

# Reliability analysis of a Po River embankment affected by underseepage

## Analyse de fiabilité d'une digue du Fleuve Pô affectée par sous-infiltration

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**ABSTRACT:** Backward erosion piping (BEP) is an internal erosion process that may develop as a consequence of seepage processes through the aquifers underneath river embankment structures. This phenomenon can be identified in the field by observing the formation of sand boils on the ground surface at or near the embankment toe. Along the Po River, more than 130 sections are affected by periodical sand boils, especially located in its middle-lower stretch. The paper presents the results of reliability analyses referred to BEP phenomena in a riverbank section located in Mazzorno Sinistro, in the province of Rovigo (Italy), whose stability has been threatened by periodical reactivations of this phenomenon. Fragility curves, describing the probability of BEP initiation with hydrometric level, are elaborated based on a two-dimensional model of the groundwater flow through the river section and on the application of the First-Order-Second-Moment (FOSM) method, assuming the main parameters as probabilistic variables. The soil profile and the probability distribution of the geotechnical parameters were obtained from the interpretation of several recent in situ and laboratory tests. Model results are compared to those obtained by applying the USACE theory and to the evidence of past reactivations.

**RÉSUMÉ:** Les canalisations d'érosion vers l'arrière (BEP) sont un processus d'érosion interne qui peut se développer à la suite de processus d'infiltration dans les aquifères situés sous les structures de digues fluviales. Ce phénomène peut être identifié sur le terrain en observant la formation de bouillons de sable à la surface du sol près du pied de la digue. Le long du Pô, plus de 130 sections sont affectées par des ébullitions de sable périodiques, en particulier dans sa partie moyenne-inférieure. L'article présente les résultats d'analyses de fiabilité relatives au phénomène BEP dans une section riveraine située à Mazzorno Sinistro, dans la province de Rovigo (Italie), dont la stabilité a été menacée par les réactivations périodiques de ce phénomène. Les courbes de fragilité, décrivant la probabilité d'initiation du BEP avec le niveau hydrométrique, sont élaborées sur la base d'un modèle bidimensionnel de l'écoulement des eaux souterraines à travers la section de la rivière et de la méthode du premier ordre-second-moment (FOSM), en supposant les principaux paramètres comme variables probabilistes. Le profil du sol et la distribution de probabilité des paramètres géotechniques ont été obtenus à partir de l'interprétation de plusieurs récentes essais in situ et en laboratoire. Les résultats du modèle sont comparés à ceux obtenus en appliquant la théorie bien connue de l'USACE et aux preuves de réactivations passées.

**Keywords:** Backward erosion piping; river embankment; fragility curve; USACE method; probabilistic analysis.

## 1 INTRODUCTION

Backward erosion piping (BEP) is an internal erosion process that excavates pipes in sandy foundations beneath river embankments, typically during high-water events. Its initiation and progression require a peculiar soil stratigraphy: a sandy foundation (aquifer), in hydraulic communication with the river, confined above by a fine-grained layer (blanket), characterized by a lower hydraulic load. The differential water head between the blanket and the aquifer can result in the concentration of hydraulic gradients in pre-existing holes or local discontinuities in the top layer, where flow velocities and gradients may become higher enough to drag sand particles to the surface forming sand volcanoes around the exit

hole (sand boils). The ongoing process produces a progression upstream of erosion channels that weaken the foundation, potentially leading to embankment failure or breach, particularly during prolonged high-water periods or repeated reactivations. The Po River (Italy) counts more than one hundred sand boils of remarkable size (Gottardi et al., 2015). This generates the need to develop reliable strategies to quantify the potential risks for these critical structures connected to BEP. The analyses presented in this paper have been developed on a Po River embankment section located in Mazzorno Sinistro (RO), affected by three historic sand boils (Bertolini, 2024). The current study focuses on the most active one, located landside, within a ditch (Figure 1).

