Analysis by FE vertical slices model of a buried culvert using Duncan-Chang hyperbolic soil model

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ABSTRACT: This paper introduces a new approach, referred to as the FE vertical slices model, which is specifically designed to study the response of a box culvert under distributed load acting on the soil's surface. The applied finite element procedure treats the culvert and its surrounding soil as a pseudo three-dimensional medium, which is divided into vertical slices. Each of these slices is considered as a plane stress problem, with the soil obeying the nonlinear hyperbolic model. To validate the effectiveness of this method, a comparative analysis is conducted with other numerical methods. A limited parametric investigation is carried out to assess how the dimensions of the box culvert and its embedment influence the distribution of vertical pressures.

1 INTRODUCTION

Box culverts play a crucial role in transporting essential substances necessary for our everyday activities. These infrastructures encompass a range of types, including water, sewer systems, gas, electricity, communication cables, as well as offshore pipelines, oil lines, coal pipelines, subway tunnels, and heat distribution lines. With advancements in engineering science, it is now feasible to design these underground structures with a precision level comparable to that achieved in the design of buildings and bridges.

Box culverts are typically categorized according to the material they are constructed from or the method of installation. In terms of material, culverts can be classified as either rigid or flexible. Rigid culverts are commonly constructed from reinforced concrete, which may be precast or cast-in-place. Flexible culverts, on the other hand, can be made from a variety of materials including steel, cast iron, aluminum, plastic, or other materials. Installation methods for culverts vary and may include placement under embankments and within trenches. Furthermore, box culverts can be installed as single-cell structures or as multiple cells depending on their intended use.

Many researchers have undertaken soil pressure measurements on box culverts under embankment installations in the field. They demonstrated that the soil pressure distribution on the top slab is parabolic with a high soil structure interaction factor at the edge and a low factor at the center (Dasgupta and Sengupta 1991; Yang 2000; Pimentel et al. 2009).

This type of soil culvert interaction problem has been investigated previously through centrifuge and numerical studies. For example, Abuhajar et al. (20015a) investigated experimentally and numerically the effect of different soil properties, soil column height, and culvert geometry on the response of box culverts. Stone et al. (1991) and Stone & Newson (2002) performed centrifuge tests on different box culverts with different configurations and relative culvert soil stiffness, their results demonstrated the significant impact of the culvert soil relative stiffness on the soil arching. Kim & Yoo (2005) examined soil response under the culvert and its effect on the factors. Kang et al. (2008) studied the effect of side friction of a box culvert on the factors and related them to the ratio of fill height to culvert width.

Abuhajar et al. (2009) analyzed numerically centrifuge test models performed by Stone & Newson (2002) and made similar observations.

In this paper, a computer program NABCULVERT (Nonlinear Analysis of Box CULVERT) has been written specifically to examine the response of box culvert under distributed load applied at the surface of soil. After model validation, a parametric study was carried out to study the soil fill height to culvert width ratio. The results of this restricted parametric study are presented graphically.

2 THE PSEUDO 3D FE METHODOLOGY

The finite element vertical slices model (VSM) proposed by Amar Bouzid et al. (2005) is based upon the combination of 2D finite element and finite difference methods. It approximates a full 3D problem into a series of 2D vertical interacting panels (slices) which can be solved using a standard 2D FE discretization, while allowing the slices interaction to be accounted for using the finite difference method by replacing the inter-slice interactions by fictitious body forces. which proved its ability to capture the elastic behavior of many soil/structure interaction problems.

This method is extended with non-linear behavior of soil using the hyperbolic model proposed by Duncan and Chang (1970). Referred to as the Nonlinear Finite Element Vertical Slices Model (NFEVSM), this numerical approach involves discretizing the 3D soil/structure medium into a sequence of vertical slices, with each slice represented by a 2D boundary value problem. The methodology integrates both finite element (FE) and finite difference (FD) methods to analyze simultaneously the embedded structure and the surrounding soil substructure.

As more details of about the theoretical developments of this numerical procedure have been given and thoroughly explained in another paper (Otsmane & Amar Bouzid (2018), Amar Bouzid et al. (2018)), only a short description will be given here.

The three-dimensional aspect of the problem, which consists of several vertical panels (slices) on different thicknesses is accounted for by coupling the shear forces between the slices. The three-dimensional soil/structure problem presented in Figure 1 shows a soil/ structure interaction problem and the vertical slices model where the different slices are acted upon by external forces and body forces (Fig. 1).

