

Why natural log of soil permeability is a linear function of void ratio for 0-400 μ m soils having a sandy-silty-clayey matrix?

Pourquoi le logarithme décimal de la perméabilité des sols est une fonction linéaire de l'indice des vides pour des sols 0-400 μ m ayant une matrice sablo-limono-argileuse ?

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ABSTRACT: Many authors working on soil permeability put forward the proportionality of its value with the term $e^3/(1+e)$, where e is the void ratio. Others working on soils having a sandy-silty-clayey matrix, consider that the natural logarithm of their permeability is proportional to e . These two relations are very different. The link between them will be presented. Then, the way to calculate the soil permeability through that of the 0-400 μ fraction will be detailed.

RÉSUMÉ: De nombreux auteurs travaillant sur la perméabilité des sols avancent que leurs perméabilités est proportionnelle au terme $e^3/(1+e)$, où e est leur indice des vides. D'autres travaillant sur des sols à matrice sablo-limono-argileuse considèrent que le logarithme décimal de leurs perméabilités est proportionnel à e . Ces relations sont très différentes. Le lien qui les relie est présenté. Puis sera détaillée la manière de calculer la perméabilité du sol au travers de celle de la fraction 0-400 μ .

Keywords: Soil permeability; void index; liquidity limit; plasticity index; volume-specific area.

1 INTRODUCTION

When working on soil permeability k , geotechnical engineers use relationships linking its value to void ratio e . Chapuis, for example propose its proportionality to the term $e^3/(1+e)$ and Tavenas put forward a linear relationship between the natural logarithm of k and e .

In order to make the comparison possible between the two relationships, a general geotechnical framework is at first proposed in order to classify the type of soil we are working on. Then, the importance of the notion of the equivalent vertical stress σ'_{eq} will be presented. Comparing the correlation for 0-400 μ m soils due to Terzaghi between compressibility index C_c and the liquidity limit w_l to the relationship in a oedometer test between C_c and the vertical pressures σ'_1 and $\sigma'_{7.5}$ explaining water-contents equal to liquidity limit and 7,5% will be deduced the value of $\log \sigma'_{7.5}$. Then, still in an oedometer test diagram, we

will develop a relationship between void ratio at a vertical pressure of σ'_{eq} defined in paragraph 3 and a relationship between soil dry density and liquidity limit and σ_{eq} . It will then imply a linear relationship between $\log(e^3/(1+e))$ and e for 0-400 μ m soils. For 0-D soils, a difficulty will remain, that is the evaluation of the volume-specific surface as proposed by Chapuis and appearing in the general relationship giving k . Will then be presented improved ways compared with those suggested by Chapuis and Locat, soil permeability being deduced from the assessment of the permeability of the 0-400 μ fraction.

2 PROPOSED GENERAL GEOTECHNICAL FRAMEWORK.

We work on soils having a 0-D particle size distribution curve, D being the lowest diameter of the

maximum particle size. The passing value at 400µ is written %400µ.

It is then important to distinguish those where D is equal to or less than 400µm, and when D is greater than 400µm, those for which 400µ-D grains are scattered in the 0-400µ matrix and those where they are in contact. When scattered, then a consolidation process explains their density. In both cases, we can notice that soil permeability of the 0-D soil is the permeability of the 0-400µ fraction, adjusted by the surface of the 0-400µ fraction passed through by the waterflow:

$$k_{0-D} = \%400\mu(\gamma_{d0-D} / \gamma_{0-400\mu}) k_{0-400\mu} \quad (1)$$

Secondly, the dry density of the total soil γ_{d0-D} , of the 0-400µ fraction $\gamma_{d0-400\mu}$ and of the grains γ_s are linked in both cases by the relationship:

$$1/\gamma_{d0-D} = \%400\mu / \gamma_{d0-400\mu} + (1-\%400\mu) / \gamma_s \quad (2)$$

For 0-400µm soils, grains 400µ-D scattered, they are generated by a consolidation process, through a first loading at a preconsolidation pressure σ'_p followed by an unloading at σ'_{v0} kPa, vertical effective pressure. The soils are supposed not to be cemented. They don't include organic matters and are not saline. Their sensitivity St is lower than 4.