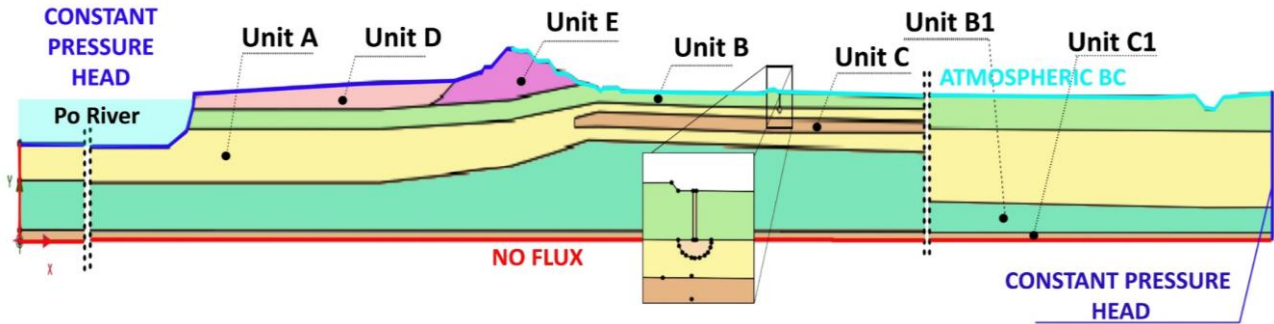


Figure 2. Soil stratigraphy of the investigated river section as adopted in the numerical modelling. The vertical dimension is magnified by double to ease its reading.

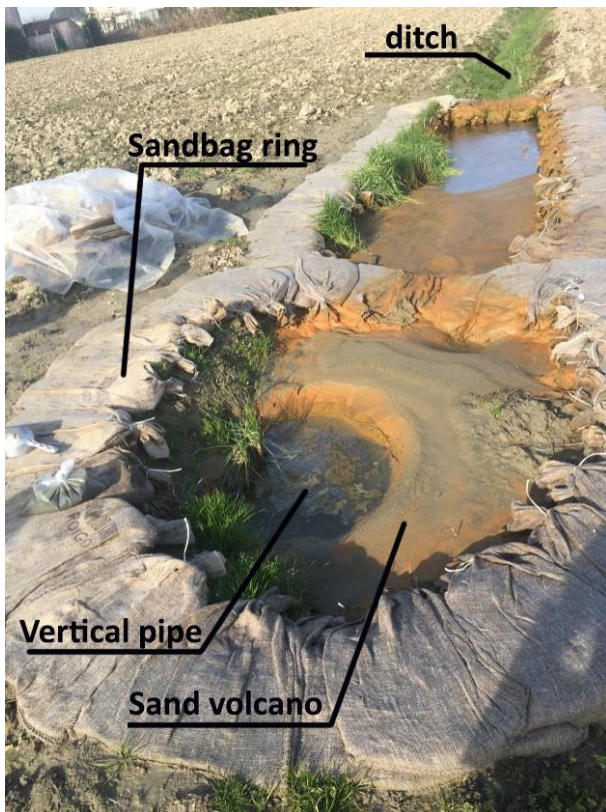


Figure 1. Photo of the sand boil reactivated in Mazzorno Sinistro in November 2019. The sandbag ring was positioned as an emergency measure to stop the progression of the erosion.

Fragility curves were calculated to assess the likelihood of BEP initiation, considering the probability distribution of the hydraulic soil properties obtained by laboratory and in-situ tests and multiple possible exit-hole configurations, and correlating it with river water levels. This involved employing two different strategies: the numerical results of a 2D finite element model simulating seepage beneath the river embankment and the analytical results obtained by applying the well-established Blanket Theory (USACE, 2000). The results of both strategies have been elaborated using a probabilistic approach, applying the First-Order-Second-Moment (FOSM) method (USACE, 1999).

## 2 THE 2D NUMERICAL MODEL

A 2D finite element (FE) model has been developed to simulate the saturated groundwater flow beneath the examined embankment section. Its primary objective was to compute the distribution of pore water pressure in proximity and within the exit hole of the historic sand boil, correlating it with incremental variations in river water head ( $H$ ). The soil stratigraphy adopted in the model was deduced from in situ tests while the external profile of the section is from a topographic survey (Figure 2). The soil units involved in the process are the aquifer (unit A) and the overlying blanket (unit B). A peculiarity of the investigated site is the presence of an interbedded cohesive layer (named unit C) within unit A, which greatly reduces the thickness of the filtration section (from 9.4m to 6.9m) around the exit hole. The spatial domain, 36m x 1556m (length x height), has been discretized in 22524 triangular elements and 183541 nodes, with an average element size of 1.9m, refined up to 2cm in the proximity of the sand boil pipe and of the model boundaries subjected to imposed pressure heads.

