

Assessment of stiffness parameters of stone columns

Évaluation des paramètres de rigidité des colonnes ballastées

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ABSTRACT: This paper presents theoretical formulations for determining stiffness parameters of stone columns that can be evaluated from the results of load tests, such as: Young's moduli, confined moduli (oedometric), triaxial moduli and moduli of subgrade reaction. Theoretical examples and real load test measurements are presented, where different stiffness parameters of stone columns are compared, and recommendations are provided for proper characterization of these stiffness parameters, aimed to facilitate their use in numerical calculations with advanced constitutive models, as well as for simplified analytical calculations. As conclusions, based on these theoretical assessments and the experience, it is indicated that regardless of the installation method, the moduli of stone columns should be evaluated under triaxial conditions and adopting values from 20 to 100 MPa, approximately. Moduli higher than 120 MPa may imply risky overestimations.

RÉSUMÉ: Cet article présente des formulations théoriques pour déterminer les paramètres de rigidité des colonnes ballastées qui peuvent être évalués à partir des résultats d'essais de charge, tels que: les modules de Young, les modules confinés (œdométriques), les modules triaxiaux et les modules de réaction du sol de fondation. Des exemples théoriques et des mesures d'essais de charge réelles sont présentés, où différents paramètres de rigidité des colonnes ballastées sont comparés, et des recommandations sont fournies pour une caractérisation appropriée de ces paramètres de rigidité, visant à faciliter leur utilisation dans les calculs numériques avec des modèles constitutifs avancés, ainsi que pour des calculs analytiques. En conclusion, sur la base de ces évaluations théoriques et de l'expérience, il est indiqué que quelle que soit la méthode d'installation, les modules des colonnes ballastées doivent être évalués dans des conditions triaxiales et en adoptant des valeurs de 20 à 100 MPa environ. Des modules supérieurs à 120 MPa peuvent impliquer des surestimations risquées.

Keywords: Stone columns; triaxial modulus; load tests; settlements.

1 INTRODUCTION

Stone columns installed by deep vibrators have been used successfully in numerous of projects since the 1960s to reinforce soft or loose soil with high compressibility, speed up the consolidation process, decrease settlement, increase bearing capacity, and mitigate soil liquefaction. This deep vibro technique has a specific European Standard (EN 14731:2005, Ground treatment by deep vibration).

The so-called bottom-feed method consists of gravel insertion into the ground through a system of tubes attached to a vibrator coupled at the bottom. The whole rod is introduced into the ground aided by lateral displacement, vibration, vertical pull-down force and compressed air. Once the required depth is reached, the tubes stabilize the cavity and allow the gravel feeding. The general motions of deep vibrator

compact the gravel by successive lowering and uplifting moves until the column is finished (Figure 1a). Other methodologies and equipment are also available, like the Rammed Aggregate Piers (RAP) firstly appeared during the 1990s. Figure 1b describes this method, which consists of extraction drilling and gravel feeding from the working platform. The compaction is only carried out vertically, through tamping by impact produced by a plate in contact with the gravel. The stability of the drilled cavity is highly important to implement this method, especially when the water table or very soft soil are found.

Attempts are usually made to evaluate the stiffness of stone columns by means of relatively quick plate load tests, where the induced settlements produced by the load increments are measured. Based on such results, the first approach to evaluate the column stiffness is the modulus of subgrade reaction (k_{col}),

defined as the ratio of the applied stresses to the measured settlements. Nevertheless, the determination of column moduli (stress - strain relationship) is a more complex procedure, strongly affected by the boundary conditions and loading procedure.

