How many tailings dam cross-sections should be analysed?

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ABSTRACT: The outer wall of an upstream tailings dam must be shown to be strong enough to withstand failure. Sampling of different cross-sections along the wall coupled with slope stability analysis can be used to illustrate this. This paper presents a parametric study to quantify the number of cross-sections that should be sampled to reduce the likelihood of overlooking a weak section. For most levels of spatial variability, sampling at least four cross-sections resulted in an acceptable (less than 1%) chance of overlooking a weak section. To achieve the same level of safety when calculating three-dimensional safety factors from extrapolated two-dimensional cross-sections, required significantly more cross-sections to be sampled (2 - 4 times as many). For this reason, extrapolating two-dimensional cross-sections to conduct three-dimensional analysis is seen as an ineffective way to assess a tailings dam's overall stability. Rather weak zones found by two-dimensional analysis ought to be thoroughly examined in-situ so that variability along the slope's axis can be included in a three-dimensional model tailored to that zone.

1 INTRODUCTION

Recent catastrophic failures of upstream tailings dams have raised concerns regarding stability. The importance of the upstream wall (or shell) in resisting failure has long been acknowledged (Donaldson 1960). Demonstrating that the upstream wall is competent relies on in-situ testing, typically using Cone Penetration Testing (CPT) of the outer prism. At present, there is limited guidance on the optimal number of sections to sample for adequate information to perform a safe stability analysis. This study aims to offer practical recommendations on the number of sections required.

Most slope stability analysis rely on two-dimensional (2D) cross-sections; however, three-dimensional (3D) analysis are becoming more common as commercial software packages become available. While 3D factors of safety are often higher than 2D factors of safety, a reliance on 3D factors of safety to show a factor of safety is adequate is often considered unwise as the safety is likely marginal (Fell et al. 2015). Nevertheless, 3D analyses are likely to be increasingly used and understanding the detail required in models to safely predict factors of safety is needed.

Hicks et al. (2008) and Hicks and Spencer (2010) modelling long embankments with a single stochastic material model (i.e., a random field) showed that for low material variability, failure surfaces encompass the entire length of the slope; whereas for high material variability, failures were discrete. A more extensive site investigation is therefore necessary to locate weak zones when material variability is high. These authors do not provide any details on how extensive investigations should be.

Silva et al. (2008) listed factors of safety and probabilities of failure associated with several slope stability projects. Using this database, four project types differentiated by coefficient of variation (CV) of factors of safety are proposed. Type 1 (best practice) are associated with $0\% < CV \leq 7.5\%$, Type 2 with $7.5\% < CV \leq 10\%$, Type 3 with $10\% < CV \leq 15\%$, and Type 4 (poor practice) with $CV > 15\%$. This study sought to provide guidance for the number of sections required for each type of project. Spatial variability was modelled as a variable internal geometry (i.e., a variable boundary between two material regions). This is analogous to failing to correctly define the extent of the upstream wall.

2 PARAMETRIC STUDY

A parametric study followed the schema illustrated in Figure 1. Component cross-sections had a constant height of 30 m, an outer slope of 1V:2H and a base length of 150 m. Each component cross-section had a different stepped internal boundary (random ordinates between 15 m and 75 m from the outer wall, with either 6, 8, 12 or 20 equal height layers). The outer section was defined with drained strength parameters ($\phi' = 31°$, $c' = 10$ kPa and $\gamma = 19$ kN/m^3) and the internal section of undrained strength parameters ($c_u = 60$ kPa and $\gamma = 20$ kN/m^3). An example of one of the 750 component cross-sections developed for this study is shown in Figure 2.

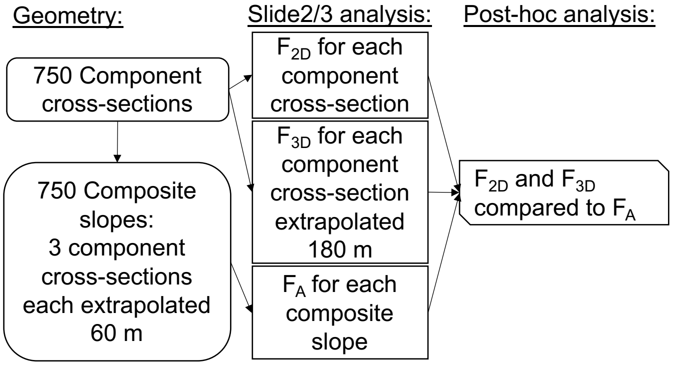

Figure 1. Parametric study schema

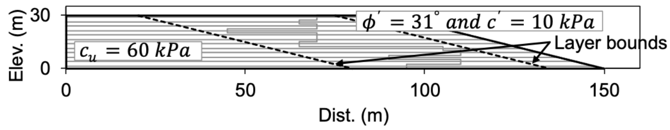

Figure 2. Example of a component cross-section

A 2D factor of safety (F_{2D}) was calculated for each component cross-section using the Spencer (1967) formulation as implemented in Rocscience's Slide 2 using non-circular slip surfaces minimised using the auto refine search method. Figure 3 shows an example output from this stage.

