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Simulation of sand-fines mixtures behaviour based on bounding surface plasticity and equivalent void ratio

Simulation du comportement des mélanges sable-fines basée sur la plasticité de la surface de confinement et de l'indice de vide équivalent

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ABSTRACT: Binary materials are sand-fines mixtures where the mechanical behaviour transitions from a configuration where the entire forces are transmitted between the grain contacts (no fines) to an array where all voids are filled with fines and contribute to the force transmission. The fines content controls this complex transition, and as such, it has been incorporated into the definition of the equivalent void ratio concept. The macromechanic modelling of sand-fines mixture soils is challenging. Conventional constitutive models are formulated and calibrated either to work for granular or fine-grained soils. This article presents an application of the boundary surface plasticity model proposed by Dafalias and Manzari in 2004, incorporating the equivalent void ratio concept. The model is tested with a series of monotonic triaxial results on Vietnam silica sand and non-plastic fine sand mixtures. Investigations about the inclusion of the equivalent void ratio as a promising option to model binary mixtures before undertaking substantial changes in the structure of well-established constitutive models are explored. Recommendations on the critical void ratio line based on the experimental observations are also provided.

RÉSUMÉ: Les matériaux binaires sont des mélanges de sable et de fines dont le comportement mécanique passe d'une configuration où toutes les forces sont transmises entre les contacts des grains (pas de fines) à une configuration où tous les vides sont remplis de fines et contribuent à la transmission des forces. Cette transition complexe est contrôlée par la teneur en fines et, en tant que telle, elle a été incorporée dans la définition du concept de taux de vide équivalent. La modélisation macromécanique des sols à mélange de sable et de fines est un défi. Les modèles constitutifs conventionnels sont formulés et calibrés soit pour les sols granulaires, soit pour les sols à grains fins. Cet article présente une application du modèle de plasticité de surface limite proposé par Dafalias et Manzari (2004), incorporant le concept de rapport de vide équivalent. Le modèle est testé avec une série de résultats triaxiaux monotones sur des mélanges de sable siliceux du Vietnam et de limon non plastique. Les recherches sur l'inclusion du rapport de vide équivalent en tant qu'option prometteuse pour modéliser les mélanges binaires, avant d'entreprendre des changements substantiels dans la structure de modèles constitutifs bien établis, sont explorées. Des recommandations sur la ligne de rapport de vide critique, basées sur les observations expérimentales, sont fournies.

Keywords: Equivalent void ratio; bounding surface plasticity; binary mixtures; sand-fines mixtures; triaxial simulation.

1 INTRODUCTION

Sand behaviour is influenced by sundry aspects such as grain size distribution, grain shape, sample preparation methods, preloading history, drainage conditions, partial saturation among others. Many research works published in the last decades have studied those effects using extensive experimental campaigns based mostly on triaxial tests for monotonic and cyclic loading conditions (e.g., Wichtmann and Triantafyllidis (2016). Based on those observations on clean sands, different constitutive models have been developed, calibrated, validated and tested in boundary value problems. However, granular soils are mixed with finer non-plastic fractions in many depositional environments. Those silty sands are idealised as a binary combination of coarse particles (sand) and fine particles (fine sands, silts), and new definitions of fundamental state variables like the equivalent void ratio have been proposed to consider the effect of the fine fraction on the mechanical transfer of forces between the coarse grains. Experimental observations have demonstrated that for binary mixtures, the critical state line (CSL) becomes a function of fines content (Thevanayagam, 1998; Yang et al, 2006; Rahman and Lo, 2008; Rahman et al., 2011; Goudarzy et al. 2022). Applying the critical state soil mechanics framework to sand with fines is challenging because a specific critical state line (CSL) is required for each fine content (f_c). At a specific f_c value, the rotation of the CSL with respect to change in f_c reverses. This specific fines content value is known as the threshold fines content (f_{thre}). It indicates a change in soil fabric from "fines-in-sand" to "sand-in-fines" (Rahman et al., 2011). An alternative way to consider the changes in the CSL induced by the fines content in binary mixtures is to use the concept of equivalent granular void ratio e^* (Rahman et al., 2011; Thevanayagam, 1998; Nguyen et al. 2017; Chang and Deng, 2019; Tamang et al., 2023; Tafili et al., 2022) instead of the conventional void ratio e:

$$e^* = \frac{e+(1-b)f_c}{1-(1-b)f_c}; 0 < b < 1 \qquad (1)$$

where b denotes the portion of the fine grains that contribute to the active intergranular contacts. Parameter b depends on grain size disparity and grain characteristics and there are different approaches to estimate its value (Rahman et al., 2011; Thevanayagam 1998; Nguyen, Benahmed and Hicher, 2017; Chang and Deng, 2019; Tamang et al., 2023).