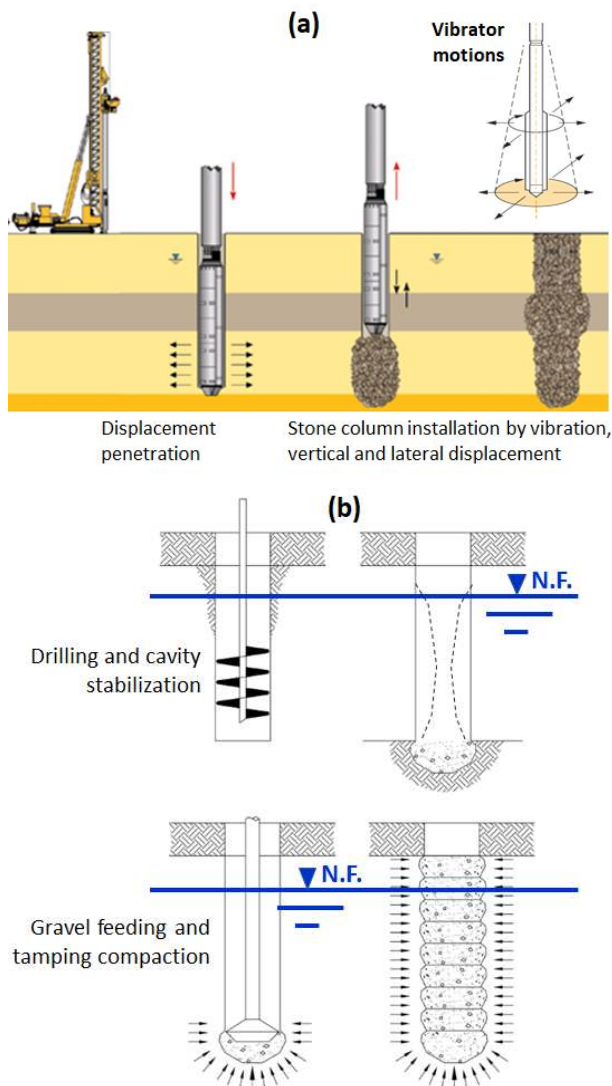


Figure 1. Methods: (a) vibro stone columns by bottom-feed; (b) Rammed Aggregate Pier with top-feed.

2 STIFFNESS EVALUATION

A range of moduli between 170 and 300 MPa is sometimes erroneously assumed for the stone columns. This assumption attributes abnormally high values for the moduli of stone columns, which is not in accordance with the actual values reported in the literature and practitioners' experience.

To properly assess the modulus of the stone columns, loading and boundary conditions must be correctly assumed. It is highly recommended to take reference values of moduli from the literature or conduct triaxial laboratory tests.

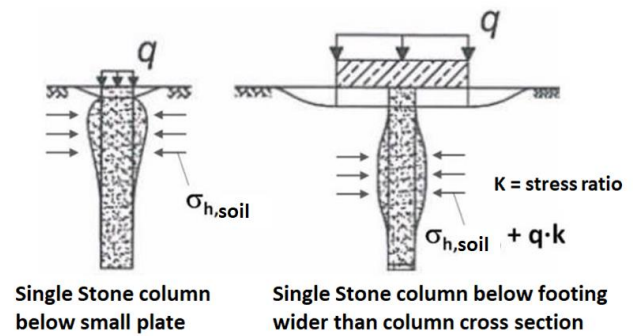


Figure 2. The triaxial conditions of stone columns below foundations and plate load tests.

Indeed, triaxial conditions correspond to the most representative state that stone columns undergo to support the foundation loading. Figure 2 shows the compression triaxial state usually produced in stone columns and surrounding soil under a foundation structure (similar to laboratory triaxial tests). Stone columns mainly deform through lateral bulging when subjected to compressive loading in triaxial conditions and are supported by the horizontal confining stress exerted by the surrounding soil. This behaviour has been demonstrated and extensively described by several researchers (Priebe, 1976; Priebe, 1995; Barksdale and Bachus, 1983; Goughnour and Bayuk, 1979; Wehr, 2004; Kirsch and Kirsch, 2010; Greenwood and Kirsch, 1983; Hu, 1995; Castro and Sagaseta, 2009).

The most common range of moduli values of stone columns in triaxial conditions varies from 20 to 100 MPa, approximately, according to many researchers who carried out comprehensive analyses by laboratory testing, field measurements, and numerical modelling (Han and Ye, 2001; Balaam and Booker, 1981; Castro, 2017; Bachus and Barksdale, 1983; Kirsch and Kirsch, 2010). The lowest moduli are obtained in the softest soils which offer the lowest confining stresses. It is worth mentioning that instead of the stone column stiffness, its friction angle and its diameter (i.e. the area replacement ratio) are the parameters with the largest impact on the improvement achieved with the treatment (Herle et al., 2007; Priebe, 1995).

