A novel approach to estimate a phreatic surface in tailings facilities

D. Calitz & R.V. Venning-Pridham
SRK Consulting, Johannesburg, South Africa

ABSTRACT: The phreatic surface plays a crucial role in assessing the safety and stability of tailings storage facilities (TSF's). Current methods of estimating the phreatic surface uses discrete pressure measurements from either vibrating wire piezometers (VWP's) or standpipe piezometers often coupled with nearby CPTu test results. This study explores a novel approach of continuously estimating the phreatic surface utilising the conformal transformation equations related to Kozeny's basic parabola and discrete pressure measurements along a 2-dimensional cross-section. This approach offers a way to estimate the shape of the phreatic surface profile along the cross-section as a function of these discrete pressure measurements – made by pressure measuring instruments such as VWP's and standpipe piezometers. For the purposes of this report the approach was applied on VWP's and compared to phreatic surfaces inferred from CPTu campaigns and seepage models. An error function was then used to evaluate its potential performance.

1 INTRODUCTION

The position of the phreatic surface plays a crucial role in determining the stability of tailings embankments, particularly when subjected to both static and seismic loads. Cone Penetration Test (CPTu) campaigns are often employed to observe the phreatic surface at various cross-sectional positions, offering an accurate snapshot of the surface's location at a specific moment. However, because CPTu tests are infrequent, there is a need for methods that can provide reliable estimates of the phreatic surface between tests.

This paper presents a novel approach to estimating the phreatic surface by solving Kozeny's (Kozeny, 1953) basic parabola and the associated conformal transformation equations from a single pressure measurement. By utilizing multiple pressure measurements, it becomes possible to draw more robust conclusions, assuming homogeneous soil conditions. It is important to note that while permeability is not a factor in this geometric approach, anisotropy does affect the results. This method offers a promising alternative to more conventional estimation techniques and aims to provide a more confident estimate of the phreatic surface despite the inherent variability over time.

2 SEEPAGE

2.1 *Background on different methods*

Various methods are available for estimating the phreatic surface resulting from unconfined seepage problems, such as those encountered in tailings facilities. One common approach is the Finite Difference Method (FDM), which iteratively solves seepage problems over a discretized mesh that represents the soil. While FDM is effective and fast, it requires careful consideration of boundary conditions, and its setup can be computationally intensive.

Additionally, many other computational methods exist for solving seepage problems under discretized boundary conditions, requiring a proper definition of those conditions.

Van Der Berg & Rust (1995) demonstrated that pressure measurements, when combined with finite element analysis, could provide accurate estimations of the phreatic surface. However, these methods often suffer from the complexity of defining the boundary conditions and setting up the system.

In contrast, Kozeny's (Kozeny 1953) work offers a closed-form solution for his basic parabola, and this paper leverages his geometric approach to fit known seepage problems more efficiently. By applying Kozeny's method, the phreatic surface can be estimated with reduced computational effort and fewer assumptions than traditional methods.

2.2 *Conformal transformation, solution*

Conformal mapping can be used to transform an unconfined flow region (x,z) such as that representing a flow net, into a simple rectangular potential-flow domain (Φ,ψ) and vice versa.

The functions *x(Φ,ψ)* and *z(Φ,ψ)* derived from the conformal transformation equations related to Kozeny's basic parabola (Knappet & Craig, 2012) can be used to solve the geometric flow net problem depicted in Figure 1.

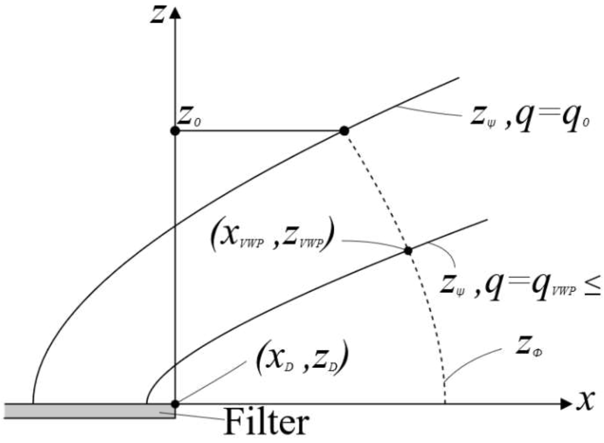

Figure 1. A geometric solution to the flow net, with known boundary conditions.

Conditions:

- The equipotential line z_Φ passes through the point (x_{vwp}, z_{vwp}).
- Let z_0 be the z value (elevation) at which the equipotential line passing through the VWP intercepts the phreatic surface.
- Let P_{vwp} be the pore pressure measured at the coordinate (x_{vwp}, z_{vwp}) in kPa.
- Consider, that $z_0 - z_{vwp} = H_{vwp}$, where H_{vwp} can be calculated as P_{vwp} (kPa) divided by the hydrostatic pore-fluid pressure gradient of *9.81 kPa/m.*
- Consider, $x \leq x_D$ as the discharge surface
- Consider that $z = z_D$ is the location of zero potential i.e. the datum.

Since the assumption for this model is that the soil is a homogeneous isotropic soil varying the permeability k, does not influence the shape of the solution, i.e. the flow paths and the equipotential line remain identical, as the k only governs an increase or decrease in q, such that the geometry is identical.

Using these conditions the following functions and formulae can be derived:

$$z_\psi = \frac{1}{k}\sqrt{q^2 - 2kqx} \tag{1}$$

$$z_\Phi = \frac{z_0}{q_0}\sqrt{k^2 z_0^2 + 2kq_0 x} \tag{2}$$

$$z_0 = z_{vwp} + H_{vwp} \tag{3}$$

$$q_0 = \frac{z_0^2 k\left(x_{vwp} \pm \sqrt{x_{vwp}^2 + z_{vwp}^2}\right)}{z_{vwp}^2} \tag{4}$$

$$q_{vwp} = k\left(x_{vwp} \pm \sqrt{x_{vwp}^2 + z_{vwp}^2}\right) \tag{5}$$

where x = horizontal distance from drainage point; z = height above datum (drainage point); k = permeability; q = discharge; H_{vwp} = VWP pressure head; z_0 = height at which the equipotential line intercepts the phreatic surface.; q_0 = discharge at phreatic surface

These equations can be used to mathematically draw a flow net that meets the specified conditions. The solution comprises of two key components:

1. *Flowlines (z_ψ):* represent flow paths for a specific discharge q. The discharge parameter can be tailored to achieve specific objectives:
   a. Substituting q as q_{vwp} ensures the flowline passes through the VWP/instrument.
   b. Substituting q as q_0 defines the flowline as the phreatic surface.
2. *Equipotential lines (z_Φ):* represents an equipotential line that intersects both the VWP/instrument and the phreatic surface flow line (z_ψ, $q = q_0$) such that the difference in hydraulic head H_{vwp} is defined such that $z_0 - z_{vwp} = H_{vwp}$.

3 DATASET

A monitored cross-section from an undisclosed facility was used to evaluate the proposed method. The cross-section has five VWP's installed at various locations (refer to Figure 3).

The results were compared against the phreatic surface observed through a CPTu campaign conducted on the cross-section, refer to Table 1. The CPTu campaign consisted of five tests (Table 1 and Figure 2) conducted along the cross-section over the course of ten days – for this reason the average pressure reading for each VWP over this period was used in application of the proposed method. Table 2 provides more information regarding the parameters used for each VWP.

The position of the phreatic surfaces observed with the CPTu campaign are summarised in Table 1.

Table 1. CPTu phreatic surface elevation

Position Name	Chainage	Phreatic-Surface Elevation
	meters	mamsl*
Position A	54.90	1251.71
Position B	81.92	1255.50
Position C	108.94	1261.40
Position D	135.96	1263.60
Position E	162.99	1270.03

* The elevations are provided in meters above mean sea level (mamsl).