The boundary conditions (BC) applied to the model are summarized in Figure 2. A no-flux boundary is applied to the base of the model since located within a low permeable clayey layer (Unit C1) not affected by the BEP phenomenon. The left-side of the model corresponding to the centerline of the river, for symmetry reason is associated to a no flux condition. On the berm and the riverside slope, it is imposed a Dirichlet condition in the form of a series of constant prescribed pressure heads simulating hydrometric levels of the river. In particular, the hydraulic loads are imposed to this boundary in 15 steady-state increments. To compare the 2D model's results with USACE ones, the loads have been referred to the landward phreatic level. Thus, hydraulic loads ( $H$ ) equal to 0m and 8.85m correspond to the level of the phreatic water table and the crest, respectively. The landward water table in proximity of the sand boil is located  $\sim 0.65$ m below ground level, as revealed by the installed piezometric monitoring system. The

boundary conditions applied to the right side of the model are meant to reflect the groundwater condition of far-field which have been set as a series of constant hydraulic heads according to field monitoring data provided by a piezometer installed in unit A, 1123m distance from the embankment toe, landside. Atmospheric boundary conditions are applied to the outer slope and the landside field.

The described 2D numerical model has been adopted to generate fragility curves. Six random variables have been adopted to this scope: the permeability coefficients of units A, B and C ( $k_A$ ,  $k_B$ ,  $k_C$ ), the diameter of the vertical exit hole ( $d_{exit}$ ), the diameter of the hemisphere located at the base of the pipe and simulating the soil volume eroded by previous reactivations ( $d_{sphere}$ ) and the permeability coefficient of both the exit hole and of the hemisphere ( $k_{exit}$ ). Table 1 reports the mean value and the standard deviation of these random variables. CPT tests have been used to estimate the distribution of the hydraulic conductivities, which turned out to be well interpolated by a Gaussian distribution. A previous calibration of the 2D model over past sand boil reactivations suggested an optimal configuration of the existing exit pipe with a permeability coefficient equal to double the one of Unit A ( $k_A$ ), then it varies in the range [ $k_A$ ;  $3 \times k_A$ ]. The average diameter of the exit hole is assumed equal to 0.2m, in accordance with field evidences (Marchi et al., 2021; Garcia Martinez et al., 2020a; 2020b) observed in another river embankment section located along the Po River (Bertolini et al., 2022a; 2022b). A value of 0.1m is assumed as the diameter of first activation and thus as lower boundary for the variation range of this random variable. As a result, the gaussian variation interval explored in the probability analysis results within the limits: average value +/- the standard deviation, that is [0.1;0.3]. Since the hemisphere is a stratagem of modelling whose geometry is not directly measurable but function of the erosive process occurred in the past reactivations, a diameter range of attempt between 0m and 4m has been adopted. The assessment of reactivation of the sand boil was carried out by comparing the critical hydraulic gradient of Terzaghi's (1960) formulation ( $i_{crit} \sim 0.969$ ) with the average gradient within the vertical pipe, deduced from the numerical model. In particular, the average hydraulic gradient ( $i_{avg}$ ) is estimated as the ratio between the pressure head loss between the top and bottom of the pipe and the length of the pipe itself ( $\sim 2.68m$  in the selected location) while the critical hydraulic gradient of reactivation ( $\sim 0.969$ ) is defined as the gradient causing sand particles flotation within the open vertical pipe which occurs when the drag forces on the particles equal their submerged weight. A critical hydraulic gradient of

first activation ( $\sim 0.87$ ) required to cause heaving of the landside top stratum prior to the piping have been also adopted.

A series of 13 simulations were conducted, which have been used to generate the fragility curves shown in Figure 3.

Table 1. Mean values of input parameters for the 2D numerical model of piping reactivation. In square brackets the SD.