Regarding the friction angle of stone columns, it is particularly interesting the investigation carried out by Duncan et al. (2007) about the same kind of material used for the stone columns (sizes 10–40 mm, approx.), which shows that moderately dense to very dense gravel may have a ϕ_{col} of approximately 55° for a very reduced level of lateral confining stress, of approximately 28 kPa (possibly in layers located at a depth of between 1.5 and 2.5 m); however, the same gravel with the same level of relative density experiences a significant decrease of ϕ_{col} to values between 40° and 47° , when the pressure is greater than

approximately 80 kPa (i.e., at depths of more than 4.5 to 7.0 m, approx.).

Figure 3 shows the degradation law of ϕ_{col} with the confining pressure, in accordance with the study by Duncan et al. (2007). It is particularly important to assess the risks that may arise if the design ϕ_{col} value is erroneously interpreted regarding the level of confining pressure to which they would be subjected, which not only depends on the initial stress state of the soil, but also on the stress state to which the soil and columns will be subjected due to the application of foundation loading.

Although the modulus in triaxial conditions (E_{TX}) is the most representative and widely used module to characterize stone columns, the subgrade reaction modulus, defined as the ratio between the vertical pressure applied on the foundation and the resulting settlement ($k_{col} = \Delta\sigma/s$), with density units (kN/m^3 , t/m^3), is also usually considered.

Thus, based on the vertical pressures ($\Delta\sigma$) and the resulting settlement (s), it is possible to determine the subgrade reaction modulus of the columns (k_{col}) and the soil (k_s) by means of loading tests using small plates (0.30 to 1.0 m). For that reason, and because it is an affordable kind of test regarding execution times and costs, the execution of these plate loading tests is considered as an attempt to assess the stiffness of stone columns, or more precisely, of the soil-column system.

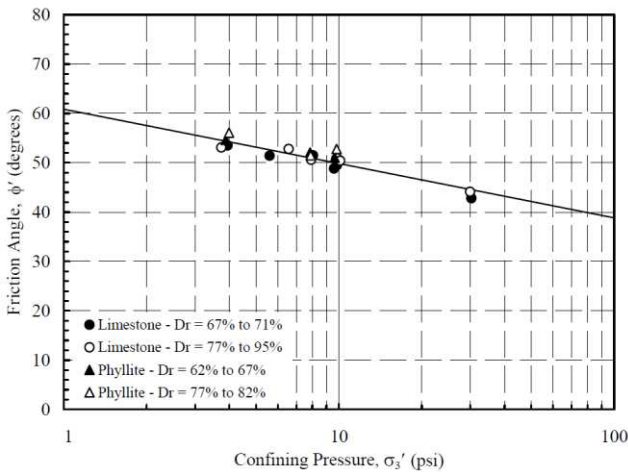


Figure 3. Duncan et al. (2007) investigation on internal friction angles on coarse aggregate type AASHTO #57.

However, these tests involve a number of limitations that complicate the proper assessment of column stiffness, such as: (i) small load plate tests only reach small depths (0.6 to 1.5 m); (ii) plate diameters usually only cover the column diameter, which limits the analysis of the real columns behaviour below the foundations which have greater area than the columns, Figure 2; (iii) plate tests are performed by fast loading and unloading procedures to complete the test in a few

hours (e.g. NLT-357/98 or ASTM D-1143 Standards), therefore, the results mainly correspond to undrained conditions, which do not properly reproduce the elastic-plastic strains that govern the behaviour of stone columns and surrounding soil in the long-term; (iv) appropriate calculation procedures must be implemented to convert the measured reaction modulus (k_{col}) into the triaxial modulus (E_{TX}), as described in following sections.

Based on loading tests with a similar diameter to that of the stone columns, several researchers have compiled the "k_{col}" values, proposing empirical correlations between "k_{col}" and SPT blow-count, without the need of performing load plate tests. Figure 4 shows the correlation proposed by Sehn and Blackburn (2008), with recommended reaction modulus values for stone columns, considering both granular and cohesive soils around the columns.