This article explores the convenience of incorporating different definitions of equivalent void ratio in the description of the CSL in the SaniSand bounding surface plasticity model proposed by Dafalias and Manzari (2004) to reproduce the experimental behaviour of binary mixtures observed on a series of monotonic triaxial tests conducted on the Vietnam silica sand.

2 MATERIALS AND METHODS

2.1 Materials

The behaviour of binary mixtures can be controlled by coarse-grained, fine-grained (non-plastic or a small amount of plastic), or a combination of both fractions. Two different grain size distributions of silica sands from Vietnam were selected to apply alternative definitions for the equivalent void ratio e^*. Figure 1 presents coarse and fine sand grain size distribution curves for the sand-fines mixture. The main properties of both materials are summarized in Table 1. Vietnam silica sand is characterized by a specific gravity (G_s) of 2.66, the plasticity index (PI) is zero, and the mean grain sizes (D_{50}) of the coarse sand and fine sand are 0.904 mm and 0.081 mm, respectively. The coarse sand has a uniformity coefficient of C_u = 1.49 and a coefficient of gradation of C_c = 0.98, while the fine sand has a uniformity coefficient of C_u = 1.89 and a coefficient of gradation of C_c = 1.13.

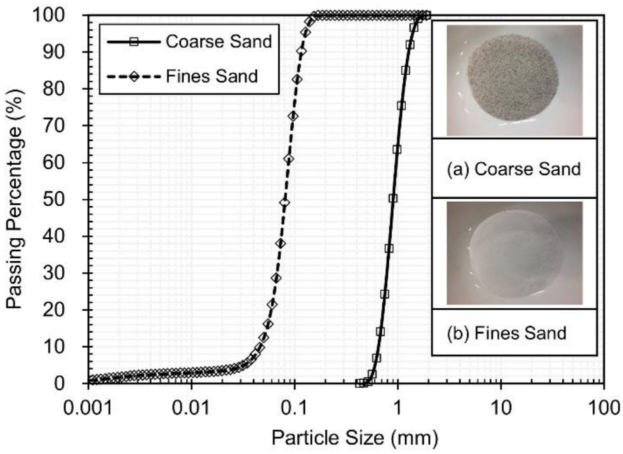

Figure 1. Particle size distribution curves of Vietnam Silica Sand: (a) coarse sand; (b) fine sand.

Nine strain-controlled monotonic triaxial tests were conducted on reconstituted specimens of Vietnam silica sand. Four drained and one undrained tests were performed on Vietnam sand with no fines, whereas three drained and one undrained tests were undertaken on mixtures with 15% fines content. A summary of initial conditions and critical values for each triaxial test is presented in Table 2, where the monotonic triaxial test under drained conditions is named TMD, and the triaxial test under undrained conditions is named TMU.

Table 1. Physical properties of Vietnam silica sand.

Mixtures	Coarse-dominated			χ
	D_{50} [mm]	e_{max} [-]	e_{min} [-]	D_{50}/d_{50} [-]
Vietnam silica sand	0.904	0.976	0.621	11.2
	Fines-dominated			
	D_{50} [mm]	e_{max} [-]	e_{min} [-]	
	0.081	1.323	0.671	

2.2 Model calibration

Five different models (shown in Table 3) based on the concept of the equivalent granular void ratio e^* and the parameter b were calibrated to describe the critical state line obtained experimentally for specimens with fines content f_c = 0% and f_c = 15%. Figure 2 shows in the e or e^*-log(p') plane the critical state points obtained from the interpretation of drained and undrained triaxial tests, and the equivalent void ratios for the specimens with f_c = 15%. From the comparison of different approaches to calibrate e^*, it is clear that the model after Thenvanagan et al. (2002) produces the best fitting. The blue curve in Figure 2 represents the CSL used in the SaniSand model (Dafalias and Manzari, 2004), and was fitted considering the critical void ratios obtained for Vietnam silica sand specimens with f_c = 0% and for the e^* calculated after Thevanayagam et al. (2002) for the f_c = 15% results.