3 EQUIVALENT VERTICAL PRESSURE

The equivalent pressure σ'_{eq} is the vertical pressure that would lead through a normally consolidated process to the same soil void ratio than that due to the loading and unloading process. Its value is linked to σ'_p and σ'_{v0} through the relationship:

$$\sigma'_{eq} = \sigma'_{v0}^{(Cs/Cc)} \cdot \sigma'_p^{((Cc-Cs)/Cs)} \quad (3)$$

Cc compression index and Cs recompression index, measured by an oedometer test, then for D less than 3,2mm.

4 DRY DENSITY, LIQUIDITY LIMIT AND EQUIVALENT VERTICAL PRESSURE

For 0-400µm soils, one correlation is well known between Cc compressibility index and liquidity limit wl due to Terzaghi:

$Cc = 0,009 (wl - 10)$ that can be written with little difference:

$$Cc = 0,0085(wl - 7,5), \text{ wl in } \% \quad (4)$$

Looking at an oedometer diagram, we can write:

$$Cc = \gamma_s / \gamma_w (wl - 7,5) / 100 / \log(\sigma'_{7,5} / \sigma'_i) \quad (5)$$

where σ'_i and $\sigma'_{7,5}$ are vertical effective stresses leading to void ratios corresponding to a saturated water content equal to liquid limit and to 7,5%, γ_s and γ_w being grains and water density.

Comparing (4) with (5), it appears that for $\log \sigma'_{i=1}$ corresponding to $\sigma'_i = 10$ kPa, then $\log \sigma'_{7,5} = 4,2$. We can then write, using the second relationship hereafter:

$$Cc = \gamma_s / \gamma_w (w_{noc} - 7,5) / 100 / (\log \sigma'_{7,5} - \log \sigma'_{eq}) \quad (6)$$

$$\gamma_w / \gamma_d = \gamma_w / \gamma_s + 0,075 + 0,00315 (wl - 7,5) (4,2 - \log \sigma'_{eq}) \quad (7)$$

or, with σ'_{eq} in kPa:

$$eoc = \gamma_s / \gamma_w (0,075 + 0,00315 (wl - 7,5) (4,2 - \log \sigma'_{eq})) \quad (8)$$

or:

$$eoc = 0,20 + 0,0085 (wl - 7,5) (4,2 - \log \sigma'_{eq}) \quad (9)$$

For 0-D soils, we will have to make a correction on γ_d through relationship (2).

5 GENERAL RELATIONSHIP GIVING k

For a water at a temperature of 10 degrees Celsius, Kozeny and Carman general equation giving k in m/s is:

$$k = 1,5 \cdot 10^{-6} (1/vssa)^2 e^3 / (1+e) \quad (10)$$

where vssa is the volume-specific surface area (m^{-1}), calculated through the gradation curve as proposed by Chapuis and Locat, and developed in paragraph 7.

For a water at a temperature t, k given by (9) must be multiplied by the expression:

$$\alpha = 0,737 + 0,025 t + 0,00016 t^2 \quad (11)$$

due to the water dynamic viscosity variation with temperature. In Martinique island, for a water at 30 degrees, k must be multiplied by 1,63.

6 CALCULATION OF $\log(e^3/(1+e))$ AS A FUNCTION OF w_l AND e FOR 0-400µ SOILS

Through relationship (9), we calculate values of e for varying values of σ'_{eq} and liquid limit w_l . Then we calculate $\log(e^3/(1+e))$ and compare its value to e .

The results are summarized in Table 1.

Table 1. values of e for varying values of σ'_{eq} and liquid limit w_l .

w_l	σ'_{eq} kPa	50	100	150	200
0,20	E	0,46	0,43	0,41	0,40
	$\log(e^3/(1+e))$	-1,16	-1,24	-1,30	-1,33
0,40	e	0,89	0,81	0,76	0,72
	$\log(e^3/(1+e))$	-0,43	-0,53	-0,60	-0,66
0,60	e	1,32	1,18	1,10	1,05
	$\log(e^3/(1+e))$	-0,007	-0,12	-0,19	-0,25
0,80	e	1,74	1,56	1,45	1,37
	$\log(e^3/(1+e))$	0,28	0,17	0,09	0,03
1,00	e	2,17	1,93	1,79	1,69
	$\log(e^3/(1+e))$	0,51	0,39	0,31	0,26

