Repeatability of triaxial tests and reproducibility of shear parameters

H.M. Shiferaw & B. Schneider-Muntau
Universität Innsbruck: Institute for Infrastructure, Computational and experimental soil mechanics, Innsbruck, Austria

ABSTRACT: Test repeatability and data reproducibility is an issue in many geotechnical laboratory tests due to factors including soil inherent inhomogeneity and laboratory test procedures. This paper presents the repeatability of consolidated undrained triaxial tests and reproducibility of shear strength parameters, from different failure stress combinations of repeated tests, on plain and cement-treated clay soil. The study involved conducting triaxial tests on remolded specimens with varying soil-cement ratios, with each test repeated on a comparable specimen. Effective shear strength parameters were estimated from regression of 21 different failure stress combinations for each soil-cement ratio considered. Statistical and machine learning methods were applied to analyse result variability. Findings revealed that test repeatability was highly impacted by specimen density uniformity, with cement-treated specimens showing greater variation. Average effective shear strength parameters estimated from the 21 different failure stress combinations indicated scattering. K-Means clustering showed overlapping of values across the different soil-cement ratios considered. The results of the study offer valuable insights into the repeatability of shear strength tests and the factors contributing.

1 INTRODUCTION

Inherent soil variability, sample preparation and testing procedure induce variations in results of geotechnical soil laboratory tests (Li et al. 2015, 2019, Moreno-Maroto & Alonso-Azcárate 2017, Weidinger & Ge 2009). Such variations in test results also cause limitations in the accuracy and reproducibility of parameters estimated from regression models, such as shear strength parameters from shear tests (Maneejuk & Yamaka 2021, Moiseev 2017, Schneider-Muntau et al. 2018). Bareither et al. (2008) investigated repeatability of direct shear test among different laboratories and reported high variability of failure envelops with the angle of internal friction varying by a magnitude as much as 18.2o for a same soil. A comparative study on shear strength parameters of a similar soil sample from 8 different laboratories reported a 16% and 55% standard deviation among the friction angle and cohesion estimates of the different laboratories (Schwiteilo & Herle 2017). Additionally, Schneider-Muntau et al. (2018), observed a variation of 8° in the average effective friction angle and 43 kPa in the cohesion estimates of a well-graded sand gravel mixture across 42 different stress combinations of 6 drained triaxial tests from a single laboratory.

Repeatability of shear tests and cement improved samples have not been investigated so far and are part of this study. In Consolidated Undrained (CU) triaxial tests of cement improved soils, stress paths and failure stresses of comparable tests specimens tested under similar test conditions can indicate variations. Evaluating the repeatability of the tests and the reproducibility of the shear strength parameters from regression models of different failure stress combinations will give an insight on to what extent results could vary and, on the factors contributing. In this research, a series of CU triaxial tests were performed on plain and cement treated clay soil specimens where two trials were considered at each test conditions.

2 MATERIALS AND METHODS

2.1 *Specimen and test procedure*

An inactive silty clay soil from the surrounding of Innsbruck, Austria and ordinary Portland cement were used to prepare test specimens. Thirty, 72 mm in height and 36 mm in diameter, remoulded test specimens were prepared considering five soil-cement ratios, 0, 7, 10, 13 and 16% cement by dry weight of the soil. Oven dried soil was mixed with required amount of cement with hand using spoon for 5 minutes, then the required amount of water was added and mixed for 5 more minutes before remoulding the test specimens. The cement treated specimens were allowed to cure for 28 days in a box full of wet saw dust before being tested.

Attention was given to maintain comparable initial densities for each pair of test specimens to be able to evaluate test repeatability. Equal percentage of water content, by dry weight of the soil-cement mix, and compaction energy was used for the preparation of all test specimens. Cement treated specimens, however, indicated a considerable variation of densities after curing. The samples initially indicated swelling, due to the saw dust moisture. After the 28 days curing period, the specimens indicated shrinking when outed from the saw dust while being prepared for test (Shiferaw & Schneider-Muntau 2025). Samples were saturated using a back pressure of 900 kPa and afterwards consolidated. Conventional three levels of effective consolidation pressures, i.e. $\sigma_c' = 100$ kPa, $\sigma_c' = 200$ kPa, $\sigma_c' = 300$ kPa, were considered. The shearing was performed at a deformation rate of 0.4 mm/min.

The average effective peak and residual shear strength parameters were estimated from regression analysis of failure stresses using the *s - t* method where failure was considered as maximum stress obliquity condition and critical state was assumed to be reached at 20% axial compression. In the *s - t* method, *s* and *t,* for each failure stress, given by Equations 1 and 2 respectively, are plotted on a *s - t* plane (as in Fig. 1) and a linear regression line is fitted to the scatter plot. φ' and c', given by Equations 3 and 4 respectively, are determined from the intercept *d* and slope δ of the fitted regression line. A total of 21 different stress combinations from the six tests (eight combinations of three, six combinations of four, six combinations of five and one combination of six) were considered for each soil-cement ratio to fit a linear regression line. Critical friction angles φ_c' were estimated by forcing the regression lines of the residual stresses through the origin (zero intercept).

$$s = \frac{\sigma_1' + \sigma_3'}{2} \tag{1}$$

$$t = \frac{\sigma_1' - \sigma_3'}{2} \tag{2}$$

$$\sin \varphi' = \tan \delta \tag{3}$$

$$c' = \frac{d}{\cos \varphi'} \tag{4}$$

where σ_1' and σ_3' are effective vertical and confining stresses, respectively.

2.2 *Statistical and machine learning evaluation of scattering*

Descriptive statistics (mean, Standard Deviation (SD), and range) of the average effective shear strength parameters determined from the 21 different failure stress combinations were calculated and evaluated for understanding of scattering. The 5% and 95% confidence limits for the estimated effective shear strength parameters were also evaluated considering the Student's t-distribution. Confidence limits to linear regression lines result in confidence hyperbolas, see e.g. Schneider-Muntau et al (2018). For this, the confidence hyperbolas of the linear regression lines fitted in the s - t method were linearized in the range of the investigated stress levels to estimate the limits of the shear parameters (e.g. s = 156 kPa to s = 499 kPa) (Fig. 1). A linearization of the confidence bands (Fig. 1, blue lines) provides an upper and lower limit of the shear parameters. Small intercepts of regression lines lead to negative values for the 5% confidence limit after linearization of the lower confidence band. In such cases, since negative cohesion is unrealistic, the lower limit of the linearization is set to zero to result in the 5% limit of the effective cohesion of zero (Fig. 1, red line) (Fellin & Oberguggenberger 2012). For estimating the limits for the critical effective friction angle, according to critical state theory, the lower limits of the linearization for both the 5% and 95% limits were set to zero. This procedure was adopted to estimate the limits in this study. From the slopes and intercepts of the linearized confidence bands, the limits for the shear strength parameters were estimated using Equations 3 and 4.

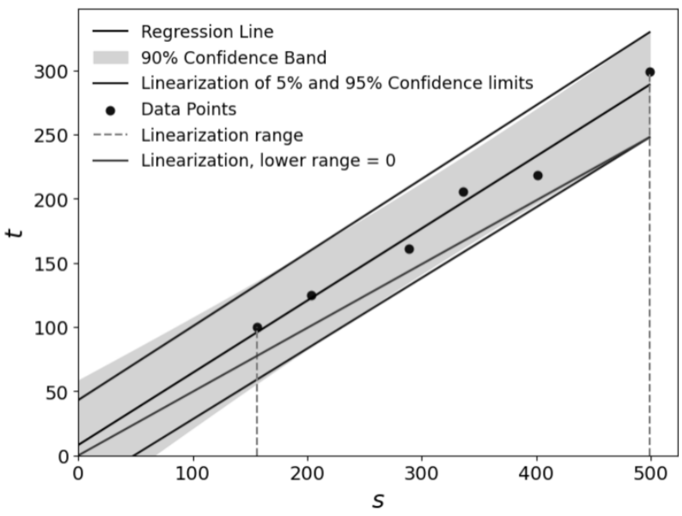

Figure 1. Regression line (mean value), 90 % confidence band and linearization of upper and lower bounds. Exemplary shown for the test results on the plain soil peak strengths evaluation.

As there are five soil-cement ratios considered in this study, the estimated effective shear strength parameters were clustered into five groups and checked for any overlap using the unsupervised machine learning technique of K-Means clustering. In K-Means clustering, data points in a same cluster are considered more similar to each other than to the data points of the other clusters.

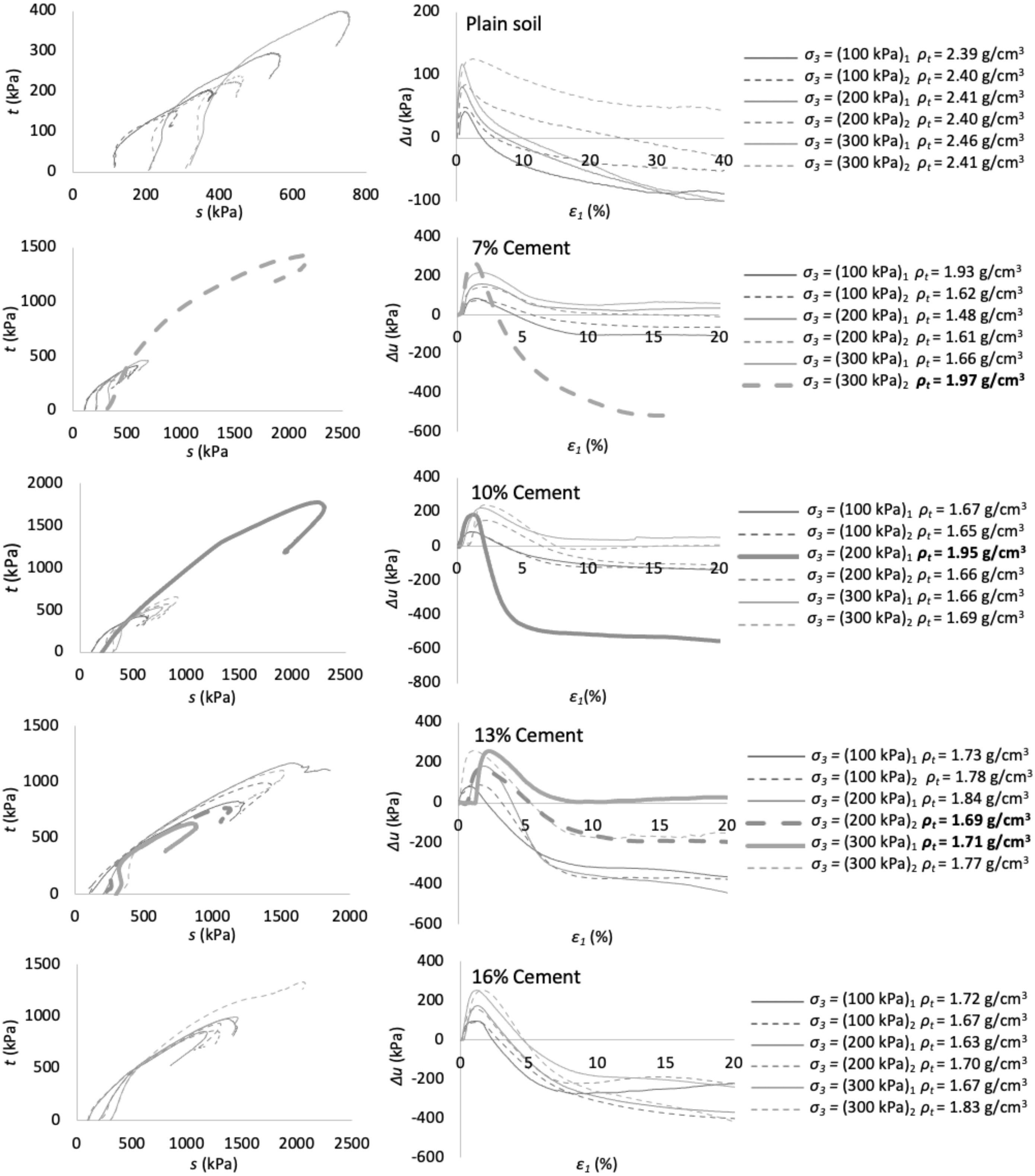

Figure 2. Triaxial test results (thicker lines indicate specimens with higher deviation in densities).

3 RESULTS AND DISCUSSION

3.1 *Test results*

Test results of all soil-cement ratios considered are presented in Figure 2 in $s - t$ and $\varepsilon_1 - \Delta u$ plots where ε_1 and Δu are vertical strain and excess porewater pressure, respectively. The total densities of tested specimens after curing (just before testing) are also indicated in the figure.

3.2 *Repeatability of results*

Based on the results, the stress paths of the repeated tests indicated notable deviations. The deviations were influenced greatly by the differences in densities of the test specimens after curing. For instance, for one of the 7% cement treated specimens, tested at an effective consolidation pressure of 300 kPa, the test specimen total density was 1.97 g/cm^3, higher than the rest, and the stress path indicated a considerable divergence from the rest. The maximum stress obliquity and the corresponding deviator stress were thus considerably large. The same phenomenon was observed for one of the 10% cement treated specimens tested at an effective confining pressure of 200 kPa, whose density was 1.95 g/cm^3. Moreover, two of the tested specimens indicated a lower density, after curing, where each resulted in small failures stresses as indicated on Figure 2 (13% cement treated specimens of densities 1.71 g/cm^3 and 1.69 g/cm^3 tested at an effective consolidation pressure of 300 and 200 kPa respectively). Those large and small values notably differ from the rest and resulted in negative cohesions (steep regression lines) when included in $s - t$ regressions. In such cases, the regression lines were forced through the origin (zero intercept) to result in a cohesion value of zero.

The variation in stress paths and thus failure stresses between the two specimens of the repeated tests, and the increased variation for the cement treated specimens is mainly attributed to sampling and variation in density of test specimens after curing. Inhomogeneity due to inherent soil variability, breakage of soil-cement structure during shearing, strain localization and shear band formation, and development of suction pressure generally contribute to test result variations (Gylland et al. 2014, Sadrekarimi & Olson 2010).

3.3 *Shear strength parameters determined from the different failure stress combinations*

The effective shear strength parameters determined from the 21 failure stress combinations indicated scattering. The means, ranges and SDs of the effective shear strength parameters estimated from the 21 stress combinations, for each soil cement ratio considered, are summarised in Table 1.

Table 1. Descriptive statistics of the average effective shear strength parameters determined from the 21 failure stress combinations for the five soil-cement ratios considered.

	Density (g/cm^3)	c' (kPa)			φ' (^{O})			φ_c' (^{O})		
	Range	Mean	Range	SD	Mean	Range	SD	Mean	Range	SD
Plain soil	2.39 - 2.46	14.1	0.0 - 33.8	11.3	33.3	28.1 - 36.6	3.0	26.3	24.4 - 27.5	0.7
7% Cement	1.48 - 1.97	21.8	0.0 - 81.1	32.6	60.3	41.0 - 85.6	12.9	38.5	37.0 - 39.1	0.7
10% Cement	1.65 - 1.95	24.3	0.0 - 82.7	35.4	63.1	44.6 - 75.8	13.0	37.7	37.2 - 38.3	0.2
13% Cement	1.69 - 1.84	13.5	0.0 - 115.7	34.2	68.1	54.3 - 72.2	4.8	39.0	38.0 - 39.8	0.6
16% Cement	1.63 - 1.83	90.3	47.0 - 160.8	27.3	57.5	46.0 - 62.8	4.7	38.9	37.9 - 39.9	0.6

The SDs and ranges for the average effective shear strength parameters of the cement treated specimens were generally higher than the plain soil as given in Table 1. Effective critical friction angle was more consistent with smaller SDs than the peak friction angle. The increase in scattering of results for cement treated specimens can be an indication of increased inhomogeneity in density of specimens and added effect of cementitious bond breakage.

3.4 *Confidence limits*

The 5% and 95% confidence limits are affected by sample size and data dispersion. For instance, for sample sizes n = 5 and n = 6, the critical value of the Student's t-distribution t_{crit}, are 3.182 and 2.776 respectively at 90% confidence level. In this study, one regression line fitted to the maximum number of available data points, excluding the ones forced through the origin due to negative intercepts, was selected and considered in confidence limit evaluation at each soil-cement ratio. As such, for the plain soil, 7, 10, 13 and 16% cement treated specimens, linear regression lines fitted to 6, 5, 5, 4 and 6 data points were selected in confidence limits evaluation. For the effective critical friction angle, the results from linear regression model fitted to all 6 data points are considered in confidence interval evaluation for all soil-cement ratios. The limits are given in Table 2. The difference between the 95% and 5% limits (width of 90% confidence band) tends to be larger for the plain soil but no clear relation was observed between the width and cement content. A consideration of always all 6 data points for peak strength, resulting in an even t_{crit} of 2.776, did not lead to narrower confidence bands. This can be explained by the notable influence of the pronounced scattering of peak values in the mentioned cases. The slopes of the linearized upper confidence band of the critical friction angle were steeper as the lower stress limit was set to zero. This has resulted in larger 90% confidence band width for the critical effective friction angle.

3.5 *K-Means clustering*

The K-Means clustering result for the shear strength parameters is given in Figure 3. The 5 different groups clustered by the algorithm are identified by the different shapes/colours on the plot in no order. As can be observed from the figure, there were overlaps of grouping for the effective peak shear strength parameters and the expected clusters according to cement ratios was not confirmed by the algorithm. The overlaps could be interpreted as follows: some of the effective shear strength parameters are closer in value to results of other soil-cement ratios than to the results of the soil-cement ratios they were determined at. The K-means algorithm clustered the average effective critical friction angle into the considered five soil cement ratios effectively without an overlap between the different cement contents (Fig. 3).

Table 2. 5% and 95% confidence limits of effective shear strength parameters determined for the different soil-cement ratios considered.

	Density (g/cm^3)	φ' (^{O})			c' (kPa)			φ_c' (^{O})		
	Range	5%	95%	Width	5%	95%	Width	5%	95%	Width
Plain soil	2.39 - 2.46	31.9	34.7	2.8	0.0	35.8	35.8	22.6	29.4	6.8
7% Cement	1.48 - 1.97	43.1	45.3	2.2	44.5	81.4	36.9	38.5	40.0	1.5
10% Cement	1.65 - 1.95	48.0	47.6	-0.4*	53.3	93.0	39.7	36.6	39.3	2.7
13% Cement	1.69 - 1.84	55.6	56.1	0.4	42.2	168.1	125.9	36.9	42.6	5.7
16% Cement	1.63 - 1.83	58.9	60.1	1.2	37.3	127.7	90.4	37.9	41.9	4.0

*Slope of linearized upper bound was less than the linearized lower bound.

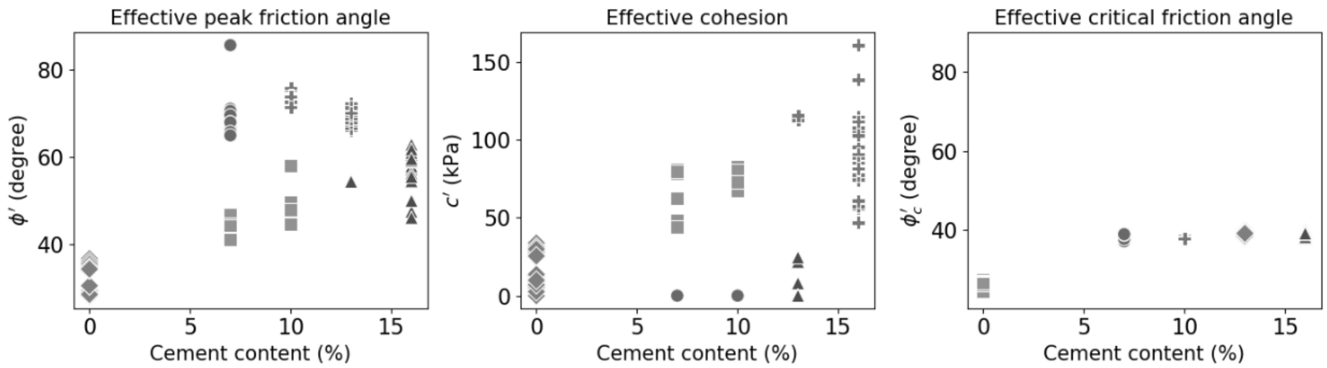

Figure 3. K-Means clustering for the effective shear strength parameters (different shapes/colors indicate different groups in no order).