Table 2. Test program and critical state results for the Vietnam Silica sand.

Test	f_c [%]	p'_0 [kPa]	e_0 [-]	e_{cri} [-]	p'_{cri} [kPa]	e^*_{cri} [-]
TMD	0	100.0	0.716	0.921	218.8	-
TMD	0	200.0	0.695	0.879	386.3	-
TMD	0	300.4	0.684	0.829	679.9	-
TMD	0	400.0	0.685	0.810	694.0	-
TMD	15	100.6	0.638	0.704	254.0	0.904
TMD	15	200.2	0.633	0.670	475.3	0.865
TMD	15	399.6	0.629	0.649	860.7	0.842
TMU	0	149.0	0.672	0.672	1134.3	-
TMU	15	150.3	0.639	0.639	912.8	0.831

Table 3. Prediction of b-value for the f_c = 15% binary mixture of Vietnam silica sand.

References	Function b()	Calibrated b
Thevanayagam et al. (2002)	constant	0.3
Rahman and Lo (2008), Rahman (2009), and Rahman, Lo, and Baki (2011)	$b(f_c, r)$	0.219
Nguyen, Benahmed, and Hicher (2017)	$b(f_c, e, \chi)$	Not a constant (0.702 ~ 0.711)
Chang and Deng (2019)	$b\left(\frac{d_{50}}{D_{50}}\right)$	0.23
Tamang et al. (2023)	$b(\chi)$	0.7

The SaniSand version proposed by Dafalias and Manzari (2004) was implemented as a user material Fortran routine (UMAT) in IncrementalDriver (Niemunis, 2014). Table 4 presents the set of calibrated parameters for the Vietnam silica sand. Calibration was based on the simulation of drained triaxial test on the sand fraction (f_c = 0%), following the recommendations presented in Wichtmann et al. (2019).

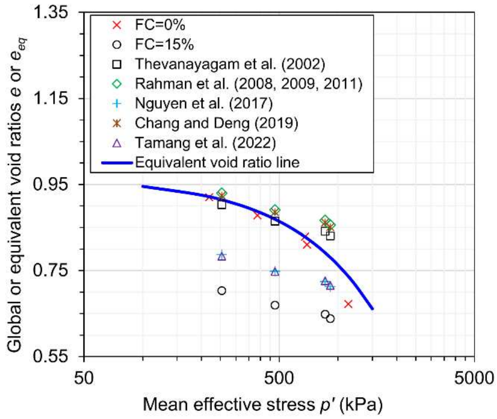

Figure 2. Critical state line and equivalent void ratios for f_c = 15% using different approaches to estimate b.

Figure 3 shows the comparison between experimental results and the simulation of drained triaxial tests with different initial mean pressures for the sand specimens with no fines (f_c = 0%). The description of initial stiffness, peak shear strength and volumetric strain is adequate between triaxial results and SaniSand simulations.