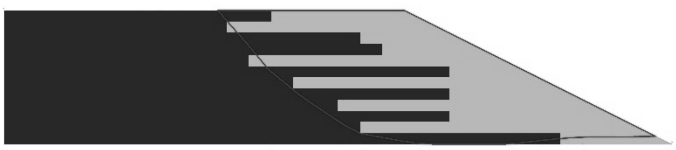

Figure 3. 2D component cross-section (F2D = 0.994)

Component cross-sections were then extrapolated 180 m and a 3D factor of safety (F3D) determined. The Spencer formulation with extensions by Huang et al. (2002) and Cheng and Yip (2007) as implemented in Rocscience's Slide 3 was used with an ellipsoidal slip surface optimised using the cuckoo search method (see Figure 4 for typical output). 750 composite slopes were then formed by joining three component cross-sections each extrapolated 60 m (see Figure 5). Components were chosen systematically, to maximise the number of different composite cross-sections and ensure no repeated combinations. End boundaries were not periodic, that is the search zone was limited to the three components. The factor of safety obtained for a composite slope was assumed to represent the actual stability of the slope (F_A) for which three possible F_{2D} and F_{3D} were available depending on which section of the slope was sampled and analysed.

Composite slopes were divided into four variability groups (see earlier discussion of Silva et al. (2008)) based on the coefficient of variation of the three component F_{2D}. In total there were 306 Type 1 composite slopes, 138 Type 2 composite slopes, 207 Type 3 composite slopes, and 99 Type 4 composite slopes (not equal as component slopes were generated randomly).

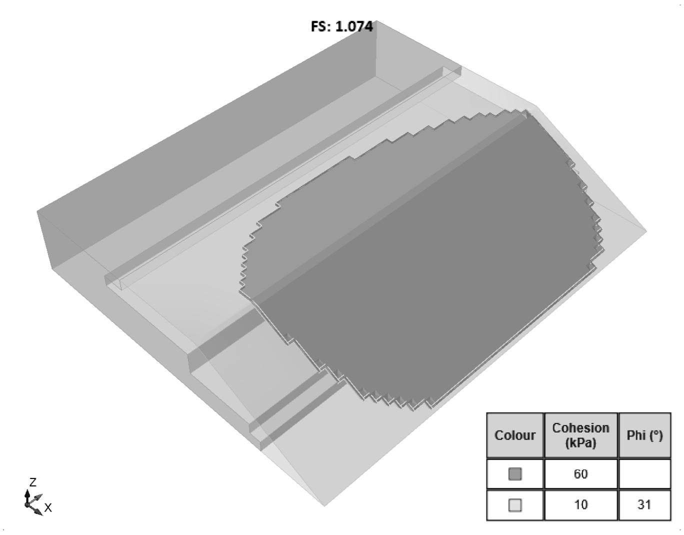

Figure 4. Example 3D component slope (F3D = 1.074)

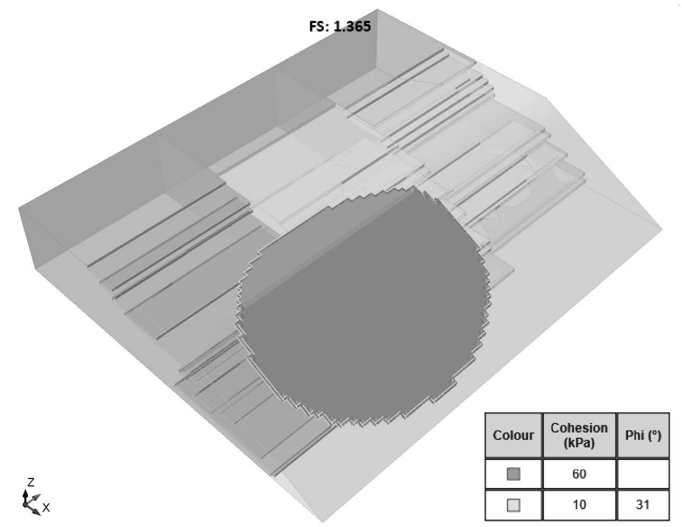

Figure 5. Example of 3D composite slope (F3D = 1.365)

For each composite slope F_A was compared to the three possible F_{2D} and F_{3D} by subtraction (i.e., $F_{2D} - F_A$ and $F_{3D} - F_A$). If the difference was negative then the sample cross-section resulted in a safe analysis (i.e., F_{2D} or $F_{3D} < F_A$), whereas if the difference was positive then the sample cross-section resulted in an unsafe analysis (i.e., F_{2D} or $F_{3D} > F_A$).

Computed differences were grouped per variability group (i.e., Type 1, 2, 3 or 4) to determine the probability of an unsafe analysis based on sampling a single cross-section (P_1). As cross-sections are mutually exclusive the probability of an unsafe analysis based on the number of additional cross-sections (P_n) sampled could then be determined using:

$$P_n = (1 - (1 - P_1)^n) \tag{1}$$

where P_1 = probability of unsafe analysis given one cross-section; and n = number of cross-sections.

3 RESULTS

The distributions of differences (i.e., F2D – FA and F3D – FA) for the four variability groups are shown in Figure 6 using histograms. Bars associated with negative differences (i.e., safe samples) are oriented downwards, whereas bars associated with positive differences (i.e., unsafe samples) are oriented upwards.

Figure 6a shows that for Type 1 slopes (i.e., low spatial variability), if only one section is sampled there is a 1.2% probability (the small darker bar orientated upwards) that this will be unsafe if a 2D factor of safety is calculated (i.e., 1.2% probability that $F_A < F_{2D}$). However, if a 3D factor of safety is determined by extrapolating the single 2D cross-section there is a 43% probability (sum of multiple lighter bars orientated upwards) that this will be unsafe (i.e., 43% probability that $F_A < F_{3D}$). As variability increases (i.e., Type 2, 3 then 4 as shown in Figure 6b, c and then d) the proportion of unsafe samples increases.

Table 1 summarises how the probability of sampling an unsafe cross-section increases as variability increases (consider rows where only 1 section is sampled). Such that for Type 4 slopes, the probability of sampling an unsafe section, if only one section is sampled and analysed in 2D, is 35%; whereas if the cross-section is extrapolated and analysed in 3D there is a 63% probability that the resulting factor of safety will be higher than the actual 3D factor of safety of the composite slope.

The number of samples required to reduce the probability of sampling an unsafe section to below 1% was then determined. Table 1 shows that if 2D factors of safety are determined then 2 cross-sections are required for Type 1 slopes, 2 for Type 2, 3 for Type 3 and 5 for Type 4. However, if 3D factors of safety are determined from extrapolated 2D cross sections significantly more sections are required: 6 for Type 1 slopes, 7 for Type 2, 8 for Type 3 and 11 for Type 4.