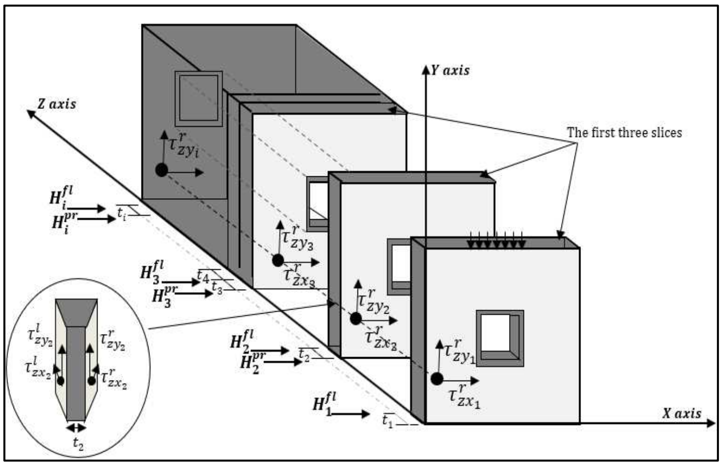

Figure 1. The Vertical Slices Model showing the interacting slices subjected to external and body forces.

The response of an individual slice to an external loading is governed by the following equilibrium equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + f_x = 0, \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + f_y = 0 \quad (1)$$

f_x and f_y are considered to be body forces in this model. For a given slice i, these forces can be mathematically expressed as:

$$f_{xi} = \frac{\tau_{zxi}^{l} - \tau_{zxi}^{r}}{t_i}, \; f_{yi} = \frac{\tau_{zyi}^{l} - \tau_{zyi}^{r}}{t_i} \quad (2)$$

Where $\tau_{zx_i}^{l}$ and τ_{zyi}^{l} are shear stresses acting at the left interface of slice i, whereas $\tau_{zxi}^{r}$and $\tau_{zy_i}^{r}$are shear stresses acting at the right interface of slice $i, t_i$is the slice thickness.

According to the standard formulation in the displacement based finite element method, the element stiffness matrix in slice i can be written as:

$$\int_v (B^t D\, B + N^t\, C^{pc}\, N)\; a_i dv = \int_v (N^t\, C^{pr}\, N)\; a_{i-1} dv + \int_v (N^t\, C^{fl}\, N)\; a_{i+1} dv + P_i \quad (3)$$

P_i in equation (3), is the external forces vector to which the slice i is subjected and a_{i-1}, a_i

a_{i-1} and a_{i+1} are the element nodal displacement vectors of slices $i-1$ and $i+1$ respectively. I is the identity matrix and D is the matrix corresponding to a plane stress problem.

The other parameters are:

$$C^{pc} = c_i^{pc} I, C^{pr} = c_i^{pr} I \text{ and } C^{fl} = c_i^{fl} I \quad (4)$$

With:

$$C_i^{pr} = \frac{2}{t_i(\frac{t_{i-1}}{G_{i-1}} + \frac{t_i}{G_i})}, C_i^{fl} = \frac{2}{t_i(\frac{t_{i+1}}{G_{i+1}} + \frac{t_i}{G_i})}, C_i^{pc} = C_i^{pr} + C_i^{fl} \quad (5)$$

$$c_i^{pr} = \frac{2}{t_i\left(\frac{t_{i-1}}{G_{i-1}} + \frac{t_i}{G_i}\right)} \quad , c_i^{fl} = \frac{2}{t_i\left(\frac{t_{i+1}}{G_{i+1}} + \frac{t_i}{G_i}\right)} \quad , \quad c_i^{pc} = c_i^{pr} + c_i^{fl}$$

Expression (3) may be re-written in a compact form as:

$$A_i\, a_i = f_i^{pr} + f_i^{fl} + p_i \quad (6)$$

This equation cannot be resolved straight-fully, since the right-hand terms are not available explicitly at the same time. Thus, it must be resolved according to an updating iterative process:

$$A_i^k\, a_i^k = f_i^{pr^k} + f_i^{fl^k} + p_i \text{with} (k = 1,2, \ldots \ldots \ldots, k_{max}) \quad (7)$$

Where, k denotes for the iteration number and $k_{max}$is determined by a certain convergence criterion. The nonlinearity in VS Model stems from the implementation the hyperbolic model proposed by Duncan and Chang (1970) for modeling the soil. In fact, they found out that both tangential modulus E_i and ultimate stress deviator $(\sigma_1 - \sigma_3)_{ult}$ are dependent on the minor principal stress σ_3. More precisely they suggested for the initial tangent modulus the following formula:

$$E_i = k\, p_a \left(\frac{\sigma_3}{p_a}\right)^n \quad (8)$$

Where k is dimensionless factor termed 'modulus number', n is a dimensionless parameter called 'modulus exponent' and p_a is the atmospheric pressure used to make$k$and$n$non-dimensional.