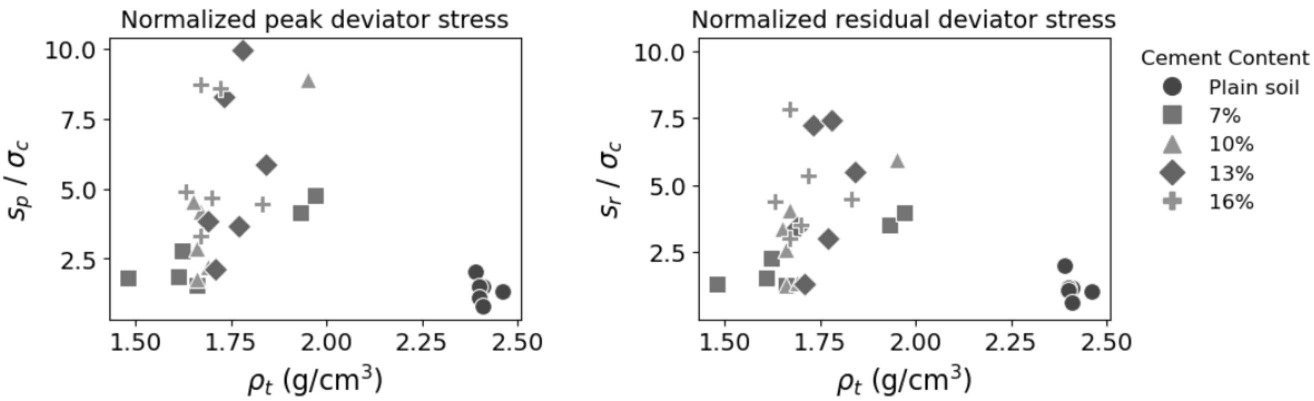

Figure 4. Scatter plot of peak and residual deviator stresses, normalized by consolidation pressure, versus density

On Section 3.2, the influence of density on test results within each soil-cement ratio was discussed. Moreover, density was found to play a more significant role than cement content on the results of some of the cement treated specimens when results are compared across the different soil-cement ratios. Specimens with higher density indicate higher stresses irrespective of the cement content. The peak deviator stress s_p, recorded for a 7% cement treated specimen which had a density of 1.97 g/cm^3, was 1424 kPa while for a 16% cement treated specimen of density equal to 1.83 g/cm^3, s_p was 1340 kPa. Despite the more than double increase in cement content, and specimens being tested at the same confining pressure, the 16% cement treated specimen indicated less s_p. Similar result of decreasing in s_p with decrease in density, despite an increase in cement content, was observed for 10, 13 and 16% cement treated specimens with densities of 1.95, 1.84 and 1.70 g/cm^3 and tested at σ_c = 200 kPa that led to s_p of 1778, 1172, and 935 kPa respectively (Fig. 1).

To evaluate the role of density, s_p and residual deviator stress s_r of the 30 specimens, normalized by the corresponding σ_c, were plotted against density of the specimens ρ_t (Fig. 4). As can be observed from the scatter plots of Figure 4, there is a tendency of increasing in s_p/σ_c and s_r/σ_c with density for the cement treated specimens, irrespective of the cement content. At critical state, where s_r is expected not to be influenced by initial density, the results are slightly more consistent. Clearly distinguishable are the results of the plain soil specimen. Generally, cemented soils were reported to have a unique critical state behaviour, different from plain soils, due to their unique structural property (Cruz Nuno et al. 2011).

4 CONCLUSIONS

In this study, triaxial tests conducted on plain and cement treated specimens under similar conditions yielded in varying results, with the variations being notably influenced by the uniformity of the specimen densities. Difference in densities caused difference in results and the effective shear strength parameters derived from different failure stress combinations across repeated tests also showed variations. 5% and 95% confidence limits determined for the effective shear strength parameters through linearization of confidence bands provided additional information on the accuracy of the estimates.

K-Means clustering indicated that the shear strength parameters derived from different cement dosages can overlap, potentially leading to misleading conclusions. This understanding is especially crucial when estimating shear strength parameters for cemented soils, where determining the appropriate dosage is a key objective.

Repeating tests and using different stress combinations to estimate shear strength parameters can offer valuable insight into the accuracy of the results. Additionally, evaluating confidence limits for these parameters enhances the understanding of the accuracy achieved.

Conducting a more thorough investigation with an increased number of tests and various stress combinations is recommended to expand the sample size and provide a more comprehensive assessment of reproducibility.

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