Looking at the values displayed in Table 1 and shown in Figure 1, it appears that they obey to the general relationship:

$$\log(e^3/(1+e)) = A + Be \quad (12)$$

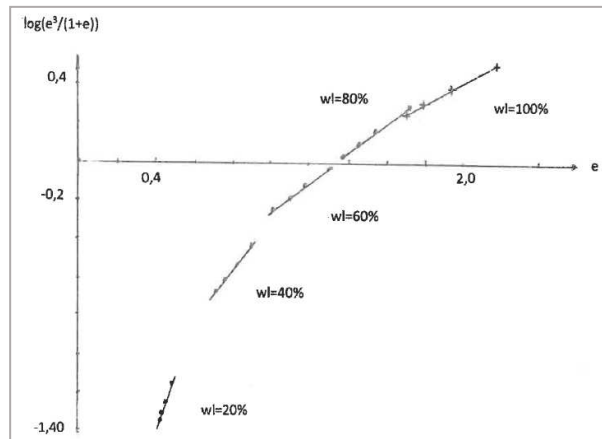


Figure 1. Relations between $\log(e^3/(1+e))$ and e .

Table 2. Values of A and B

w_l	0,20	0,40	0,60	0,80	1,00
A	-2,444	-1,658	-1,200	-0,884	-0,638
B	2,762	1,382	0,907	0,710	0,528

It appears that A and B are hyperbolic functions of w_l , as shown by Figure 2:

$$A = -0,706/(w_l + 0,0883) \quad (13)$$

$$B = 0,529/(w_l - 0,009) \quad (14)$$

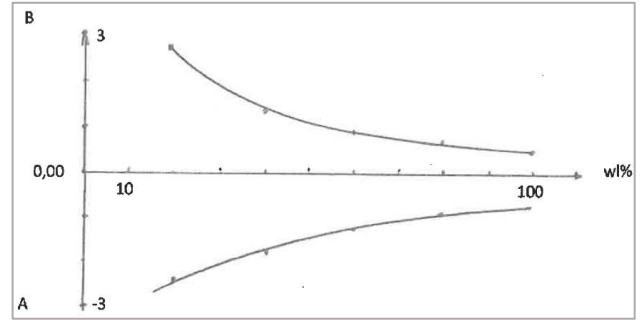


Figure 2. Values of A and B function of w_l .

Then taking the natural logarithm of each member, relationship (10) becomes:

$$\log k = 6,176 - 2\log vssa + \log(e^3/(1+e)) \quad (15)$$

and using expressions (12), (13) et (14):

$$\log k = 6,176 - 0,706/(w_l + 0,0883) - 2\log vssa + 0,529e/(w_l - 0,009) \quad (16)$$

In the proposed framework of thinking, it appears that the natural logarithm of permeability is indeed a linear function of void ratio e for 0-400µm soils.

7 CHAPUIS AND LOCAT CALCULATION OF VSSA

We need to have the soil particle size distribution and the soil dry density γ_{d0-D} . We will use relationships (1) and (10), and for the latter $vssa$ and e are those of the 0-400µ fraction.

7.1 Soils having a non-plastic matrix, PI of the 0-400µ fraction being less than 10.

Chapuis and Legaré (1992), suggest to calculate $vssa$ through the grain size curve, and the expression:

$$vssa = 6 \sum (Pnod_i - Pnod_{(i+1)}) / (d_{(i+1)}) \quad (17)$$

$(Pnod_i - Pnod_{(i+1)})$ is the difference between the percentage by weight smaller than size d_i and that larger than next size $d_{(i+1)}$, $vssa$ unit being m^2/m^3 . The grain size curves always have a minimum measurable particle size d_{min} . In the method of Chapuis and Legaré, an equivalent size d_{eq} must be defined for all particles smaller than d_{min} through the expression:

$$d_{eq}^2 = d_{min}^2 / 3 \quad (18)$$

the percentage in weight dedicated to d_{eq} being the difference between 100% and the percentage value

corresponding to d_{min} . Secondly (figure 3), in the fine size zone, the grain size distribution has to be modified to follow a minimum slope of 20% (Chapuis).