Probabilistic variables	
Mean values [SD]	
$k_A=3.19E-05$ m/s [3.66E-07]	$d_{pipe}=0.2m$ [0.1]
$k_B=4.69E-09$ m/s [3.21E-09]	$d_{sphere}=2m$ [2]
$k_C=7.86E-10$ m/s [8.11E-10]	
$k_{exit}=6.38E-05$ m/s [3.19E-05]	

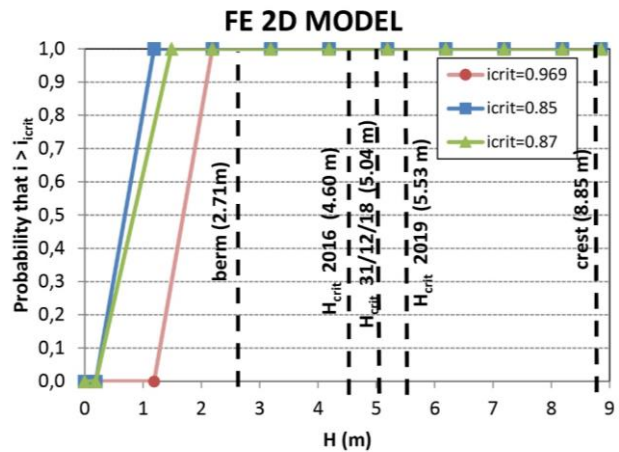


Figure 3. Cumulative probability of BEP initiation elaborated adopting the FE 2D numerical model.

A probability of 1 (i.e., certain BEP occurrence) is registered for hydraulic loads greater than 2m. It is also interesting to notice that an increase of 1m (from  $\sim 1m$  to  $\sim 2m$ ) in the hydraulic load turns a likelihood of occurrence from 0 to 100%. The most significant random variables (for  $H=8.85m$ ) are the permeability of the exit pipe ( $\sim 55\%$ ) and the pipe diameter ( $\sim 42\%$ ), while the remaining have low or null significance. In 2018, a reactivation of the sand boil was registered for a minimum hydraulic load of 5.04m. Another two reactivations of the same sand boil occurred a few years earlier, but the exact day of the BEP trigger and the (minimum) critical operating hydraulic load at the time are unknown. The peaks of these high-water events, under which surely the sand boil was active, were registered for hydraulic loads equal to 4.60m and 5.53m in the 2016 and 2019 events, respectively. The cumulative probability curve of BEP initiation forecasts correctly the three past activations but overestimates the minimum critical hydraulic load

being equal to  $H=2.5\text{m}$  the flood peak registered in May 2023 (25/5/23) which did not induce sand boil reactivation.

### 3 THE USACE APPROACH

The Blanket Theory was introduced within the technical manual USACE (1956) and is also described in the 2000 edition (USACE, 2000). The solution for semi-pervious top stratum of uniform thickness that extends infinitely landward of the levee, sketched in Figure 4 (case 7a of the methodology), was adopted for this study.

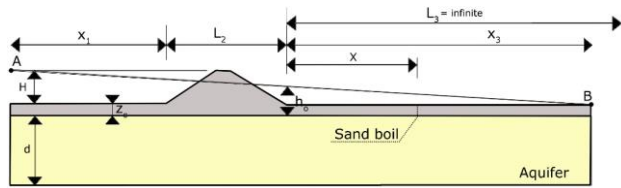


Figure 4. Scheme 7a of a section affected by BEP phenomenon characterized by a semi-pervious top stratum of uniform thickness riverside and landside (modified from USACE, 2000).

Table 2. Mean values of input parameters for the probabilistic safety assessment of the investigated section using USACE approach. In square brackets the standard deviation.

USACE APPROACH	
Deterministic variables	Probabilistic variables
$x_1 = 131.8 \text{ m}$	$d = 6.13 \text{ m}$ [2.47]
$L_2 = 66.4 \text{ m}$	$z_b = 2.68 \text{ m}$ [0.06]
$L_3 = \text{infinite}$	$k_A/k_B = 6818$ [78]
$x = 96.2 \text{ m}$	

The safety factor against (first) boiling or heaving computed according to the USACE approach is defined as the ratio of the exit gradient ( $i_{\text{exit}}$ ) at the sand boil location to the critical gradient ( $i_{\text{crit}}$ ). The exit gradient is a function of several variables, including the thickness of the aquifer and the blanket ("d" and "z<sub>b</sub>," respectively), the ratio of permeability between the aquifer and the top stratum ( $k_A/k_C$ ), the embankment base length ( $L_2$ ), and the residual water pressure head at the embankment toe ( $h_0$ ). Table 2 reports the "exact" values of the deterministic variables and the probabilistic distributions (in terms of mean value and standard deviation) of the stochastic variables assumed for the analysis. Among the latter, the thickness ranges of variation for the aquifer (d) and the blanket ( $z_b$ ) have been retrieved from CPT test data. In particular, the range of variation of "d" is obtained from the interpretation of all the CPT tests performed in the section, while  $z_b$  from a selected