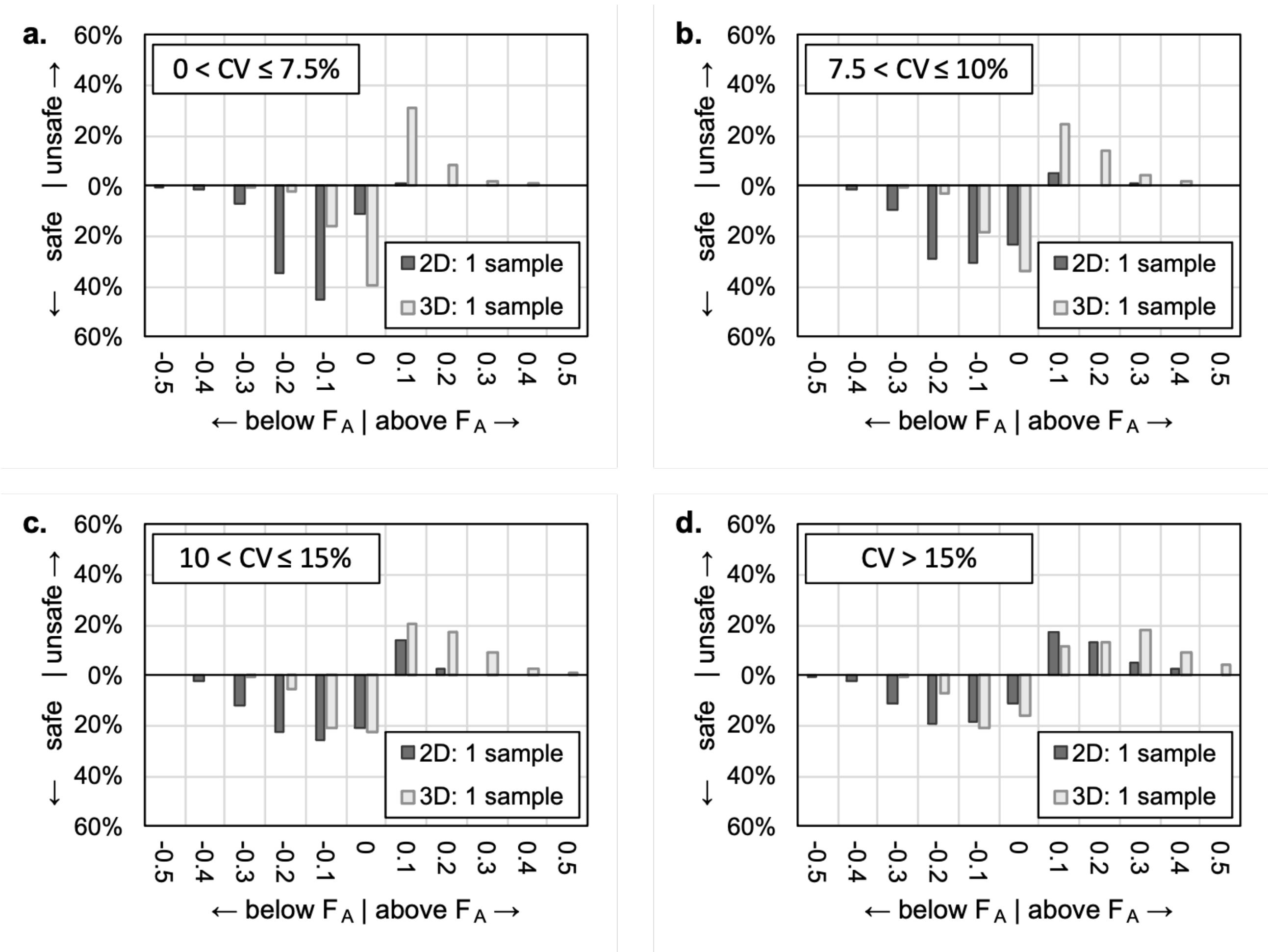

Figure 6. Distribution of differences associated with different variability groups: a. Type 1, b. Type 2, c. Type 3, and d. Type 4

Table 1. Samples required to reduce probability of unsafe samples

Variability group	F_{2D}		F_{3D}	
	Number of sampled cross-sections	Probability that sampled cross-sections will be unsafe	Number of sampled cross-sections	Probability that sampled cross-sections will be unsafe
Type 1 (0 < CV ≤ 7.5%)	1	1.2%	1	43%
	2	0.01%	6	0.6%
Type 2 (7.5 < CV ≤ 10%)	1	6.5%	1	49%
	2	0.4%	7	0.7%
Type 3 (10 < CV ≤ 15%)	1	14%	1	54%
	3	0.3%	8	0.7%
Type 4 (CV > 15%)	1	35%	1	63%
	5	0.5%	11	0.6%

4 PRACTICAL IMPLICATIONS

To use the information summarised in Table 1 to propose the number of cross-sections that should be sampled (i.e., investigated in-situ) an engineer would need an estimate of the expected variability of factors of safety that would result if multiple cross-sections were analysed. To avoid this problem of infinite regress, it is considered prudent to start with at least four cross-sections.

Sections investigated should be chosen to locate weak sections. Depositional history should be considered in choosing these sections (e.g., locations with poor drainage, areas with weak or impermeable foundation soils or zones with poor beach control). If weak zones are not easy to identify, equally spaced cross-sections are considered adequate.

Two-dimensional factors of safety should be determined for each cross-section and the coefficient of variation determined. If this coefficient of variation is greater than 15% it would then be prudent to investigate additional cross-sections, especially if weak sections have likely been overlooked. The four sections analysed should give an indication of where to focus additional work.

The folly of extrapolating 2D cross-sections, to obtain 3D factors of safety is clearly illustrated by the number of samples required. This is a consequence of having to find the cross-section that extrapolates to give the lowest 3D factor of safety (a problem not present when a 2D factor of safety is calculated). Should a 3D factor of safety be required, additional site investigation work should focus on the zone surrounding the section with the lowest 2D factor of safety. Spacing of these additional in situ tests should consider the size of the critical slip surface estimated from an extrapolated section. Variability along the axis of the slope can then be incorporated into the model giving greater confidence to the calculated 3D factor of safety.

Tailings dams are dynamic and weak zones identified in pervious investigations may not remain in the same place. Monitoring plans should therefore aim to investigate different cross sections over a facilities life to build up an understanding of spatial variability.

5 CONCLUSIONS

A parametric study was conducted by varying the inner geometry of a typical upstream tailings dam to provide guidance on the number of cross-sections needed for stability evaluations. For most spatial variability levels, sampling at least four cross-sections yielded an acceptable probability of obtaining a two-dimensional factor of safety lower than the three-dimensional factor of safety for the weakest composite combination.

However, achieving the same safety level when calculating three-dimensional factors of safety based on extrapolated two-dimensional cross-sections required significantly more cross-sections (2 to 4 times as many). This indicates that extrapolated three-dimensional factors of safety should not be used to assess the overall stability of a tailings dam. Instead, they should be used to evaluate weak zones identified through two-dimensional analysis, which should then be investigated in-situ in greater detail to incorporate variability along the slope axis into the three-dimensional model.

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