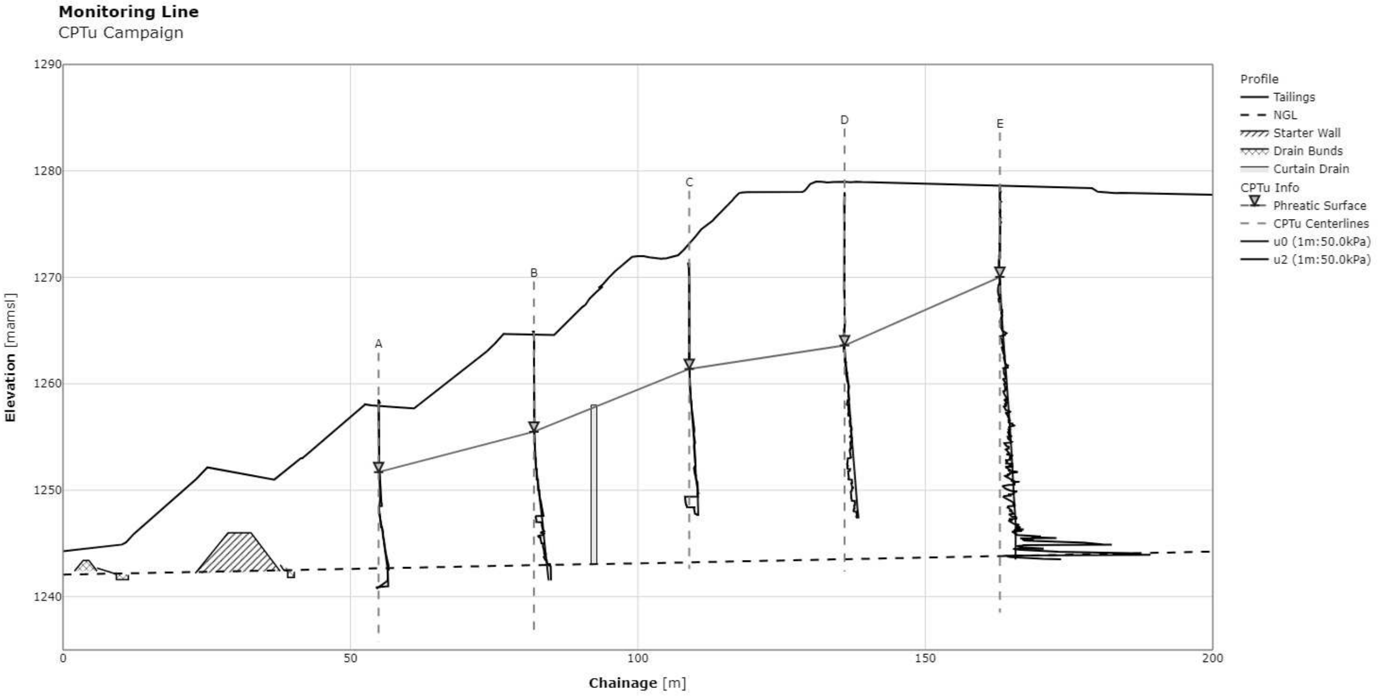

Figure 2. Phreatic surface from CPTu campaign

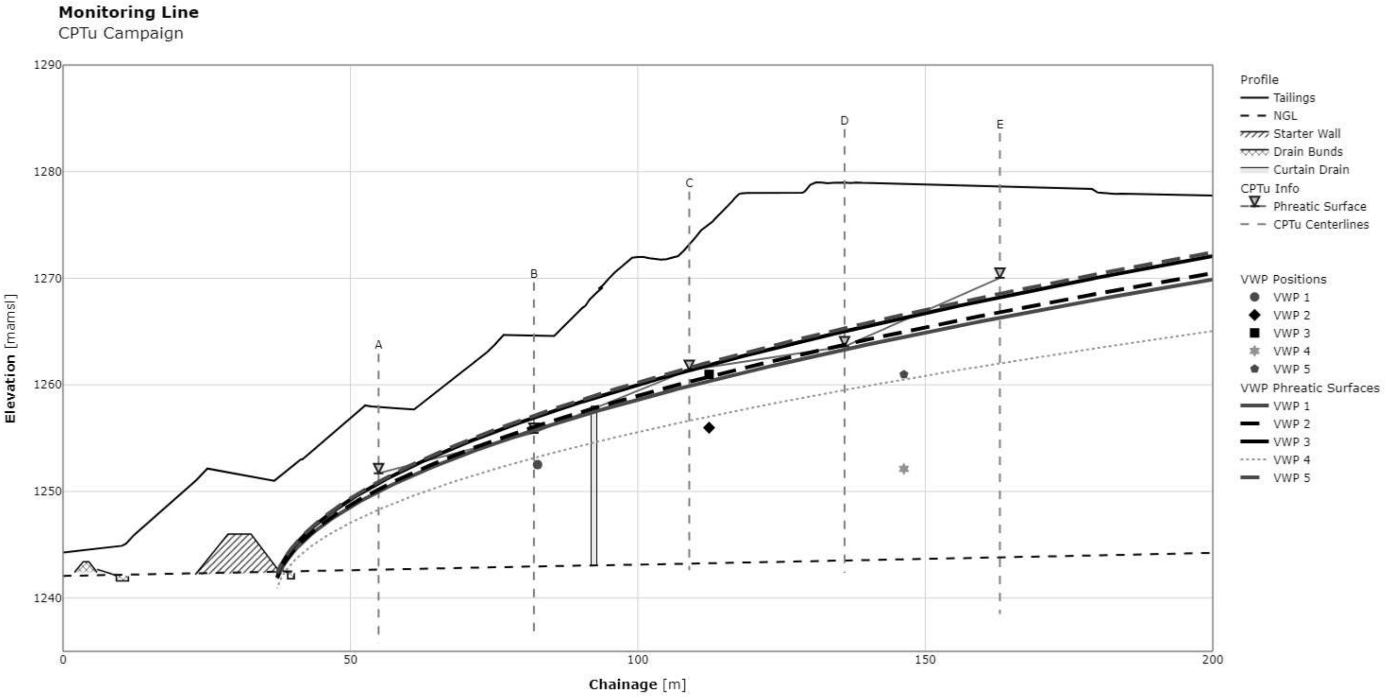

Figure 3. Phreatic surface from proposed VWP method

Table 2. VWP Information & Values

Instrument Name	Chainage	Instrument-Elevation	Pressure
	meters	mamsl	kPa
VWP 1	82.55	1252.51	31.921
VWP 2	112.39	1255.97	46.239
VWP 3	112.39	1260.98	8.557
VWP 4	146.28	1252.12	81.754
VWP 5	146.28	1260.98	54.326

4 RESULTS

The method was applied to each instrument using values provided in Table 2. The drainage location was set as x_D = 38 m and z_D = 1240 mamsl (meters above mean sea level), as this is the location of a toe drain in the cross-section.

The method results in a continuous curve representing the phreatic surface along the cross-section. Each continuous curve was constructed, and the elevation of the phreatic surface was interpreted at the various test positions for comparison to that obtained from CPTu tests at the same positions – the results are summarised in Table 3.

Figure 3 illustrates the various continuous curves obtained with the dataset alongside the linearly drawn phreatic surface elevation obtained from CPTu observations.

