A Geometric Approach to Assessing Erodibility in Cohesionless Soils

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ABSTRACT: In this study, we introduce a novel empirical method for assessing the erodibility of cohesionless soils. This method involves the calculation of a control diameter that represents the coarser fraction of the soil while considering key parameters that influence the soil's response to erosion. By comparing this control diameter to the characteristic diameter of the loosely composed finer fraction, we can effectively evaluate the soil's internal stability.

Keywords: Internal stability; Constriction size, Erosion, Geometric model.

1 INTRODUCTION

A comprehensive understanding of transportation mechanisms in porous media necessitates the development of an appropriate model for the pore network. Mass and fluid transfer within soils are influenced by several factors including the morphology, dimensions, roughness of pore channels, and the constriction size. Constrictions refer to the narrowest passages between pores, serving as the primary barriers for fine particles when subjected to seepage flow through a granular filter (Sjah & Vincens, 2013). Therefore, evaluating the soil's filtration capacity involves determining the cumulative constriction size distribution [CCSD] of the soil. The [CCSD] represents the range of constriction sizes among the coarsest particles, and their corresponding proportions. This information is crucial for assessing retention capacity of the soil, the discharge of particles and its internal stability. (Moraci et al., 2012; Taylor et al., 2016; Vincens et al., 2015).

Silveira et al., (1975) have demonstrated a correlation between the filtration potential of fines by a coarser particle and the characteristics of the void network, particularly the size of constrictions within the granular filter. Indaratna et al., 2007 proposed a method to determine the constriction control size, denoted as (D_{c35}), which represents the constriction diameter that corresponds to 35% of fines. If the value of D_{c35} is smaller than the diameter of the largest loose particle, the soil is considered internally stable. To assess internal stability of cohesionless soils against suffusion, (Dallo et al., 2013) recommended that (D_{c35}) of the soil skeleton be compared with the diameter (d_{85}) of the finest loss particles. Furthermore, Dallo & Wang (2016) used the model of the capillary tube (Kovac model) to evaluate the size of the control constriction and proposed a new limit to discriminate between stable and unstable soils inside. This approach is applicable to examining the stability against suffusion of granular soils as well.

It should be noted that these models often involve a comparison between the typical particle size of the finer looser fraction, and the control constriction size derived from the [CCSD] of the coarser fraction. Such an assessment method overlooks several significant factors which can have important impact on the process of the soil erosion. To address these limitations, our research aimed to investigate the erosion processes by conducting experiments using mixtures of sand (with a maximum particle size of d_{max}=2mm) and fines (with a maximum particle size of d_{fmax}=100µm) under various testing conditions. The primary objective was to identify the key parameters that govern suffusion development probability. Based on the contribution of these parameters, we have proposed a novel approach to evaluate the stability of cohesionless soils against suffusion.

2 EXPERIMENTAL STUDY AND DATA PROCESSING

2.1 *Tested materials*

To investigate the impact of the coarser fraction's granulometry on erosion processes, we utilized commercially sands with a defined gradation. These sands, denoted as C1, C2, C3, C4, and C5, were selected for the study. In order to simulate the presence of erodible fine particles within the voids created by the coarser skeleton, we employed silica flour. Figure 1 illustrates the grain size distribution of both materials.

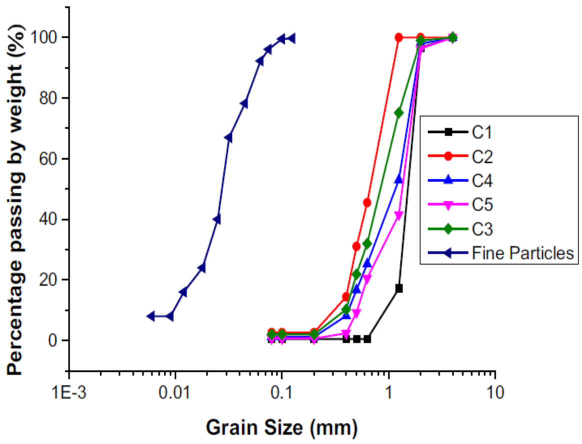

Figure 1. Grain size distribution of tested materials.