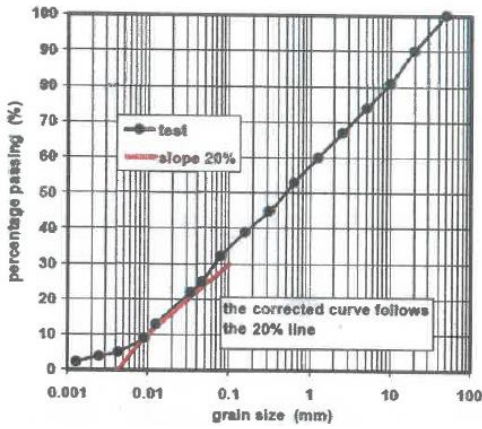


Figure 3. Chapuis proposed modification if slope flatter than 20%.

The assessment of v_{ssa} is made on the 0-400 μ fraction. The void index is that of the 0-400 μ fraction, equal to the void index of the 0-D soil, divided by the volume of the 0-400 μ grains:

$$e_{0-400\mu} = (\gamma_s/\gamma_{d0-D} - 1)/(\%400\mu \gamma_{d0-D}/\gamma_s) \quad (19)$$

7.2 Soils having a plasticity index of the 0-400 μ fraction greater than 10.

The m_{ssa} value is that of the 0-400 μ fraction calculated through Locat (1984) proposals. Chapuis gives on Figure 4, Locat correlation between mass specific surface area and percentage smaller than 2 μ m. Based on this suggestion, we propose the underlying correlation:

$$m_{ssa} \text{ in } m^2/kg = 10^3(5 + 1.2 PI) \quad PI \text{ in } \% \quad (20)$$

We know (Gress) that:

$$PI = -4(1 - \%2\mu) + 4.5 \%2\mu VB2\mu \quad (21)$$

VB2 μ being the methylene blue value of the 0-2 μ fraction.

(20) can then be expressed as:

$$m_{ssa} = 10^3 (0.2 + \%2\mu (4.8 + 5.4 VB2\mu)) \text{ in } m^2/kg \quad (22)$$

Figure N°4 gives the link between m_{ssa} value of the 0-400 μ fraction, the passing at 2 μ and the blue methylene value of the 0-2 μ fraction VB 2 μ .

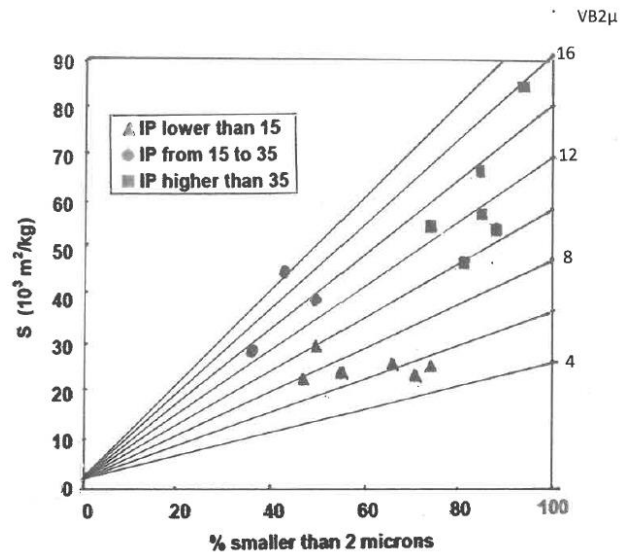


Figure 4. Locat highlighted by VB2 μ values.

The link between v_{ssa} and m_{ssa} is given by:

$$v_{ssa} = \rho_s m_{ssa}, \rho_s \text{ given in } kg/m^3 \quad (22)$$

8 CONCLUSIONS

The reason why the natural logarithm of permeability is a linear function of void ratio for 0-400 μ soils has been explained.

Furthermore, ways to estimate the soil permeability have been given in order to calculate the permeability of all 0-D soils through Kozeny- Carman expression, through that of the 0-400 μ fraction.

All these proposals are valid in the general framework of working, we have detailed. Difficulties will still be due to the quality of the samples on which we will measure dry densities and determine grain size curves.

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