Isolating the material with $0.15 \leq B_q \leq 0.29$ (Figure 2) shows the material generating these pore-pressure intervals is concentrated on the right side but not exclusively there, with minor presence in the rest of the model. How do we create the geotechnical model to be used for stability analysis? If we were to create discrete zones, or 'balls' of materials, there would be a risk of misplacing these zones or overlooking their true dimensions. So, the question arises: How can we construct a geotechnical model that accounts for the geological uncertainty? It is not feasible to assert that

where there is residual soil, there is no presence of very soft residual soil; these materials could coexist.

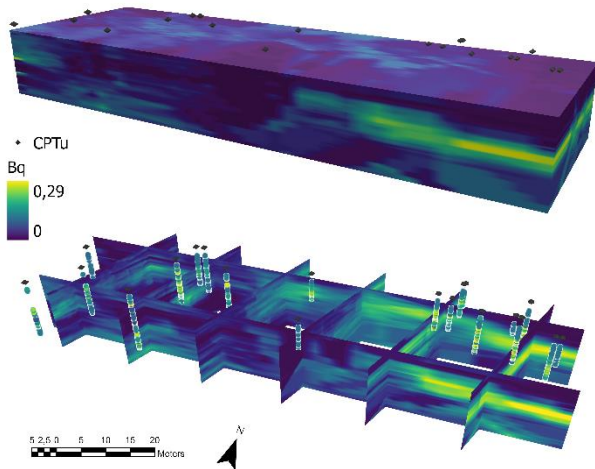


Figure 1. Recovered Model: 3D model of the subsurface colored by the pore-pressure parameter, B_q , where yellow represents high values of B_q , and blue low values.

Creating a model with distinct delineations for these materials poses a significant challenge. Our approach revolves around defining a criterion that permits the coexistence of both materials in varying proportions simultaneously. This can be achieved using a mean parameter and its associated standard deviation.

Another possible approach would be to calculate a confidence interval for the pore-pressure parameter and its uncertainty, but would be necessary to perform several simulations, applying for example Direct Sequential Simulation (DSS) as described in Soares, A. (2001), which falls outside the scope of this study.

In practice, it is more useful to have an average parameter associated with a standard deviation. Therefore, the mean value and standard deviation of each pixel along the cross-section were calculated, resulting in a mean section and a standard deviation section. The same procedure was repeated in the longitudinal direction. Figure 3 shows the mean cross-section and longitudinal section at a depth of 12.07 m, which was chosen due to its intersection with the high pore-pressure zone (yellow in the figure). Figures 3a) and 3b) depict the mean (the mean of the values of each pixel in the interpolated cube) longitudinal and cross-section, respectively. A black dashed line used for comparison at a depth of 12.07 m is also shown. Figures 3c) and 3d) illustrate the variation in the mean value of the parameter B_q along this line in each of the directions. Notice a higher variability of the mean in the cross-section.

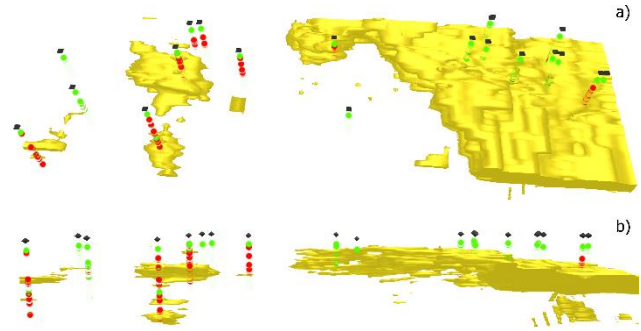


Figure 2. Mass of soil with $0.15 \leq B_q \leq 0.29$. The black dots represent the available CPTu. The CPTu data are plotted in terms of pore-pressure ratio, B_q , where the green points have $B_q < 0.15$, and the red points have $B_q \geq 0.15$.

Similarly, Figure 4 presents the associated standard deviation. In the cross-section, the area with high pore-pressure is sharp and clear, while in the longitudinal section, it appears blurry, reflecting the higher standard deviation in this direction. Figure 5 shows the mean value of the B_q parameter in black and a cautious value in red, calculated by adding half of the standard deviation to the mean, for a depth of 12.07 meters in both sections.

Since, as illustrated in Figure 1, we cannot isolate a specific area where the very soft soil exists, nor can we disregard its coexistence with other materials, one possible approach to addressing this weaker soil in the geotechnical model for obtaining the dam's safety factor may involve considering that this material represents a certain percentage of the total soil mass.

Considering that B_q is the parameter describing the excess of pore-pressure during CPTu, and A_f is the parameter describing excess of pore-pressure at failure in a triaxial test, even though a theoretical relationship between them has not been established, we can say that, albeit distinct, both parameters measure the response of the same soil when subjected to rapid loading.