2.2 *Experimental setup and testing procedure*

The tested samples were subjected to hydraulic stress by descending flow using a constant head permeameter (Figure 2). For the preparation of the samples, firstly, the sand was mixed for three minutes with a moisture content below the optimum Proctor level. As the mixing continues, fines are gradually incorporated into the mix as required. The mixing process was extended for an additional ten minutes. After mixing, the soil was compacted into the cylindrical mold in three layers, applying a specific energy. Mechanical consolidation was then carried out by applying vertical load using a hydraulic press. Subsequently, a depression was applied to the sample using a vacuum pump. Following saturation, the sample was subjected to hydraulic stress through a downward vertical flow. For each suffusion test conducted to evaluate the stability against suffusion, four different hydraulic gradients (i.e., 5, 15, and 20 m/m) were applied. The effluent from the sample was collected at the exit of the mold at specific time intervals. After measuring the effluent volume, the mass of eroded particles was determined by weighing them after drying.

2.3 *Results of the experimental study and data processing*

The outcomes of the diverse tests enabled us to establish a connection between soil erodibility and (1) the distribution of grain sizes within the solid component, represented by the uniformity coefficient, (2) the initial fines content, (3) the initial porosity of the solid portion, and (4) the global porosity of the soil, taking into account the porosity reduction caused by the presence of the fines particles. A thorough evaluation of soil erodibility necessitates consideration of the aforementioned parameters.

Figure 2. Experimental device

3 ASSESSMENT OF THE INTERNAL INSTABILITY BY THE CSD CRITERIA

Various methods have been suggested for determining and calculating the [CSD], including experimental, numerical, and analytical approaches (Vincent et al., 2014; Shir and Sullivan, 2016). In our research, we used an analytical technique based in the 'inscribed circle method' concept, introduced by Silveira et al., (1975). This method involves representing the porous medium within a 2D plane, where coarser particles and constrictions are represented as circles. The compacted state of the soil is represented by a configuration of three spheres (Figure 3), whereas the less compacted state is illustrated by a configuration of four circles.(Figure 4).

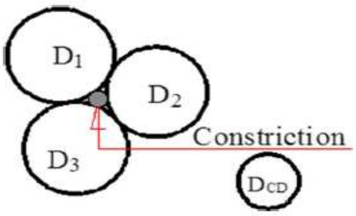

Figure 3. Particle arrangements in dens state

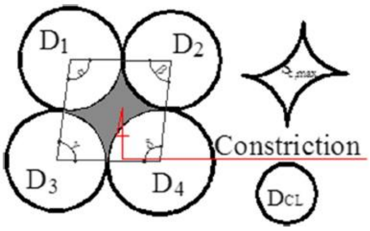

Figure 4. Particle arrangements in loosest state

In the most densely arranged configuration (Figure 3), the constriction size D_{CD}, which aligns with the diameter of the largest circle capable of fitting within three mutually touching filter particles, can be determined using formula (1):

$$\left(\frac{2}{D_1}\right)^2+\left(\frac{2}{D_2}\right)^2+\left(\frac{2}{D_3}\right)^2+\left(\frac{2}{D_{CD}}\right)^2=0.5\left[\left(\frac{2}{D_1}\right)+\left(\frac{2}{D_2}\right)+\left(\frac{2}{D_3}\right)+\left(\frac{2}{D_{CD}}\right)\right]^2 \tag{1}$$

Where D_1, D_2, and D_3 represent the diameters of the three particles constituting the cluster (Figure 3). In the least compacted state (Figure 4), the constriction size D_{CL} can be computed using the aforementioned equation. (Silviera et al., 1975):

$$D_{CL}=\sqrt{\frac{4S_{c\max}}{\pi}} \tag{2}$$

Where S_{Cmax} denotes the maximum area enclosed by the four mutually touching particles (Figure 4), which can be calculated using the subsequent formula (Silveira et al., 1975):

$$S_C = \frac{1}{8}[(D_1 + D_2)(D_1 + D_4)sin\alpha + (D_2 + D_3)(D_3 + D_4)sin\gamma - (\alpha D_1^2 + \beta D_2^2 + \gamma D_3^2 + \delta D_4^2)] \quad (3)$$

Locke et al., (2001) proposed equation (4) to compute the constriction size D_C at any given relative density R_d :

$$D_C = D_{CD} + P_C(1 - R_d)(D_{CL} - D_{CD}) \quad (4)$$

Where D_C is the constriction size associated with a specific percentage of particles P_C and D_{CD} and D_{CL} are the constriction sizes for the denser and looser cases, respectively, corresponding to the same P_C value.

