Critical State Line Testing Approach: A review

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ABSTRACT: Critical state theory, formulated in the late 1960's, is a robust criterion for the characterisation of soil behaviour and its strength characteristics, including post-liquefaction strength. The investigations of two recent tailings storage facility failures have accentuated the importance and use of the critical state line (CSL) as a basis for liquefaction assessment and constitutive modelling. Consultants and laboratories alike have, therefore, shifted focus to the accurate measurement of the CSL and the most effective way to do so. The conventional triaxial test is used for CSL determination, however, there are various approaches that can be followed. Some advocate to approach the CSL from the contractive (or wet) side of critical, while others recommend tests from both the dilative (or dry) and contractive side. Given the short time frames within which these tests are being carried out, budget constraints, and the lack of detailed-orientated testing from commercial laboratories a definite methodology will assist both consultants and laboratories to optimise these tests. Based on numerous CSL determination campaigns by the authors, this paper presents a review of CSL measurements, various considerations, and finally an approach is presented for CSL determination with the aim to assist both commercial laboratories and consultants.

1 INTRODUCTION

The critical state concept and the implementation thereof is currently widely adopted in geotechnical engineering practice. The investigations into two recent tailings storage facility failures have brought the framework into the spotlight, highlighting its importance as a foundation for liquefaction assessment and constitutive modelling. The greater geotechnical community has, therefore, been inundated with tailings related projects where the implementation of critical state concepts and the determination of the critical state line (CSL) is key to the outcomes of the project. As such, the geotechnical community, particularly in South Africa has, for the first time, seen the benefits of understanding and implementing the critical state concept not only in tailings related projects but also in general geotechnical engineering applications.

Critical state theory, formulated in the late 1960's, is a conceptual framework for the characterisation of soil behaviour and it unifies the characteristics of shear strength and volumetric change. At its core, the framework is a 3D concept as shown in Figure 1.

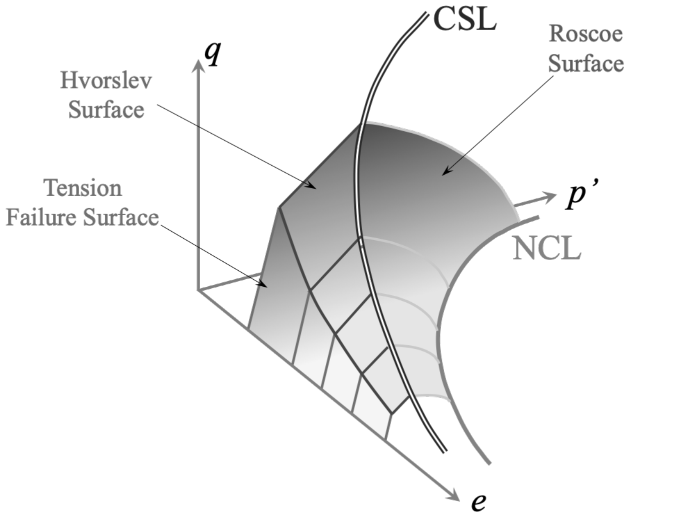

Figure 1. Critical state concept in 3D (Atkinson & Bransby 1978)

Within this framework, the CSL is used as a reference line to assess the mechanical behaviour of soils. Although geotechnical engineers are familiar with the concept and the theory, determining the CSL is not explicitly apparent in literature and open for interpretation in many instances. This is likely due to the broadness of the concept and points to the importance of the understanding of CSL theory as well as the required test methods to determine the CSL.

The successful determination of the CSL line is as highly dependent on the quality of the testing as it is on the practitioner specifying and interpreting the data. The reliability and successful implementation of CSL testing using current state-of-practice methods have recently been demonstrated (Reid et al., 2021); however, several shortcomings have also been identified. It is clear that there is sometimes a considerable amount of guesswork associated with specifying CSL testing, almost like a game of Russian Roulette, as practitioners are familiarising themselves with the testing methods and interpretation. There should, however, be less reliance on laboratories to assist from the theoretical perspective and more emphasis on the quality of the testing

This paper presents a review of CSL determination, and highlights various pitfalls and considerations based on the authors' experience. The paper aims to provide relevant insights on the different aspects of CSL testing and finally presents a methodology for CSL determination to assist both commercial laboratories and consultants.

This paper does not discuss the details of CSL theory, but only the general testing methodology. For more details on the concept, the reader is referred to the work of Schofield & Wroth (1968), Atkinson & Bransby (1978), and Wood (1990).

2 TYPICAL TRIAXIAL TESTS

To successfully specify triaxial tests for the CSL determination and constitutive model calibration, it is important to understand the different test types and the soil behaviour associated with each. A brief overview is provided in this section for ease of reference.

There are typically two types of triaxial tests used for this purpose:

1. Isotropic Consolidated Undrained Triaxial test (ICUDTx)
2. Isotropic Consolidated Drained Triaxial test (ICDTx)

These tests can be carried out on soil in two different initial states:

1. Loose - coarse grained soil, or Normally consolidated (NC) - fine grained soil.
2. Dense - coarse grained soil, or Over consolidated (OC) - fine grained soil.

Depending on the initial soil state and test type, the soil behaviour will either be dilative or contractive as indicated in Figure 2.

Figure 3 to Figure 6 shows the typical stress paths for the various test types and initial soil states in a 3D plot and projection within the critical state framework. The aim of the typical stress paths is to highlight the different soil behaviour, i.e. contractive or dilative, and the associated projected strength and volume change paths in the context of triaxial testing.

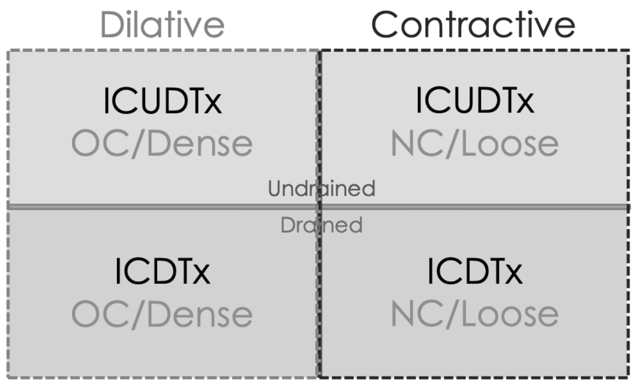

Figure 2. Typical triaxial tests, soil type and soil behaviour.

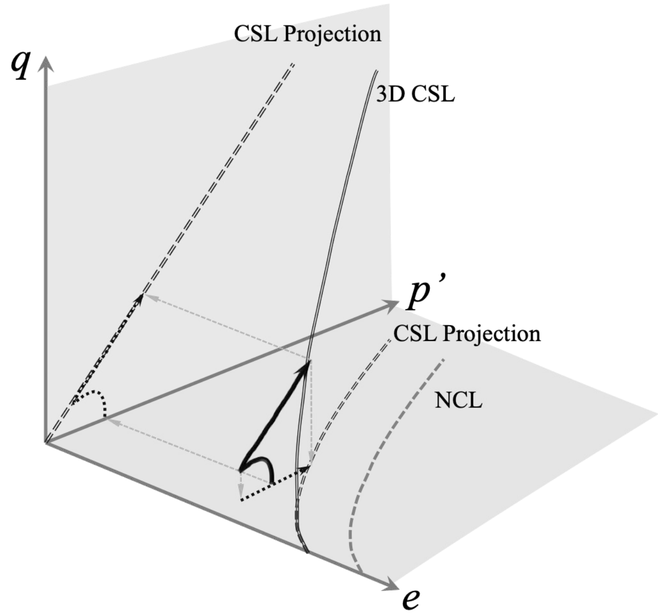

Figure 3. Typical stress path – ICUDTx – OC/Dense - Dilative

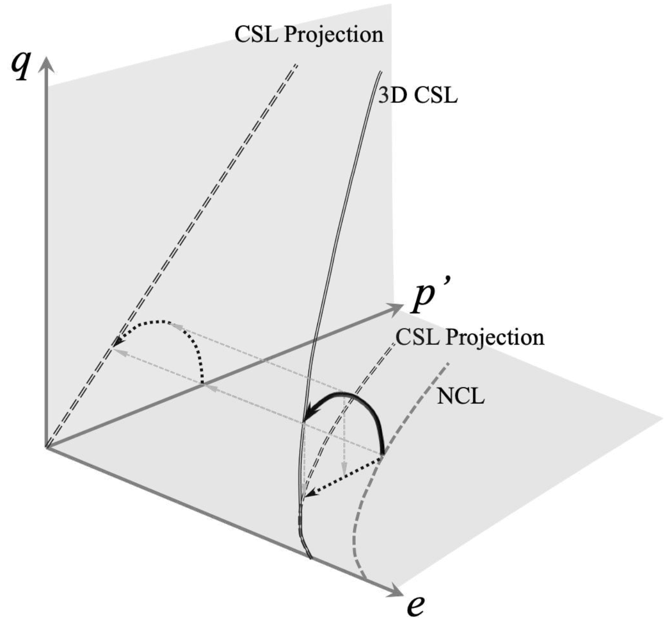

Figure 4. Typical stress path – ICUDTx – NC/Loose - Contractive

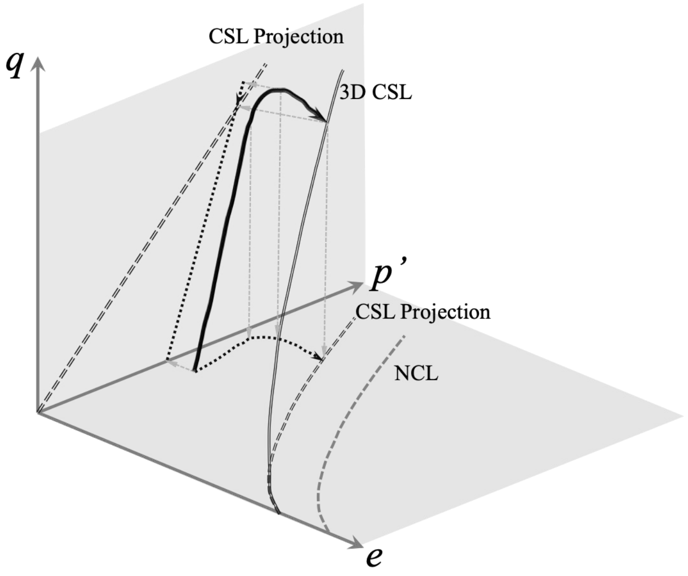

Figure 5. Typical stress path – ICDTx – OC/Dense - Dilative

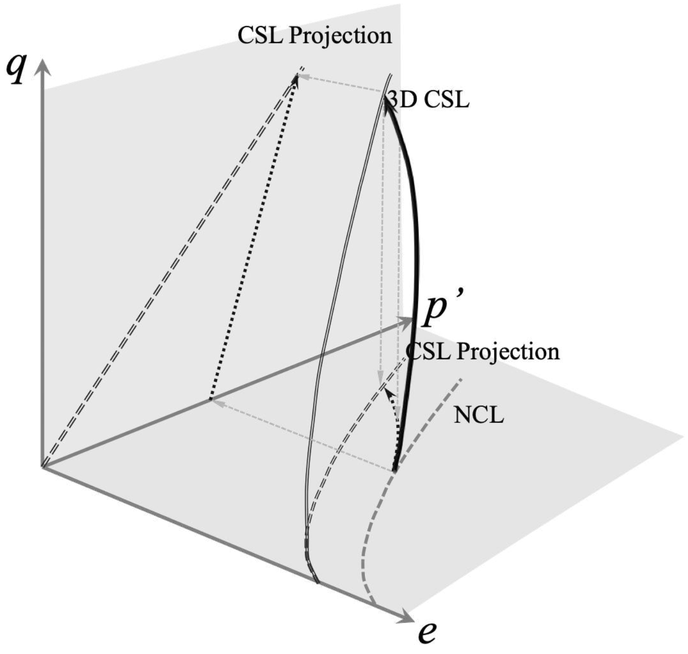

Figure 6. Typical stress path – ICDTx – NC/Loose - Contractive

3 CONSIDERATIONS

In addition to understanding the critical state concept and the various triaxial tests and stress paths, there are numerous other considerations that practitioners should be mindful of. Based on the authors' experience, these considerations are more practical, rather than theoretical, as the inability to determine the CSL typically lies in the practical execution of the testing and the laboratory's attention to detail, rather than the interpretation of results presented. The following should be considered:

3.1 *Number of tests*

There is no specific guidance for the number of triaxial tests required to successfully determine the CSL. The successful determination is based on the quality of the testing and the interpretation thereof. Higher quality test results will likely require a fewer number of overall tests. On the other hand, for research related CSL determination, a larger number of tests may be appropriate as shown in Figure 7 where Verdugo & Ishihara (1996) carried out 55 tests for the CSL determination of Toyoura sand.

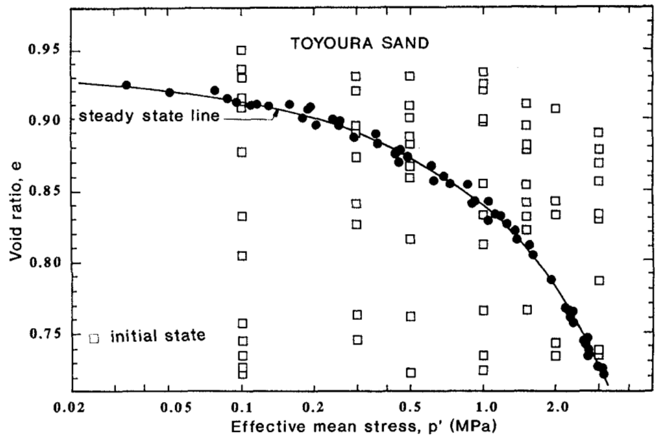

Figure 7. Toyoura sand critical state line (Verdugo & Ishihara 1996)

Jefferies & Been (2015) recommend a minimum of 10 tests approached from various initial states as shown in Figure 8.

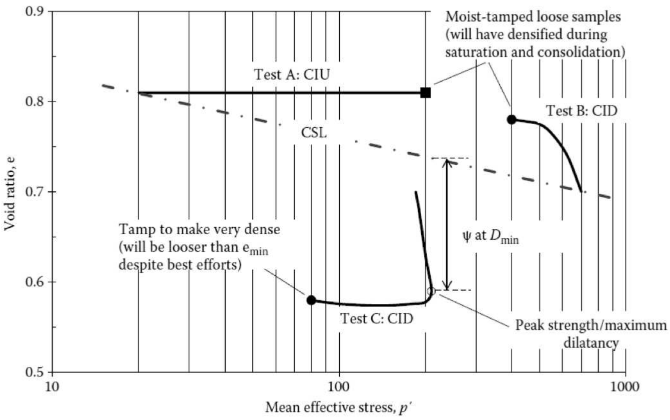

Figure 8. Conceptual minimum CSL test programme (Jefferies & Been 2015)

Through their round-robin CSL testing regime Reid et al. (2021) showed that the average number of tests required to successfully determine the CSL is 6.

Based on the authors' experience a minimum of 6 – 8 are required depending on the quality of the laboratory tests, while allowance should be made for 10 tests to mitigate this risk.

3.2 *Preferred soil behaviour*

There is a tendency to think that the CSL should only be approached from the contractive side. The reason for this is likely due to the majority of triaxial tests carried out are undrained and it is in many instances easier to prepare looser or normally consolidated specimens. Drained tests on dilative samples are as important in the CSL determination as contractive specimens. The reason being that drained dilative test results are required for constitutive model calibration. Although dilative samples do not generally reach critical state, these tests can be strategically specified to reach critical state, or very close to it.

As such, specifying triaxial tests for CSL determination should consider the objective of the testing as well as the calibration of the constitutive model. Each individual test is as important as the overall objective.

3.3 *Strain percentage*

The percentage strain to reach critical state is a topic that has received limited attention. If tests are carried out according to conventional testing standards (ASTM D 4767, BS 1377-8) the test will likely be stopped at 15% strain. Although there is no correct answer, the increased attention on CSL testing has shown that 15% is often not sufficient. The authors' personal preference is a minimum of 25% strain with a tendency to aim for 30%. There are, however, practical issues that come into play and such high strain percentages may not be possible. On the contrary, critical state may not have been reached even at strains of 30%.

The authors advise that judgement should be applied, while considering commercial aspects, duration of tests, and the capabilities of the laboratory.

3.4 *Constitutive models*

The constitutive model to be employed should also be used to guide the testing for CSL determination. Certain models require specific tests that should be included in the test programme. In addition, the theoretical framework and interpretation of the critical concept within the model should guide the test requirements. A fundamental understanding of the model and the critical state framework is key to successful CSL determination.

3.5 *Was critical state reached?*

The question that should be asked when analysing test data is whether critical state was reached, which requires a fundamental understanding of the critical state framework. Specimens may have reached critical state from a strength perspective, however, from a volume change or pore water pressure perspective this may not be the case. In addition to understanding the critical state concept, the readers are referred to the work of Torres-Cruz (2019) and Reid et al. (2021) for guidance when critical state is reached.

3.6 *Void ratio tracking*

Void ratio tracking is perhaps the most important aspect when determining the CSL. The final void ratio of triaxial specimens is vital to successful determination of a CSL with high degree of confidence. The authors' have addressed this issue in detail in a separate paper also presented at this conference (Vermeulen and Archer 2025) to which the readers are referred.

The considerations discussed in this section relates to specific elements of triaxial testing. Jefferies and Been (2015) provides the following general considerations for successful CSL testing:

- Uniform samples must be prepared
- Specimens must be fully saturated.
- The void ratio must be known accurately (to within about ±0.003).
- The measurement system must be capable of measuring low stresses as well as pore pressures at a high rate with very little system compliance.

Related to the tracking of the void ratio, is the initial prepared void ratio. