

# Influence of spatial variability of strength on bearing capacity for unsaturated soil

## Influence de la variabilité spatiale de la résistance sur la capacité portante des sols non saturés

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**ABSTRACT:** Soil, even within a homogeneous layer, exhibits spatial variation in cohesion. Additionally, experimental tests have confirmed that spatial variability in cohesion can occur from changes in soil fabric under unsaturated conditions. Its significant effects on slope stability have been studied comprehensively, but limited to bearing capacity problems. This paper addresses, therefore, an analytical approach that utilises limit state analysis to investigate the influence of spatial variability in cohesion on the bearing capacity of a rigid surface strip footing. The analytical study considered various scenarios, accounting for different suction profiles that addressed the spatial variability in cohesion beneath and alongside the footing. The numerical results demonstrated a 18.5% increase in the ultimate bearing capacity, attributed to the effects of spatial variability in cohesion (changes in soil fabric). Inspection of the obtained failure mechanisms for the modelled scenarios revealed deeper and wider failure patterns when a higher multiplier/increase was applied to a greater cohesion value.

**RÉSUMÉ:** Le sol, même au sein d'une couche homogène, présente une variation spatiale de cohésion. De plus, des tests expérimentaux ont confirmé qu'une variabilité spatiale de la cohésion peut résulter de modifications du tissu du sol dans des conditions non saturées. Ses effets significatifs sur la stabilité des pentes ont été étudiés de manière approfondie, mais limités aux problèmes de capacité portante. Cet article aborde donc une approche analytique qui utilise l'analyse à l'état limite pour étudier l'influence de la variabilité spatiale de la cohésion sur la capacité portante d'une semelle filante à surface rigide. L'étude analytique a examiné divers scénarios, tenant compte de différents profils de succion qui traitaient de la variabilité spatiale de la cohésion sous et le long de la semelle. Les résultats numériques ont démontré une augmentation de 18,5% de la capacité portante ultime, attribuée aux effets de variabilité spatiale de la cohésion (changements dans la structure du sol). L'inspection des mécanismes de défaillance obtenus pour les scénarios modélisés a révélé des schémas de défaillance plus profonds et plus larges lorsqu'un multiplicateur/augmentation plus élevé était appliqué à une valeur de cohésion plus élevée.

**Keywords:** Soil fabrics; spatial variability; unsaturated soil; numerical analysis; bearing capacity.

## 1 INTRODUCTION

Soil, even within a homogeneous layer, exhibits spatial variation in strength. Additionally, spatial variability can arise from changes in soil fabrics under unsaturated conditions or due to the drying and wetting phenomenon. Understanding the characteristics of spatial variability is essential for assessing the reliability of geotechnical engineering problems. Researchers have studied the spatial variability in soils, emphasizing its significance in introducing uncertainties within the field of geotechnical engineering (see Tan et al., 2020).

Cho (2010) conducted a numerical study, extended the traditional limit equilibrium method of slices for the purpose of analysing the probabilistic stability of slopes, into the variations in soil strength parameters and their associated uncertainties. The results showed

that the proposed method adequately addressed the causes of failure that arise from differences in soil characteristics. Jiang et al. (2018) used conditional random fields to analyse slope reliability and modelled the spatial variation in undrained shear strength of soil for a slope stability problem. The method successfully incorporated the data, from site investigation to model the differences, in soil properties using conditional random fields.

The preceding studies have indicated a significant effect of spatial variability on slope stability, but their focus was restricted to addressing bearing capacity problems. Changes in soil fabric can significantly affect shear strength (cohesion) (see Toll and Ong, 2003, Shwan and Smith, 2015 and Shwan, 2015). This paper addresses the effects of spatial variability (referring to variations in cohesion resulting from fabric changes due to suction) on the bearing capacity

of a rigid surface footing under different suction profiles, using an upper-bound theorem approach.

## 2 PROBLEM GEOMETRY AND MATERIALS

The schematic diagram of a rigid surface strip footing (1 m width) is depicted in Figure 1a with the obtained failure mechanics, not restricted to the boundaries. While, Figure 1b shows the soil water retention curve (SWRC) for a simulated soil, plotted using an SWRC equation proposed by Shwan and Smith (2014):

$$S_r = e^{-\kappa(s-AEV)} \quad (1)$$

where  $S_r$  is the degree of saturation,  $\kappa$  is a fitting parameter ( $\text{kPa}^{-1}$ ) accounts for soil types (see Shwan, 2023a, 2023b, and 2023c),  $s$  is the suction ( $\text{kPa}$ ) and  $AEV$  is the air entry suction. Equation 1 was validated against experimental data and fitted to the Fredlund and Xing (1994) model, (see Shwan, 2015, and 2017).

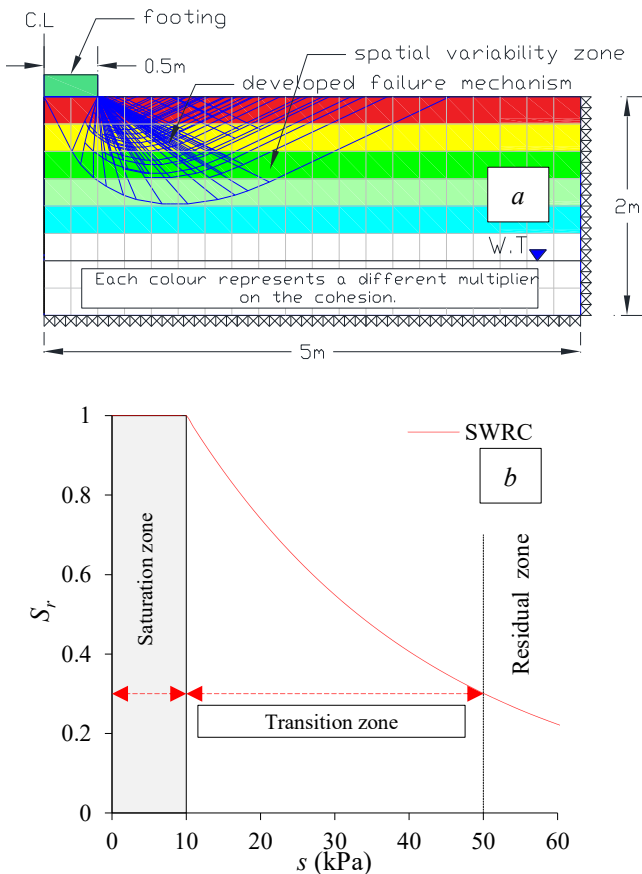


Figure 1. (a) Schematic diagram of the modelled footing, and (b) Simulated Soil-Water Retention Curve (SWRC).

The modelled suction range, applied by changing the water table level, ranged from 0 to 60 kPa.

The required parameters for the analysis were  $\kappa = 0.03 \text{ kPa}^{-1}$  and  $AEV = 10 \text{ kPa}$ , obtained through the SWRC in Figure 1b. The angle of shearing resistance ( $\phi$ ) and cohesion ( $c$ ) values of  $40^\circ$  and  $5 \text{ kPa}$ , respectively, were used. In addition, dry, unsaturated and saturated unit weights were  $14, 16$  and  $18 \text{ kN/m}^3$ , respectively.

## 3 MODELLING SPATIAL VARIABILITY

Soils exhibit different increases in effective stress under unsaturated conditions, (Bishop and Blight, 1963) attributed to different suction levels and soil structure, enhancing soil fabrics. Spatial variability in strength was, therefore, modelled in the UNSAT-DLO approach, a robust upper bound theorem approach modified by the author to account for the effects of unsaturated conditions on strength, using various scenarios. Spatial variability was attained by applying different multipliers ( $\mathbf{M}$ ) to cohesion ( $\mathbf{M} \times c$ ), attributed to fabric changes, in each grid beneath the footing using different cases, see Figure 1a.

The suggested cases were categorised into two groups: Category 1 (C1), Figure 2, and Category 2 (C2), Figure 3. The case where cohesion exhibits no spatial variability (no changes in soil fabric, such as in fully dry and saturated conditions) was also simulated for comparison. Each category comprised three scenarios: SCE1, SCE2, and SCE3. Each colour in Figures 2 and 3 represents a spatial variability zone in cohesion where a different  $\mathbf{M}$  value was utilised (i.e.  $\mathbf{M} \times c$ ). The values of  $\mathbf{M}$  used in the analysis are depicted in the legends of Figures 2 and 3, located in the last column of the colored grid. For the soil (grid) beneath the footing, a higher  $\mathbf{M}$  value was applied, attributed to a higher applied suction which caused a higher enhancement in cohesion. As the soil was closer to the water table level (resulting in a lower suction value), a smaller  $\mathbf{M}$  value was utilised. In C2, higher  $\mathbf{M}$  values were applied compared to C1, as shown in Figure 3. SEC3 in Figure 3c represented the case in which the increase in cohesion due to suction was doubled ( $c = 10 \text{ kPa}$ ) with  $\mathbf{M} = 2$ . As stated above, the fully saturated and fully dry cases ( $\mathbf{M}=1$ , indicating no changes in soil fabric) were also considered, represented by the white colour in Figures 2 and 3.