Table 3. Phreatic surface elevation at test positions

Measurement	Position A	Position B	Position C	Position D	Position E
	mamsl	mamsl	mamsl	mamsl	mamsl
CPTu	1251.71	1255.50	1261.40	1263.60	1270.03
VWP 1	1250.00	1255.74	1259.89	1263.31	1266.30
VWP 2	1250.21	1256.06	1260.29	1263.78	1266.82
VWP 3	1250.79	1256.92	1261.37	1265.03	1268.23
VWP 4	1248.29	1253.15	1256.64	1259.52	1262.02
VWP 5	1250.91	1257.11	1261.61	1265.31	1268.54

Performance:

The method's performance for each VWP is described in terms of average relative error, that is the sum of the relative errors at each position divided by the number of positions.

The values are adjusted by the elevation of the drainage location datum (1240 mamsl) as the elevations above this are the scope of prediction (otherwise errors would be minimal).

$$\mathrm{error}_{\mathrm{avg}} = \frac{\sum_{\mathrm{n=1}}^{\mathrm{n=i}}\left(\frac{|\text{predicted value}-\text{expected value}|}{\text{expected value}-\mathbf{datum}}\times 100\right)}{\mathrm{i}} \tag{6}$$

where predicted value = the value interpolated at the test position using the estimated phreatic surface; datum = the reference elevation as which potential is defined as zero (i.e. the drainage location); and expected value = the phreatic surface elevation inferred from the CPTu data at position.

The error results are as follows:

- VWP 1: 7.4%
- VWP 2: 6.6%
- VWP 3: 5.9%
- VWP 4: 22.1%
- VWP 5: 6.1%

The results indicate that VWP 1, 2, 3, and 5 demonstrated relatively low errors, ranging between 5.9% and 7.4%, suggesting consistent accuracy across these instruments and reinforcing the method's reliability. However, VWP 4 stands out with a significantly higher relative error of 22.1%, indicating a substantial deviation from the expected results.

Overall, the findings suggest that VWP 1, 2, 3 and 5 provide reliable results with relatively low errors while VWP 4 requires further investigation to identify the sources of inaccuracy.

While the method should not replace CPTu, when combined with enough pressure measuring devices, it may provide an alternative way of predicting the phreatic surface for monitoring purposes.

The engineers' level of confidence in these results may be influenced by the number of corroborating results obtained from multiple instruments.

Limitations:

- The method requires defining a drainage point (x_d, z_d).
- The method assumes that Kozeny's transformation equations can define the seepage through the cross-section.
- Currently, as applied here, the method assumes isotropic-homogenous behaviour.
- The method does not make provision for more complex drainage systems.

5 FUTURE DEVELOPMENT

Further work should consider the following unaddressed aspects:

- Incorporate anisotropy.
- Incorporated complex drainage systems.
- Compare method results to that obtained via finite difference methods.
- Investigate the effects of layering.
- Further performance investigations on more CPTu campaigns, and different facilities.

Addressing some of these aspects will lead to a better understanding of the limitations and application of the method.

6 CONCLUSION

This study introduces a novel approach for estimating the phreatic surface by using a single pressure measurement and incorporating the conformal transformation equations developed by Kozeny.

The proposed method offers a practical and efficient alternative to conventional techniques.

The method was applied to a cross-section and the results indicate reliably low errors, except for one instrument (VWP 4).

By better understanding the solution of the seepage problem the authors hope to provide more insight into the problem.

ACKNOWLEDGMENTS

The authors would like to thank and acknowledge SRK for the CPTu and VWP data.

REFERENCES

Kozeny, J. 1953. *Hydraulik: Ihre Grundlagen und praktische Anwendung* (1st ed.). Springer. pp. 414-416

Knappett, J. & Craig, R.F. 2012. *Craig's Soil Mechanics*. CRC Press. p. 67.

Van Der Berg, J.P. & Rust, E. 1995. *Monitoring of the phreatic surface in a tailings dam and subsequent stability implications.* University of Pretoria.

The paper was published in the proceedings of the 2nd Southern African Geotechnical Conference (SAGC2025) and was edited by SW Jacobsz. The conference was held from May 28th to May 30th 2025 in Durban, South Africa.