number of tests performed in proximity of the sand boil, in order to be more representative of the blanket condition at the outflow point. Moreover, the depth of the ditch ( $\sim 0.45\text{m}$ ) in which the sand boil occurred has been subtracted from the estimated  $z_b$  value, to have an exit hole configuration more representative of site conditions and similar to the one adopted in the 2D FE model. Starting from the presumed expected values and standard deviation of the random variables, the moments of the performance functions were derived using Taylor's series method (specifically, the first-order second-moment - FOSM). The obtained fragility curves are presented in Figure 5.

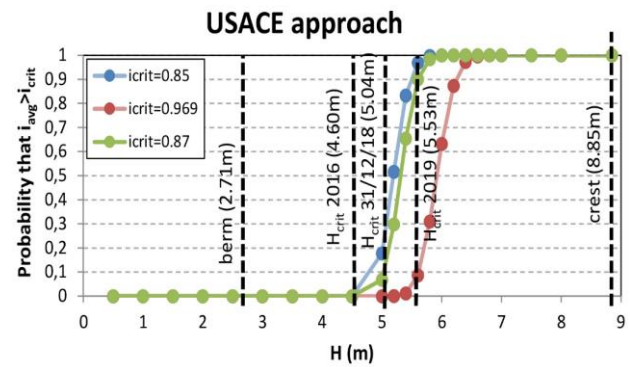


Figure 5. Cumulative probability of BEP initiation as deduced from the Blanket Theory.

The cumulative probability of BEP initiation remains null up to hydraulic loads of 4.5m and 5.5m, obtained for critical hydraulic gradients of 0.85/0.87 and 0.969, respectively. What emerges clearly is that for critical (first activation) gradients of 0.85 and 0.87, two of the three reactivations (i.e., 2016 and 2018) are not caught properly by the USACE approach, which underestimates the minimum critical hydraulic load causing sand boil activation. Similarly, the adoption of a critical (reactivation) gradient of 0.969 does not correctly predict all the three reactivations. Regarding this last observation, note that the analytical equations at the base of the Blanket Theory to estimate the seepage flow and the substratum pressure underneath the section are relative to an embankment configuration prior the possible formation of a landside open exit being these analyses usually performed in a design phase. It is also relevant to observe that the USACE solution is particularly sensitive to the thickness of the aquifer and of the blanket, with percentage of total variance equal to  $\sim 76\%$  and  $\sim 24\%$ , respectively (for  $H=8.85\text{m}$ ).

### 4 CONCLUSIONS

The study describes the results of an analytical approach (Blanket Theory) and of a numerical FE

analysis, which can be adopted to predict the probability of BEP initiation in a river embankment along the Po River, prone to periodic sand boil reactivations during high-water events.

The soil profile, as well as the physical, mechanical and hydraulic properties of the site, have been deduced from field investigations and laboratory tests. The outcomes of the two different strategies were represented as fragility curves, denoting the probability of BEP initiation with increasing hydraulic loads. Results have been discussed considering observed reactivation in several real high-water events. The Blanket Theory tends to underestimate the probability of BEP reactivation. On the contrary, the FE analysis slightly overestimates the observed behaviour. Although these analyses are not directly comparable to each other, their application to this well-documented section suggests that the FE model can produce better results. In the near future, possible new reactivations should provide additional piezometric data that would enable the improvement of the calibration of FEM parameters.

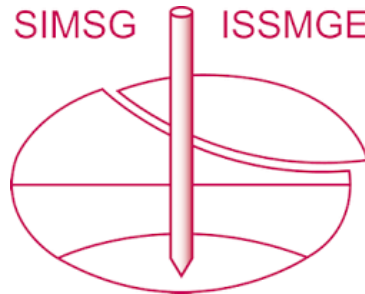
## ACKNOWLEDGEMENTS

This study was carried out within the RETURN Extended Partnership and received funding from the European Union Next-GenerationEU (National Recovery and Resilience Plan – NRRP, Mission 4, Component 2, Investment 1.3 – D.D. 1243 2/8/2022, PE0000005 - Spoke TS 2).

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