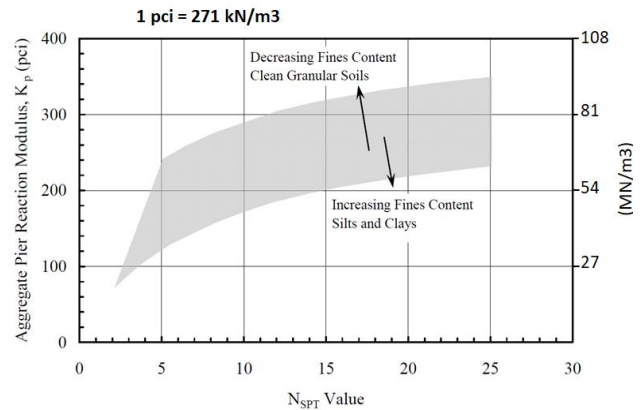


Figure 4. Correlation proposed by Sehn and Blackburn (2008) to estimate the reaction modulus for stone columns.

2.1 Stiffness formulation of stone columns

From the elasticity theory, a very simplified approach to estimate the settlement for a given soil element could be considered through Eq. (1):

$$S = \frac{\sigma_v}{E} \cdot L \quad (1)$$

where S is the settlement, E is the Young modulus of the column, L is the length of the considered element and σ_v is the axial stress applied.

Using the definition of reaction modulus (k_{col}) of a given soil element according to Eq. (2), it can be obtained a relationship between E and k_{col} :

$$k_{col} = \frac{\sigma_v}{S} = \frac{\sigma_v}{\frac{\sigma_v}{E} \cdot L} = \frac{E}{L}; E = k_{col} \cdot L \quad (2)$$

Then, under the simplified assumption that the induced stress during a field loading test has maximum value at the top of the element L, and reaches zero

stress at the bottom of the element, it can be considered that the average induced stress in the element is half of the applied stress at the top ($\sigma_{v,average} = \sigma_v / 2$). Thus, replacing σ_v by $\sigma_{v,average}$ in Eq. (2) the Eq. (3) can be obtained which represents a simplified relationship between the Young modulus (E) and the reaction modulus to evaluate the results from plate loading tests.

$$E = \frac{k_{col} \cdot L}{2} \quad (3)$$

More precisely, Boussinesq presented the closed-form solution to determine the settlement (S) under a rigid circular plate subjected to uniform load on an elastic and isotropic semi-space, as shown in Eq. (4).

$$S = \sigma_v \cdot R \cdot \frac{(1-\nu^2)}{E} \cdot I_a \quad (4)$$

where:

R: foundation radius

I_a : influence factor (in this case: $I_a = \pi / 2$)

σ_v : vertical stress

E: Young modulus ($E = \sigma_v / \varepsilon_v$; for $\sigma_h = 0$)

ν : Poisson's ratio ($\nu = \varepsilon_h / \varepsilon_v$)

From equation (4) the reaction modulus (k_{col}) can be determined as described in Eq. (5):

$$k_{col} = \frac{\sigma_v}{S} = \frac{E}{(1-\nu^2)} \cdot \left(\frac{2}{\pi}\right) \cdot \left(\frac{1}{R}\right) \quad (5)$$

Eq. (6) shows the relationship between Young's modulus (E) and the reaction modulus (k_{col}):

$$E = k_{col} \cdot (1 - \nu^2) \cdot R \cdot \left(\frac{\pi}{2}\right) \quad (6)$$

Finally, looking back to the relationship between the oedometric and Young moduli (E_{oed} / E) based on Poisson's ratio (ν), depicted in Eq. (7), it is interesting to estimate the oedometric modulus both based on the simplified modulus (8) and based on the modulus derived from the Boussinesq's solution for rigid circular plate (9).

$$E_{oed} = E \cdot \left[\frac{1-\nu}{(1-2\nu)(1+\nu)} \right] \quad (7)$$

$$E_{oed} = \frac{k_{col} \cdot L}{2} \cdot \left[\frac{1-\nu}{(1-2\nu)(1+\nu)} \right] \quad (8)$$

$$E_{oed} = k_{col} \cdot R \cdot \left(\frac{\pi}{2}\right) \cdot \left[\frac{(1-\nu)(1-\nu^2)}{(1-2\nu)(1+\nu)} \right] \quad (9)$$

The relationships between k_{col} , E_{oed} and E from the Boussinesq's solution (Equations 6 and 9) include the implicit consideration of the influence depth of the foundation load, which may correspond to the plate load tests on stone columns in triaxial conditions and subjected to the confining stress of the surrounding soil. This implies a remarkable advantage over equations obtained from the simplified approach, because it allows the estimation of the moduli without the need of assigning arbitrary values to the original length of the soil element that needs to be studied (L), as is required with equations (3) and (8). Figure 5 shows the depth affected by the loads according to the Boussinesq's solution for homogeneous semi-space (one layer) and also shows the solution of Poulos and Davis (1974) for a bilayer model.

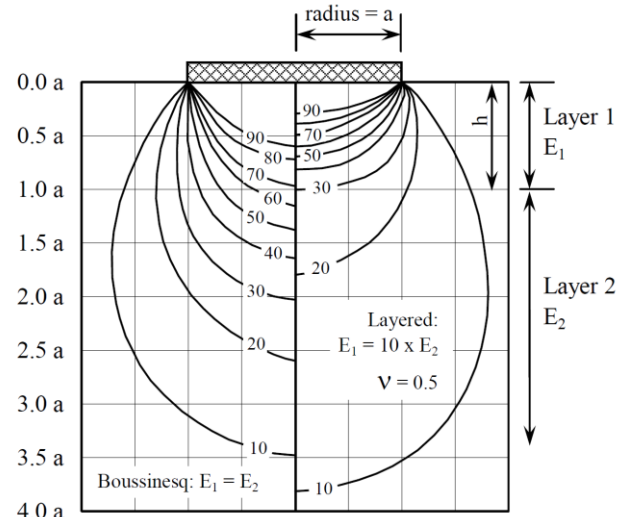


Figure 5. Influence depth of loaded rigid circular plate: (left) Boussinesq solution for homogeneous soil model; (right) Poulos and Davis (1974) for a bilayer model.

2.2 Example of moduli estimation

An example of moduli estimate for stone columns is shown, considering the following input data:

- Bearing plate diameter of 60 cm.
- Poisson's ratio of $\nu = 0.45$ for compacted gravel (Priebe, 1995).
- Reaction modulus, $k_{col} = 55 \text{ MN/m}^3$. According to Figure 4, it could match a soil with SPT = 3–5; k_{col} could also be obtained from a plate load test on a column with 230 kN load (820 kN/m^2) and resulting settlement of 1,5 cm ($820 \text{ kN/m}^2 / 0.015 \text{ m}$) $\approx 55 \text{ MN/m}^3$.

Table 1 summarizes the results when using equations related to E_{oed} , E and k_{col} , from Boussinesq's solution. The results show a triaxial or Young modulus of 21 MPa and oedometric modulus of 78 MPa. These values are far from the aforementioned range of 170–300 MPa, showing that regardless of the execution

method, such values exceed the real stiffness of stone columns.

Table 1. Example moduli estimation of stone column.

Parameters	Bousinesq (Equations 6 and 9)	Plain strain (Equations 3 and 8)
Influence depth, L	1.05 m	6.25 m
E_{oed}	78 MPa	652 MPa
E	21 MPa	172 MPa

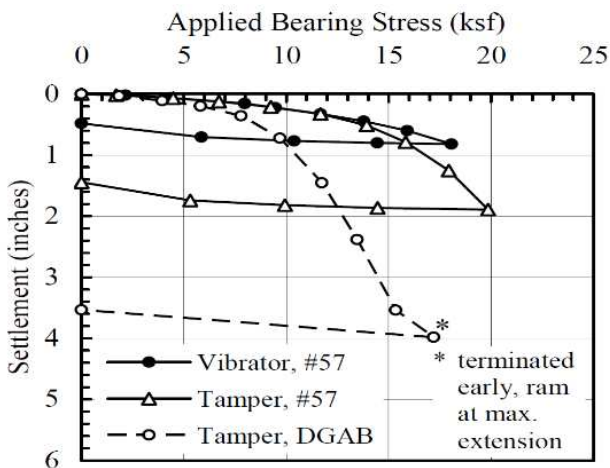
Table 1 also shows the results for the simplified approach, considering an influence depth of $L = 6.25$ m, which is necessary to obtain a modulus of the stone column of approximately $E = 170$ MPa.