The evaluation of internal stability using the [CSD] criterion is based on the determination of the control constriction size of the coarser fraction of the soil from the cumulative constriction size distribution curve. This measurement indicates the diameter of the constriction that the largest particle can pass through. Subsequently, it is compared with the diameter of a typical particle selected from the set of larger loose particles. Table 1 provides a summary of the results obtained using various models based on this criterion for the tested soils.

Table 1. Assessment of internal stability by the CSD criteria.

	Kenney & Lau (1985)	Indraratna et al., (2007)	Dallo et a.l, (2013)	Dallo & Wang (2016	Laboratory
C_1+5%F	U	U	U	U	U
C_1+10%F	U	U	U	U	U
C_1+20%F	S	U	U	U	S
C_2+5%F	S	S	S	S	S
C_2+10%F	S	S	S	S	S
C_2+20%F	S	S	S	S	S
C_3+5%F	S	S	S	S	S
C_3+10%F	S	S	S	S	S
C_3+20%F	S	S	S	S	S
C_4+5%F	U	S	S	S	S
C_4+10%F	S	S	S	S	S
C_4+20%F	S	S	S	S	S
C_5+5%F	U	S	S	S	U
C_5+10%F	U	S	S	S	U
C_5+20%F	S	S	S	S	S

U unstable, S stable

4 PROPOSITION OF A NEW MODEL FOR ASSESSING SUFFUSION SUSCEPTIBILITY

In our proposed model, based on experimental findings, we incorporate multiple factors in assessing the probability of suffusion occurrence in cohesionless soil (expressed as Sp in equation (5)). These factors include the granulometric distribution of the coarser portion ($GSD^{(C)}$), the granulometric distribution of the finer portion ($GSD^{(F)}$), the CSD of the coarser fraction, the initial content of fines (F%), the porosity of the coarser fraction (n_c), and the total porosity of the soil sample (n). Equation (5) represents this relationship:

$$S_p = f(GSD^{(C)}, GSD^{(F)}, CFC, F\%, n, n_C) \quad (5)$$

The grain size distribution of the coarser fraction ($GSD^{(C)}$) and the finer fraction ($GSD^{(F)}$) were determined through sieve analyses. The cumulative constriction size distribution (CSD) was calculated using the methods outlined by Silveira et al., (1975), assuming that the coarser fraction is relatively dense based on laboratory conditions.

The porosity values, n and n_c, were determined through experimental measurements. However, the porosity of the coarsest fraction, n_c, can be estimated by relating it to the total porosity of the medium, n, and the initial amount of fines, F, using the equation below:

$$n_C = n + (1+n)F \tag{6}$$

The interplay between the initial quantity of fines and the porosity significantly affects the erosion process, as the increase in fines leads to a reduction in the available void space within the medium, thereby decreasing porosity. This relationship can be incorporated into equation (5) by introducing the ratio (n/n_c).

The gradation of the larger particle fraction [$GSD^{(C)}$] is represented by the uniformity coefficient (C_u) of the solid component. On the other hand, the distribution pattern of the smaller particle fraction [$GSD^{(F)}$] is expressed by the characteristic diameter of the loosely composed finer particles (d_{85}), which signifies 85% of the smallest particles within the fine fraction. The cumulative constriction size distribution [$CCS^{(D)}$] is represented by the controlling constriction size (D_{C35}) as recommended by (Indraratna et al., 2007; Dallo et al., 2013; Dallo & Wang, 2016).

Based on equation (5), the probability of suffusion development can be expressed using the following relationship:

$$S_p = f(C_U, n/n_C, D_{C35}, d_{85}) \tag{7}$$

Within equation (7), the parameters C_u, n/n_C, and D_{C35} correspond to characteristics associated with the coarser fraction, while d_{85} pertains to the fraction containing erodible fines.

The evaluation of soil's internal stability, and consequently its vulnerability to suffusion, is performed as described earlier through the examination of erodible particle filtration. To facilitate this assessment, we have derived a control diameter, denoted as D*, based on the parameters characterizing the coarser fraction. By utilizing equation (7), the expression for D* is as follows:

$$D^* = (f(C_U).C_U + f(n/n_C).\frac{n}{n_C})D_{C35} \tag{8}$$

This control diameter, D*, acts as a criterion for gauging the potential for erodible particle filtration, thereby providing insights into the soil's internal stability and susceptibility to suffusion.

The control diameter can be calculated using equation (8), where $f(C_u)$ represents a function reliant to the uniformity coefficient and f(n/nc) denotes a function of porosity. These two factors, $f(C_u)$ and $f(n/n_c)$, have been determined through statistical analysis of the erosion test results conducted on the soils under investigation.

The data for all samples (the overall porosity of the medium n, the porosity of the coarser fraction nc, the uniformity coefficient C_u, D_{c35} and d_{85}) has been analyzed using commercial software for statistical analysis (SPSS Statistics. V25). The controlling diameter can be determined using the following correlation:

$$D^* = (0.054.C_U + 0.638.\frac{n}{n_C})D_{C35} \tag{9}$$

The mathematical expression for distinguishing between stable and unstable soils is given as (D*/d85 = 1). When ($D^*/d_{85} < 1$), the soil is considered internally stable, whereas ($D^*/d_{85} > 1$) indicates internal instability. The results obtained from employing this methodology on the investigated soils are succinctly outlined in Table 2.

Table 2. Analysis of test results using yhe proposal model.

	C_u	n/n_c	D_{c35} [mm]	D^* [mm]	D^*/d_{85}	Prediction stability	Test
C_1+5%F	1.81	0.88	0.221	0.146	1.71	U	U
C_1+10%F	1.81	0.74	0.221	0.126	1.48	U	U
C_1+20%F	1.81	0.42	0.221	0.081	0.95	S	S
C_2+5%F	2.44	0.94	0.077	0.056	0.66	S	S
C_2+10%F	2.44	0.86	0.077	0.052	0.62	S	S
C_2+20%F	2.44	0.69	0.077	0.044	0.52	S	S
C_3+5%F	2.50	0.93	0.092	0.067	0.79	S	S
C_3+10%F	2.50	0.85	0.092	0.062	0.73	S	S
C_3+20%F	2.50	0.66	0.092	0.051	0.60	S	S
C_4+5%F	3.39	0.93	0.109	0.085	1.00	S	S
C_4+10%F	3.39	0.85	0.109	0.079	0.93	S	S
C_4+20%F	3.39	0.67	0.109	0.067	0.78	S	S
C_5+5%F	2.89	0.90	0.131	0.096	1.13	U	U
C_5+10%F	2.89	0.79	0.131	0.086	1.02	U	U
C_5+20%F	2.89	0.54	0.131	0.066	0.77	S	S

U unstable, S stable

5 CONCLUSION

In this study, a novel approach was employed to evaluate the internal stability of cohesionless soils. Experimental investigations were conducted on sand-fine mixtures under various testing conditions. These experiments established a correlation between soil erodibility and granulometric distribution of both the coarse fraction and the fine fraction. The initial fines content, as well as the initial porosity of the coarser fraction and the global porosity of the soil, was taken into account to address the porosity reduction caused by the presence of fines. The assessment of the vulnerability of cohesionless soils to suffusion has mainly been based on the constriction size distribution criteria concept, which offers a more complete description of the soil's retention capacity process.

The constriction size distribution of the examined soils was ascertained through an analytical approach rooted in the concept of inscribed circles. An empirical formula, incorporating the main parameters influencing erosion processes, was derived to calculate a controlling diameter that characterizes the coarser fraction. Through the comparison of this controlling diameter with the diameter of a representative particle derived from the larger loosest fine particles, it becomes possible to establish the demarcation line between stable and unstable soils. In contrast to conventional models found in existing literature, the approach introduced in this study provides an enhanced evaluation of the vulnerability of cohesionless soils to suffusion.

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