## 4 NUMERICAL RESULTS

The failure mechanisms results obtained for C1 and C2 are shown in Figures 2 and 3, respectively. The case with no spatial variability ( $\mathbf{M}=1$ ), denoted as NS in the figure's legend, was superimposed with the modelled categories, denoted as SP. In Figure 2 (C1), similar

failure mechanisms were observed for both NS and SP cases, with the SP case exhibiting a higher ultimate bearing capacity ( $Q_{ult}$ ), discussed later. However, Figure 3 (C2) depicted wider and deeper failure mechanisms for cases with  $M=2$  compared to C1. Clearly, the higher  $c$  value and  $M$ , the wider and deeper failure mechanism was obtained, see Figure 3c. The implications of the obtained failure mechanisms are significant, as strength mobilisation is a possible scenario attributed to changes in fabric, therefore, spatial variability in cohesion.

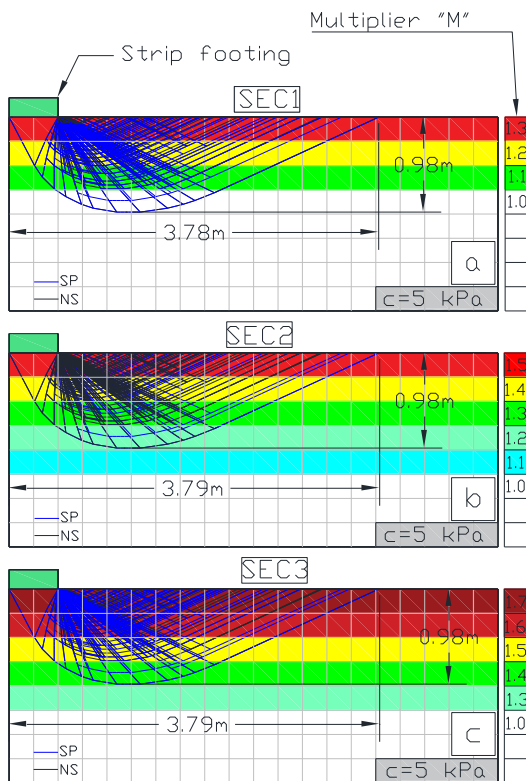


Figure 2. Results of failure mechanisms for various scenarios with different values of  $M$  applied to cohesion in category 1 for (a) SEC1, (b) SEC2, and (c) SEC3.

Figure 4 a and b shows the numerical results of the modelled scenarios previously presented in Figures 2 and 3. The x and y axes represent normalised suction and ultimate bearing capacity, respectively.

It is evident that the effect of unsaturated condition was significant compared to both the fully saturated condition ( $s=0$  kPa) and dry case (plotted as a horizontal dashed line). For example, a 3.8-fold increase in normalised  $Q_{ult}$  was obtained at  $s/AEV = 4$  for the NS case when compared with  $s = 0$  kPa ( $S_r = 1$ ). Experimental studies in the literature have reported significantly higher increases in  $Q_{ult}$  (Mohamed and Vanapalli, (2006) ( $\approx 6$  fold) and Shwan and Smith (2014) ( $\approx 10$  fold). The ultimate bearing capacity was seen then to decrease, attributed to desaturation-induced strength reduction.

On the other hand, the effect of spatial variability in cohesion on the normalized  $Q_{ult}$  was significant. A 2.1% increase in  $Q_{ult}$  was obtained when comparing the C1-SEC1 with the NS cases (Figure 4a), even though almost identical failure mechanisms were obtained in Figure 2a as stated previously.