Assuming it was possible to carefully separate the results of the triaxial tests into two groups: one corresponding to residual soil and the other to very soft residual soil, and that both soils appear in space without explanation (randomly). Also, assuming that in any failure surface that cuts through the soil mass, the available resistance there, on average, results from the spatial weighting of the occurrence of one or the other soil simultaneously along that surface. For illustrative purposes, if $B_{q,maximum} = 0.29$ corresponds to the group of samples (CIUC tests) of the very soft residual soil and $B_{q,minimum} = -0.11$ is equivalent to the group of samples (also from CIUC tests) of residual

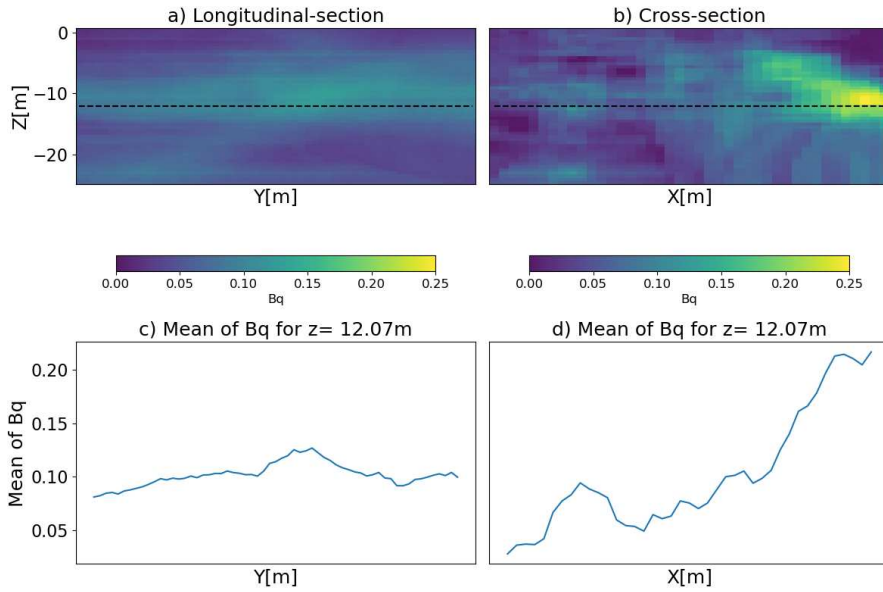


Figure 3. Mean value of B_q for the longitudinal and cross-section at the depth of $z=12.07m$.

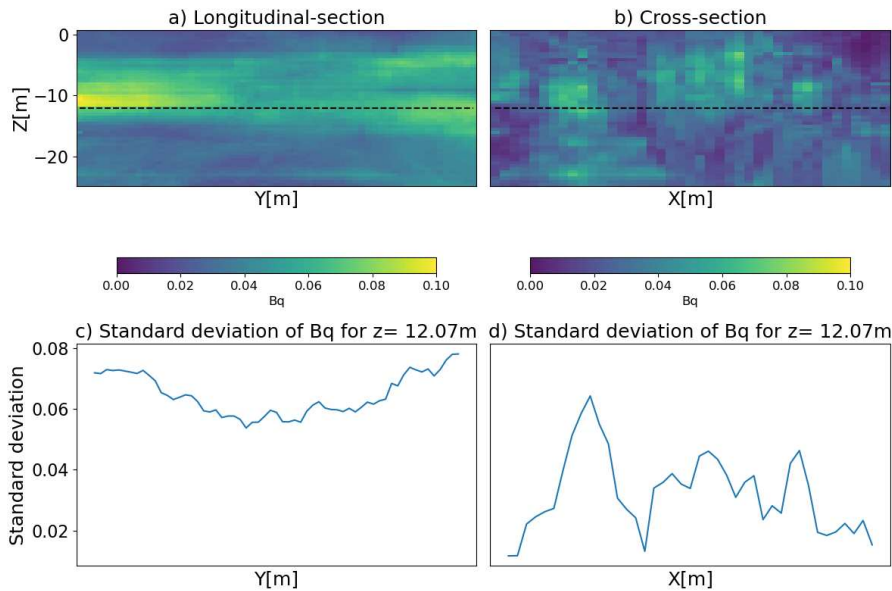


Figure 4. Standard deviation of B_q for the longitudinal and cross-section at the depth of $z=12.07m$.

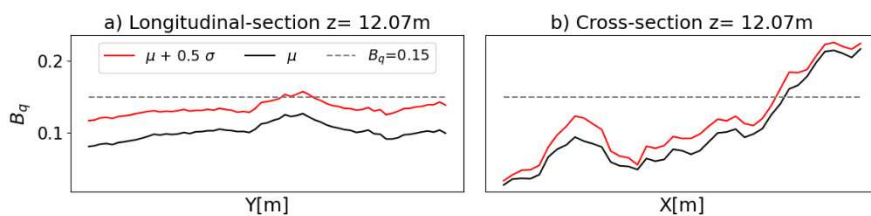


Figure 5. Mean and cautious value of B_q . In black, the mean value of the parameter B_q is presented, as shown in Figure 3, and in red, the mean value plus half of the standard deviation, with a reference to a depth of 12.07 meters.