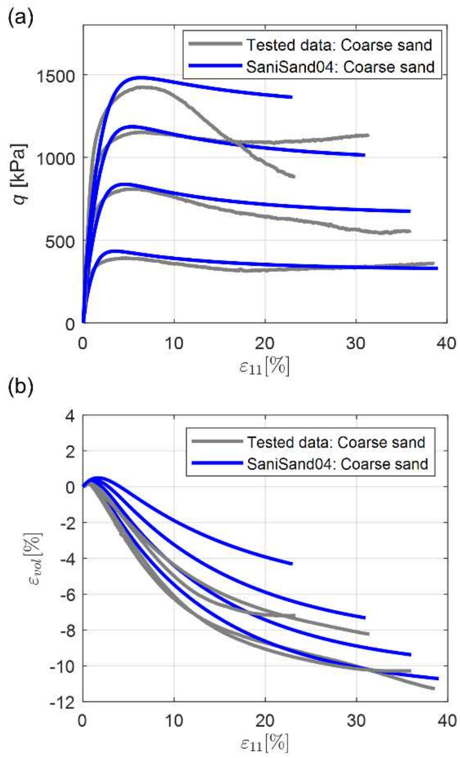

Figure 3. Comparison of experimental and simulated triaxial curves for the Vietnam silica coarse sand (f_c = 0%) under drained conditions: a) Deviatoric stress q vs axial strain ε_{11} curves. b) Volumetric strain ε_{vol} vs axial strain ε_{11} curves.

Table 4. Calibrated parameters of the Sanisand model (Dafalias and Manzari, 2004) for the Vietnam silica sand.

e_0	λ	ξ	M_c	M_e	m
0.966	0.0206	1	1.567	1.029	0.1
G_0	v	H_0	c_h	n_b	A_0
100	0.25	8	0.95	0.75	1.1
n_d	Z_{max}	c_z	b		
1.3	20	5000	0.3		

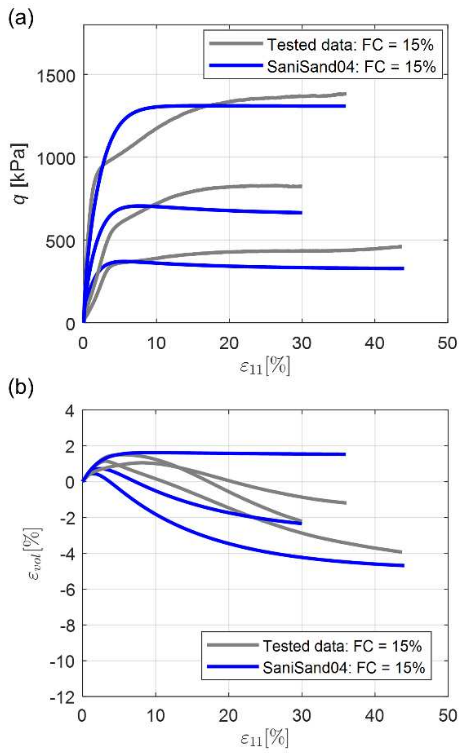

Figure 4. Comparison of experimental and simulated triaxial curves for the Vietnam silica coarse sand (f_c = 15%) under drained conditions: a) Deviatoric stress q vs axial strain ε_{11} curves. b) Volumetric strain ε_{vol} vs axial strain ε_{11} curves.

3 RESULTS AND DISCUSSION

The series of drained triaxial tests for the binary mixture with f_c = 15% was simulated, using the equivalent void ratio e^* calculated with the Thevanayagam et al. (2002) formula for fine contents of 15% as the initial void ratio e_0 in IncrementalDriver. Figure 4 summarizes the comparison between experimental results and simulations with SaniSand for three different mean initial stresses (p'_0 = 100, 200, 400 kPa), considering e^*. The increase in the fines content in the binary mixture of Vietnam silica sand is reflected in the change from strain softening behaviour observed for samples with f_c = 0% (Figure 3-a) to strain hardening for specimens with f_c = 15% (Figure 4-a). This trend is well reproduced by the numerical simulations with SaniSand and the equivalent void ratio e^*. The effect of fines content is also clearly observed on the volumetric behaviour. For the specimens with f_c = 15% a reduction of approximately 50% in the dilatancy was observed. Once again, simulations using SaniSand and the equivalent void ratio managed to reproduce the reduction in dilatancy, as shown in Figure 4-b.

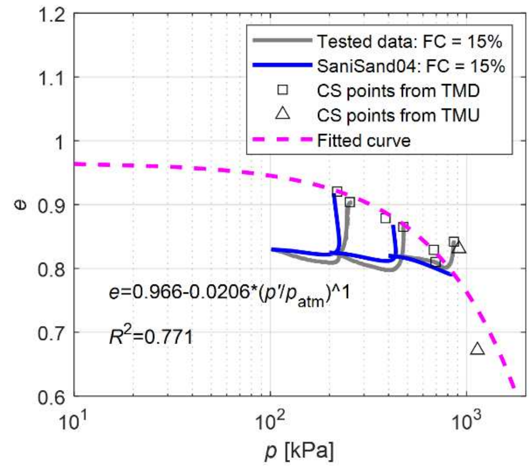

Figure 5. Comparison between experimental and simulated mean effective stress versus void ratio paths for Vietnam silica sand with f_c=15%.