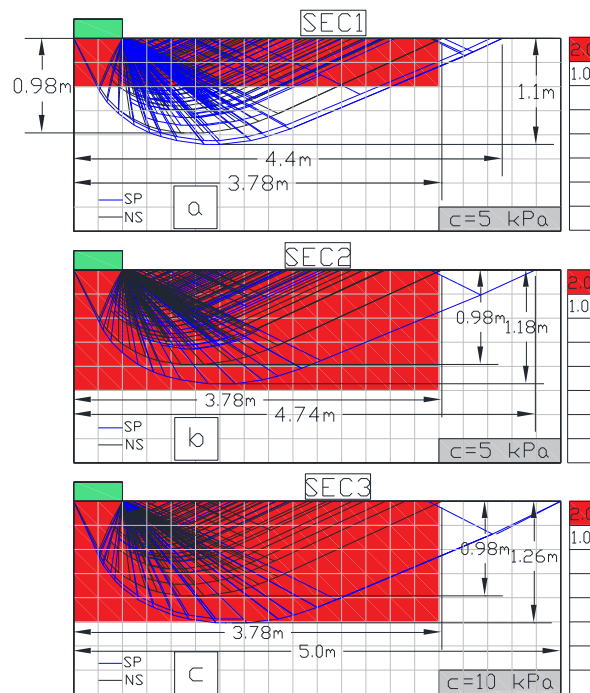


Figure 3. Results of failure mechanisms for various scenarios with different values of  $M$  applied to cohesion in category 2 for (a) SEC1, (b) SEC2, and (c) SEC3.

Figures 4a and b also exhibit other percentage increases in  $Q_{ult}$ , when comparing SP cases with their respective NS cases. The case C2-SEC3 exhibited the highest increase in  $Q_{ult}$  (Figure 4b) as higher  $c=10$  kPa and  $M = 2$  were used. The value  $M=2$  represented the scenario where  $c$  increased by a factor of 2 due to suction effects. Experimentally, a 4.1-fold increase in  $c$  for unsaturated samples, attributed to fabric changes, was reported (Reis et al., 2011).

## 5 CONCLUSIONS

Even within a homogeneous deposit, soils may exhibit spatial variation in cohesion. This paper addressed a numerical study into the effects of spatial variability in cohesion for a simulated soil, attributed to fabric changes, on the ultimate bearing capacity ( $Q_{ult}$ ) for a rigid surface strip footing using an upper bound theorem, UNSAT-DLO approach. The approach, considers the effect of suction on strength, was utilised to model two different categories. Each category comprised three different scenarios. The scenarios accounted for spatial variability by applying different

multipliers ( $M$ ) to cohesion ( $M \times c$ ), representing various improvements in soil fabric resulting from the application of different suction levels.

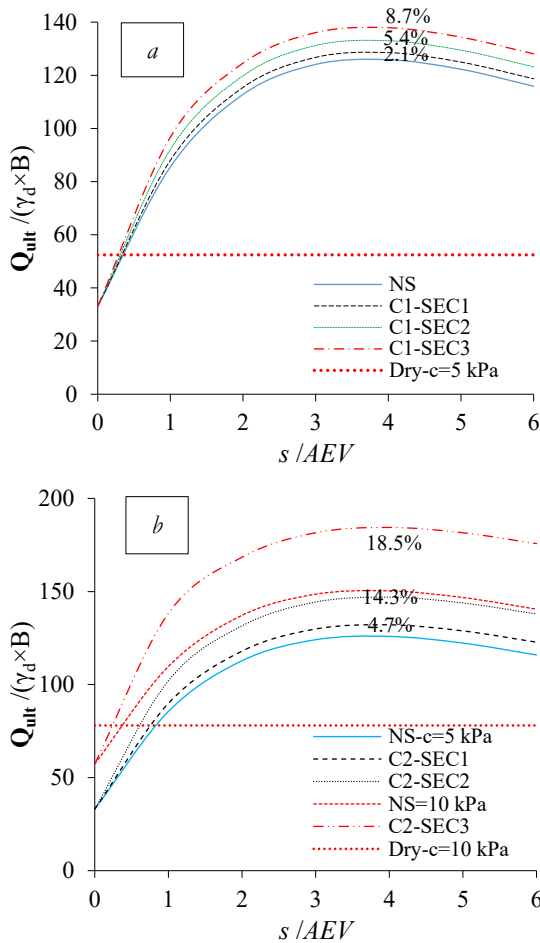


Figure 4. Normalised  $Q_{ult}$  versus normalised suction for various scenarios of the Category (a) C1 and (b) C2.

The numerical results demonstrated a significant effect of spatial variability in cohesion on  $Q_{ult}$ . Up to 8.7% and 18.5% increase in  $Q_{ult}$  were obtained for various applied  $M$  values. Finally, different failure mechanism patterns were observed in the modelled scenarios. A wider and deeper failure mechanism was obtained when a higher  $M$  value was applied to a higher cohesion value case.

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