This depth would be 20.8 times the radius of the foundation. In order to achieve $E = 300$ MPa it would be necessary to increase the influence depth to $L = 11$ m, which would be 37 times the radius of the plate.

2.3 Load tests on stone columns

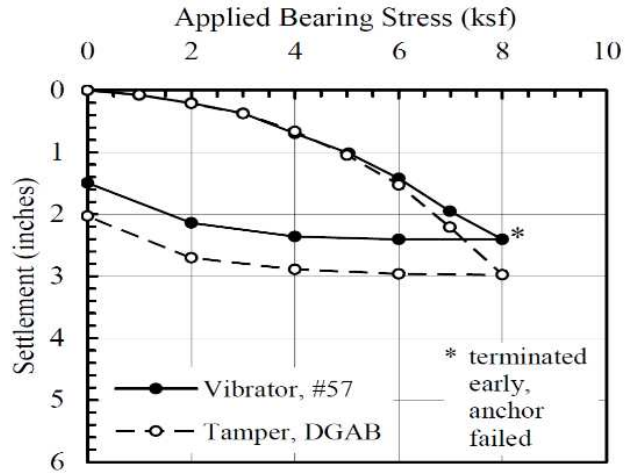
Stuedlein and Holtz (2008) performed an extensive study regarding the results of several load tests on small groups of stone columns (footings with 3 to 7 columns) and isolated columns with plate diameter of 76 cm.

Figures 6 and 7 show part of the results of this study, where very similar behaviour is observed in the columns performed with both methods (vibro bottom-feed and rammed aggregate piers).



a) 30-inch-diameter plate load tests

Figure 6. Plate Footing load tests on rammed aggregate piers (Tamper #57 and Tamper DGAB) and by bottom-feed deep vibrators (Vibrator #57), (Stuedlein and Holtz, 2008).



b) 9 ft x 9 ft footing load tests

Figure 7. Footing load tests on rammed aggregate piers (Tamper #57 and Tamper DGAB) and by bottom-feed deep vibrators (Vibrator #57), (Stuedlein and Holtz, 2008).

This similarity shows that the columns moduli are approximately the same for both execution methods, at least in the upper part of the columns, which can be assessed by plate load tests.

The dashed line in Figure 6 corresponds to a specific plate load test over one rammed aggregate pier, where relevant higher settlements is noticed. Further details of this specific load test can be found in Stuedlein and Holtz (2008).

3 CONCLUSIONS

Stone columns by deep vibrators, which are introduced into the ground by displacement and bottom-feed of gravel, have been successfully implemented for more than 60 years worldwide. This method has automatic control systems for execution parameters and one specific European Standard (EN 14731:2005).

Other procedures were developed around the 1990s, which include rammed aggregate piers as an alternative method, performed with extraction drilling, top-feed of gravel from working platform and compaction by vertical ramming.

The stone column moduli must be assessed under triaxial conditions. According to the experience and reported cases in literature the most common values of triaxial moduli vary from 20 to 100 MPa, the lowest moduli are obtained in the softest soils which offer lowest confining stresses. The consideration of column triaxial moduli higher than 120 MPa could imply a design overestimation regarding the reduction of settlement. Plate loading tests on isolated columns and small groups of columns show very similar behaviour in stone columns performed by both

methods. Tests on isolated columns with small plate diameters (30–60 cm) only allow assessments up to a depth of 1–1.5 m, so they do not represent the real behaviour of the entire column.

The Boussinesq's closed-form solution to determine the settlement (S) under a uniform loaded rigid circular plate on an elastic and isotropic semi-space can be considered for analytically interpretation of plate load tests, so that can be estimated the triaxial modulus of stone columns as a function of the measured reaction modulus during the test.

It is worth mentioning that instead of the stiffness, the friction angle and the diameter of stone columns present higher impact to maximize the improvement effects.

For adequate estimation of the friction angle of stone columns, it is essential to take into consideration the degradation law of ϕ_{col} with the confining pressure. This is particularly important to assess the risks that may arise if the design ϕ_{col} value is erroneously interpreted regarding the level of lateral confining pressure to which they would be subjected, which not only depends on the initial stress state of the soil, but also on the stress state to which the soil and columns will be subjected due to the application of foundation loading.

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