Figure 5 presents the evolution of void ratio e vs mean effective stress p' for the three drained triaxial tests with f_c = 15%. Simulations for p'_0 = 100, 200 kPa match the experimental observations, describing the transition between contractancy and dilatancy behaviour. For p'_0 = 400 kPa, the model describes well the contracting phase but then it reaches the critical state, whereas the experiments exhibit first contractancy followed by a dilatant stage that exceeds the fitted critical state line.

Additionally, simulations of two undrained triaxial tests for specimens with fine contents f_c = 0, 15% and mean confining pressure p'_0 = 150 kPa were conducted (Figure 6). Simulations for the coarse Vietnam silica sand with no fine content agree well with the description of the stiffness, but overestimate the shear strength measured in the laboratory. Similarly, the curves describing the evolution of the excess pore water pressure Δu_w vs the axial strain ε_{11} coincide well in the transition between positive and negative Δu_w up to $\varepsilon_{11} \approx$ 3% between experimental and simulation results. SaniSand results for the binary mixture with f_c = 15% exhibit in average a similar trend, but the model underestimates the shear strength and the steady state excess pore water pressure.

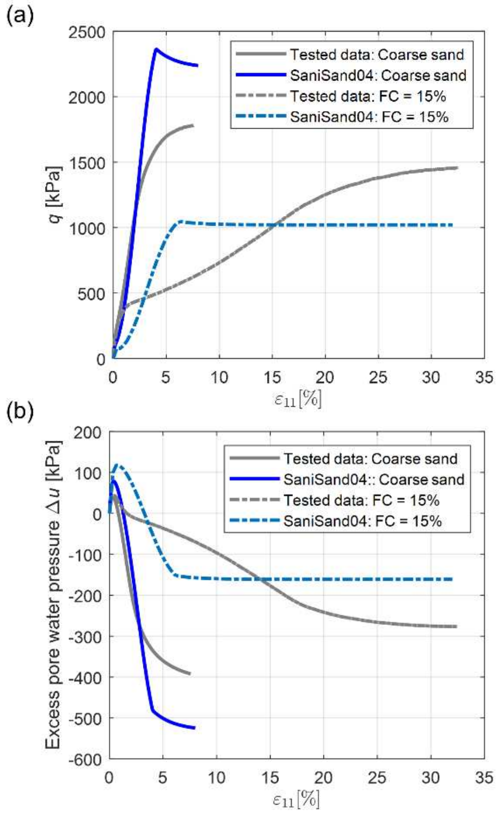

Figure 6. Simulations for the coarse sand and mixture under undrained conditions: (a) Deviatoric stress v.s. Axial strain and (b) Excess pore water pressure v.s. Axial strain.

4 FINAL REMARKS AND FUTURE WORK

This work presented experimental results of drained and undrained monotonic triaxial tests performed on coarse Vietnam silica sand and binary mixtures thereof with fines content f_c = 15%. Five different equations to estimate the equivalent granular void ratio e^* were calibrated for the available experimental set of results. The Thevanayagam et al. (2002) model delivered the best description of critical void ratios for mixtures with fine contents. Experimental points in the space $e^* - \log p'$ considering the coarse specimens and the mixtures with f_c = 15% were used to fit the critical state line, following the definition used in the SaniSand model (Dafalias and Manzari, 2004). Simulations of experiments with the popular SaniSand model incorporating the concept of e^* both for the CSL and as the initial void ratio were presented. Results showed that the concept of e^* can account for the effects of fines content on granular binary mixtures, especially well for drained triaxial tests. However, additional modifications to the constitutive model might be needed to improve the prediction of undrained tests. Further validation with other materials like the Karlsruhe fine sand is planned, considering drained and undrained triaxial tests.