soil, we can use random fields (Baecher and Christian, 2005; Griffiths and Fenton, 2007) to infer that any property X (such as undrained shear strength) related to the generation of excess pore pressure will be:

$$\begin{aligned} \bar{X}(\bar{B}_q) &= \alpha \cdot X(B_{q,minimum}) + \\ &(1 - \alpha) \cdot X(B_{q,maximum}) \approx \\ \bar{X}(\alpha \cdot B_{q,minimum} + (1 - \alpha) \cdot B_{q,maximum}) \end{aligned} \quad (1)$$

that is:

$$\alpha = \frac{B_{q,maximum} - \bar{B}_q}{B_{q,maximum} - B_{q,minimum}} \quad (2)$$

In the equations above, \bar{B}_q represents a cautious estimation of the mean value of B_q . Returning to the example, with $\bar{B}_q = 0.15$, we would have $\alpha \approx 0.35$. This would mean that in a simplified model where the entire foundation of residual soil is treated as a homogeneous and isotropic unit of an equivalent soil, the value of the variable \bar{X} would correspond to a combination of 35% of residual soil with $X(B_{q,min})$ and 65% of very soft residual soil with $X(B_{q,max})$. Regardless of the theoretical ambiguity, this example serves to illustrate that:

- In summary, any attempt at zoning the foundation, including its absence, is valid, depending solely on the representativeness of the values of the parameters assigned to each zone.
- Zoning the foundation is entirely legitimate when, as in this example, one can conclude that there is a region in space with markedly distinct properties and dimensions capable of influencing the results of stability analyses. Otherwise, we would fall into excessive pessimism in the general case and dangerous optimism in the specific case.
- Selective and biased sampling, if not subsequently subjected to any form of weighting, will likely result in a biased perception of the values of soil properties available on average in each zone of the terrain. This weighting should and can be done in field tests, considering the volumes of soil involved in the failure mechanism we are interested in analysing.
- Unless there is an explanation for the occurrence of very soft soils within residual soils, and it is demonstrated that very soft soils occur, without a known pattern, with greater or lesser frequency in the upper alteration horizon of the massif, it is not simply credible and immediately prudent to reduce zoning to the

distinction between two extreme soils, one very good and the other very bad (the case of $\alpha = 0$ or $\alpha = 1$).

4 CONCLUSIONS

By applying EBK in this context, we aim to provide a robust and data-driven method for mapping and characterizing the very soft residual soil, thereby enhancing our understanding of its influence on the safety and stability of the tailings dam. This methodology allows us to not only estimate material properties but also assess the associated uncertainties, which is essential for informed decision-making in geotechnical projects.

The pore-pressure parameter proved effective in distinguishing residual soil and the very soft residual soil. Based on known values, a geostatistical method was adopted to predict the parameter across the entire study area. This type of interpolation is widely used and is applied in geological modelling commercial software. Despite being an extremely powerful tool, it has limitations, highlighting the need for model weighting by the interpreter and its association with the uncertainty. This method allows us to have an idea of the areas where we could find soils that generate high pore-pressure values. It is not prudent to deny the coexistence of the residual soil within the very soft residual soil, as it has been observed that, even in a smaller proportion, the very soft residual soil may appear across the entire analysed extent.

However, the interpolation result suggests the existence of a zone where the very soft soil concentrates (zone on the right in Figure 1), which could be explicitly considered in the geotechnical model, thus allowing the formation of mechanisms that pass through this zone. In this case, the definition of the B_q parameter representative of the mass of the very soft soil (where, in addition to the very soft soil, soils with different B_q values and consequently different strengths are involved) could be determined as a value lower than the lowest one found in this mass. On the other hand, if the decision is made not to explicitly specify this zone and to model it as a single equivalent soil, the ability of the geotechnical model to influence the formation of failure mechanisms would be lost. However, for the same mechanism, if the resistance weighting in this zone is cautious, it is possible to arrive at the same safety factor, since on average, the available resistance would be similar.

It is not possible to assert that there are no small contributions of the very soft residual soil within any region where the more consistent residual soil may be predominant (zone on the left in Figure 1). In the

absence of an explanation for the spatial occurrence of the very soft soil and given the spatial variability of the chosen variable (B_q), it is possible to make some judgment about the spatial contribution of each of the materials at different stages of alteration. In this study, it is assumed that softer soils may randomly appear within other soils and coexist with them. The study allows for rationalizing the choice of resistance parameters in most of the foundation, where it is not possible to discern a geometric shape that encompasses soils with specific properties, evaluating in what proportion each soil appears. This weighting is purely statistical and does not allow determining how zones with different incidences of softer soils respond physically.

Despite all the limitations of statistical methods, without their application, it would not be possible to understand how to spatially weigh the properties of each soil and choose parameters based on this weighting. In the specific case under study, one could choose to specify the mass of very soft soil evident in the zone on the right of Figure 1 and homogenize the soil on the left, characterizing it with an equivalent

resistance parameter, calculated cautiously based on the proportion of each material.

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