The ultimate stress difference$(\sigma_1 - \sigma_3)_{ult}$is defined in terms of the actual failure stress difference by another parameter called failure ratio$R_f$which is given by:

$$R_f = \frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{ult}} \quad (9)$$

Using Mohr-Coulomb failure criterion, where the envelope is considered as a straight line, the principal stress difference at failure is related to the confining pressureσ_3 as:

$$(\sigma_1 - \sigma_3)_f = \frac{2c\, cos(\phi) + 2\sigma_3\, sin(\phi)}{1 - sin(\phi)} \quad (10)$$

Where c is the cohesion intercept and ϕ the friction angle. The tangent modulus E_t is given by:

$$E_t = \left[1 - \frac{R_f(1 - sin(\phi))(\sigma_1 - \sigma_3)}{2c\, cos(\phi) + 2\sigma_3\, sin(\phi)}\right]^2 k p_a \left(\frac{\sigma_3}{p_a}\right)^n \quad (11)$$

For unloading and reloading cycles Duncan and Chang proposed the following expression:

$$E_{ur} = k_{ur} p_a \left(\frac{\sigma_3}{p_a}\right)^m \quad (12)$$

Where, $E_{ur}$is the loading-unloading modulus and $k_{ur}$is the corresponding modulus number. In the Duncan-Chang's basic model the Poisson's ratio $\upsilon_s$was assumed to be constant throughout the whole process.

3 VALIDATION OF NABCULVERT

3.1 *Comparative analysis*

The deformation of the soil is the result of a complex interaction between the culvert and the surrounding soil due to the relative rigidity between the soil and the structure, is a critical consideration in the design of the culvert. Factors influencing soil deformation on culvert include the height of the soil above, the geometric configuration of the soil and the properties of the soil surrounding it. Abuhajar et al. (2015a) studied the response of culverts (pipe) to static loads and tests on small-scale physical models were carried out to analyze the soil-culvert interaction by considering the height and density of the soil above and the culvert geometry. The results of these centrifugation tests were used to verify a numerical model (FLAC2D) which was used to study in more detail the response of box culverts to static loads. The results were evaluated in terms of bending moment and soil-culvert interaction factors.

They studied the ratio between the filling height H of the soil and the width B_c of the culvert $(H/B_c = 0 - 6.13)$ for two different thicknesses t of culvert walls, called thin culverts $(t/B_c = 0.06)$ or thick $(t/B_c = 0.12)$. In addition, the variation in the ratio between the thickness and width of the culvert(t/B_c) and the change in soil properties, such as relative soil density, friction angle, dilation angle, modulus of elasticity and Poisson's ratio were examined.

The results showed that the interaction factors of the culverts were not only a function of the column height of the soil above the culvert, but also a function of the culvert thickness, the modulus of elasticity of the soil and the poisson coefficient, as a result, their results have been used to develop equations that can be used to facilitate the design of box culverts and to evaluate the values of static soil pressure and bending moment.

The same mesh and properties of the culvert (table 1) has been adopted in the computations relevant to Abuhajar et al. (2015a) (FLAC2D): 52.0m by 20.0m so the culvert has a dimension of 4.572m × 4.572m and their thickness of 0.533m, the external load is introduced into the model as a distributed load q = 100.0 kN/m applied at the soil surface above the culvert (Fig. 2).

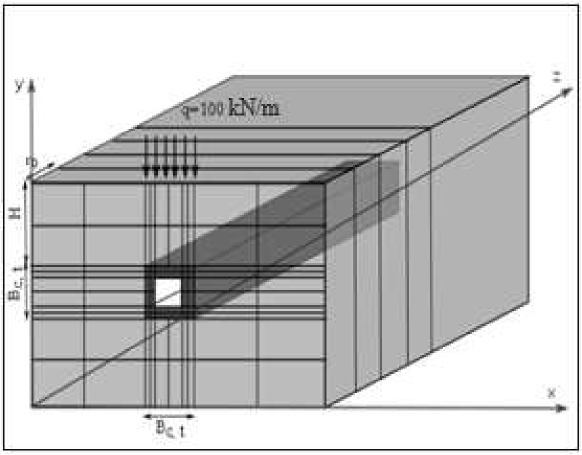

Figure 2. Model parameters

Table 1. Main modeling parameters for the dense sand soil.

Model parameters	Dense sand soil
Relative density $D_r(\%)$	90.0
Mass density $\rho(kg/m^3)$	1687.7
Elastic modulus $E_s(MPa)$	30.0
Shear modulus $G(MPa)$	11.7
Bulk modulus $K(MPa)$	22.7
Poisson's ratio υ_s	0.28
Friction angle $\phi(deg\,r\,ees)$	32.0

Figure 3 illustrates the variation of the vertical stress σ_v(kPa) as a function of the distance of the culvert $B_c(m)$. The results provided by both NABCULVERT and FLAC2D are in good agreement with a slight variation observed in the results of NABCULVERT. This is probably due to the differences in locations where the vertical stresses were calculated in the two softwares.

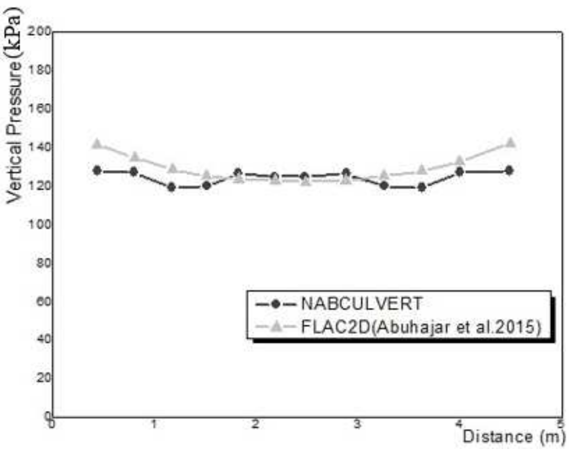

Figure 3. Variation of the vertical stressσ_v(kPa) with the distance B_c of the pipe (upper slab).

The variation of vertical pressures with the distance B_c for a variety of height ratio H/B_c is illustrated in Figure 4.

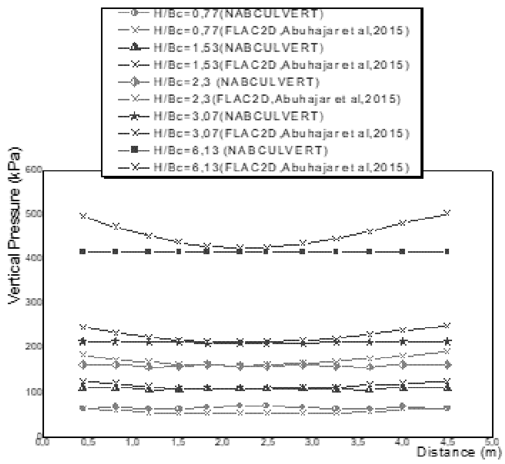

Figure 4. Comparison between NABCULVERT and FLAC2D (Abuhajar et al. 2015a)

The results of Figure 4 show a good agreement between NABCULVERT and FLAC2D. The curves provided by FLAC2D exhibit a parabolic pattern which gets more pronounced when H/B_cincreases. However, the predicted curves by NABCULVERT remain straight.

3.2 *Parametric study*

3.2.1 *The effect of H/B_con the vertical and horizontal pressure*

An application of the buried culvert under load distributed throughout the soil surface was studied (Fig. 5). To study the effect of ratio H/B_c, seven different values of H/B_c equal to 0, 0.77, 1.53, 1.76, 2.30, 3.07 and 6.2 were considered. Culvert thickness was taken as 0.5 m.

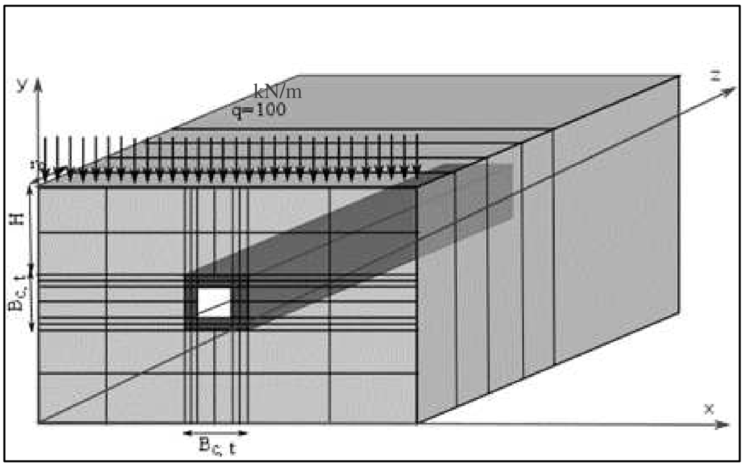

Figure 5. Application of the load q.

Figures 6 and 7 represent the effect of H/B_c on the vertical and horizontal pressure σ_y and σ_x under the effect of the distributed load q throughout the surface of the ground on the upper slab and the two lateral sides of the culvert. We observe that:

- The distribution of the vertical and horizontal stress on the upper slab increases whenH/B_c increases, the same applies to the two lateral sides of the pipe.
- The variation can be significant in the lower part with respect to the upper part for the two lateral ribs of the culvert, so symmetry on the two lateral sides of the pipe has been noticed.
- The magnitude of the vertical pressure is important that of the horizontal pressure.

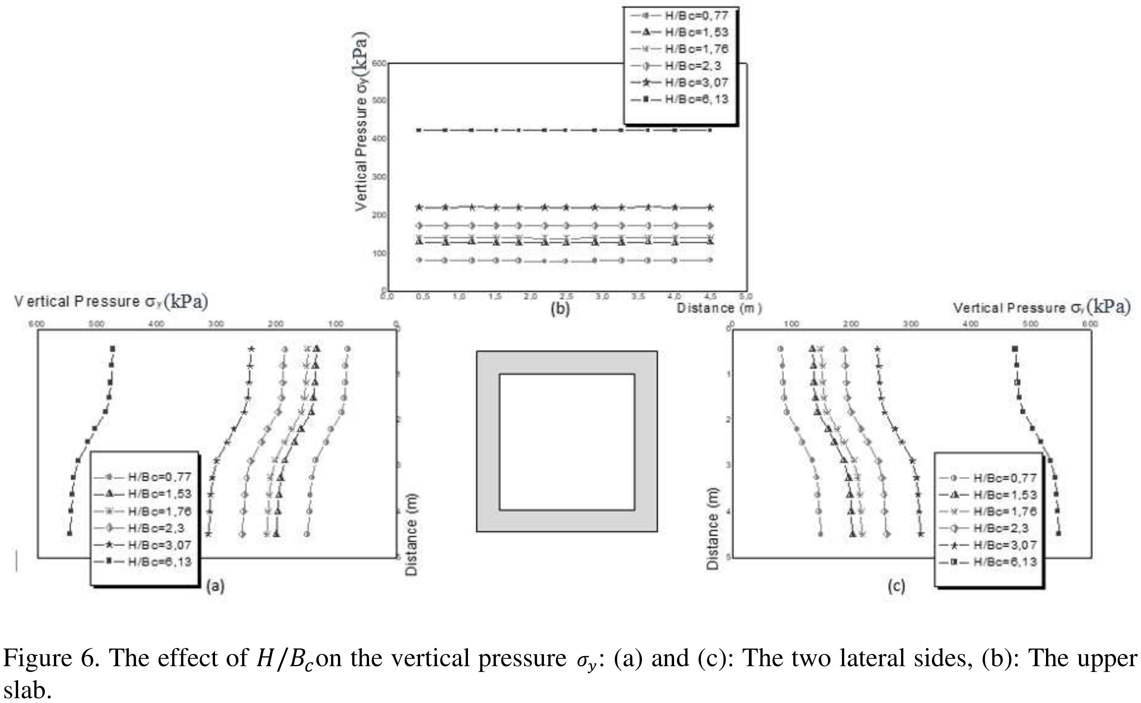

Figure 6. The effect of H/B_con the vertical pressure σ_y: (a) and (c): The two lateral sides, (b): The upper slab.

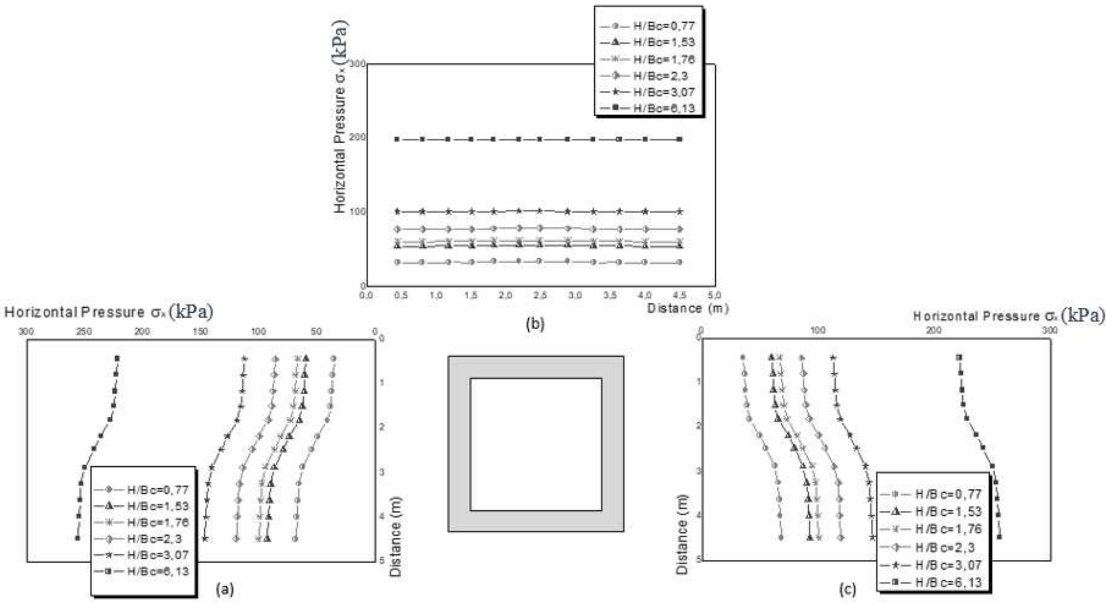

Figure 7. The effect of H/B_con the horizontal pressure σ_x: (a) and (c): The two lateral sides, (b): The upper slab.

3.2.2 *The effect of H/B_con the displacement*

To investigate the effect of H/B_cratio on the displacement, seven different values covering a range of culvert were considered, : H/B_cratio of 0, 0.77, 1.53, 1.76, 2.30, 3.07 and 6.2 with culvert thickness $t = 0.5\ m$.We note that: H/B_chas an effect on the displacement at the surface of the ground, knowing that when the ratioH/B_cincreases the displacement decreases (Fig. 8).

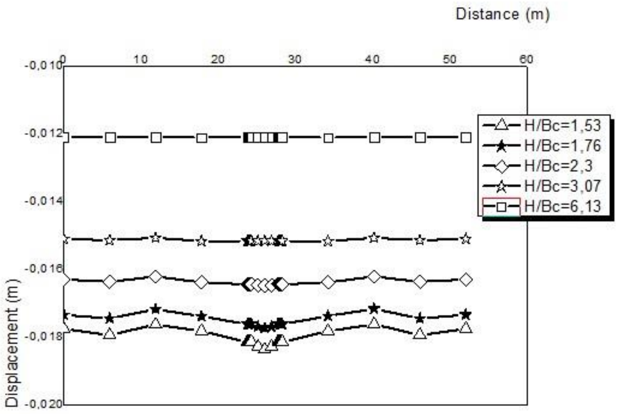

Figure 8. Effect of H/B_con the displacement.

4 CONCLUSION

The proposed model NABCULVET was applied to analyze a buried pipe under external loading, the validation of this model with the study of Abuhajar et al. (2015a) gives results that are in good agreement.

A parametric study of distributed load behavior along the entire soil surface was carried out to investigate the effect of H/B_c that can influence vertical pressure, horizontal pressure and displacement.

It can be seen that as the ratioH/B_cincreases, the distribution of the vertical and horizontal stress on the upper slab increases, the same applies to the two lateral sides of the pipe. Unlike displacement, when the ratioH/B_cincreases the displacement decreases at the level of the soil surface.

So, our NABCULVERT program is reliable because it is defined on the basis of the two powerful numerical methods which are the finite difference method and the finite element method. In addition, it includes the hyperbolic soil model, which is an excellent soil criterion. The predictions obtained are in good agreement with FLAC2D.

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