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REFERENCES

Chang, C. S., Deng., Y. (2019) Revisiting the concept of inter-granular void ratio in view of particle packing theory, *Géotechnique Letters*, Vol. 9 (2), pp. 121-129. https://doi.org/10.1680/jgele.18.00175.

Dafalias, Y., Manzari, M. (2004) Simple plasticity sand model accounting for fabric change effects, *Journal of Engineering Mechanics*, Vol. 130 (6), pp. 622-634. https://doi.org/10.1061/(ASCE)0733-9399(2004)130:6(622).

Goudarzy, M., Sarkar, D., Lieske, W., Wichtmann, T. (2022) Influence of plastic fines content on the liquefaction susceptibility of sands: monotonic loading, *Acta Geotechnica*, Vol. 17, pp. 1719-1737. https://doi.org/10.1007/s11440-021-01283-w.

Nguyen, T. K., Benahmed, N., Hicher, P.Y. (2017) Determination of the equivalent intergranular void ratio - Application to the instability and the critical state of silty sand, In *EPJ Web of Conferences*, Vol. 140, Montpellier, France, EDP Sciences, pp. 02019. https://doi.org/10.1051/epjconf/201714002019.

Niemunis, A. (2014). Incremental Driver User's manual. Available at: https://www.soilmodels.com/idriver/, accessed: 13/01/2018.

Rahman, M. M., Lo., S. R. (2008) The prediction of equivalent granular steady state line of loose sand with fines, *Geomechanics and Geoengineering: An International Journal*, Vol. 3 (3), pp. 179-190. https://doi.org/10.1080/17486020802206867.

Rahman, M. M., Lo, S. R., Baki, M. A. L. (2011) Equivalent granular state parameter and undrained behaviour of sand–fines mixtures, *Acta Geotechnica*, Vol. 6, pp. 183-194. https://doi.org/10.1007/s11440-011-0145-4.

Rahman, M. M. (2009) Modelling the influence of fines on liquefaction behaviour. PhD diss., UNSW Sydney. https://doi.org/10.26190/unsworks/22259.

Tafili, M., Knittel, L., Gauger, V. (2022) Experimentelle und numerische Untersuchungen zum Kompressionsverhalten von Sand-Schluff-Gemischen, Experimental and numerical studies on the compression behavior of sand-silt mixtures, *Geotechnik*, Vol. 45 (1), pp. 3-15. (in [German]). https://doi.org/10.1002/gete.202100006.

Tamang, B., Kim, U., Jin, J., Lee, S., Kim, Y. (2023) Undrained monotonic shear behavior of sand mixed with a small amount of fines content, *Acta Geotechnica*, Vol. 18 (6), pp. 2915-2927. https://doi.org/10.1007/s11440-022-01776-2.

Thevanayagam, S. (1998) Effect of fines and confining stress on undrained shear strength of silty sands, *Journal* *of geotechnical and geoenvironmental engineering*, Vol. 124 (6), pp. 479-491. https://doi.org/10.1061/(ASCE)1090-0241(1998)124:6(479).

Thevanayagam, S., Shenthan, T., Mohan, S., Liang, J. (2002) Undrained Fragility of Clean Sands, Silty Sands, and Sandy Silts, *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 128 (10), pp. 849-859. https://doi.org/10.1061/(ASCE)1090-0241(2002)128:10(849).

Wichtmann, T., Fuentes, W., Triantafyllidis, T. (2019) Inspection of three sophisticated constitutive models based on monotonic and cyclic tests on fine sand: Hypoplasticity vs. Sanisand vs. ISA, *Soil Dynamics and Earthquake Engineering*, Vol. 124, pp. 172-183. https://doi.org/10.1016/j.soildyn.2019.05.001.

Wichtmann, T., Triantafyllidis, T. (2016) An experimental data base for the development, calibration and verification of constitutive models for sand with focus to cyclic loading: part I—Tests with monotonic loading and stress cycles, *Acta Geotechnica*, Vol. 11 (4), pp. 739-761. https://doi.org/10.1007/s11440-015-0402-z.

Yang, S. L., Sandven, R., Grande, L. (2006) Steady-state lines of sand–silt mixtures, *Canadian Geotechnical Journal*, Vol, 43 (11), pp. 1213-1219. https://doi.org/10.1139/t06-069.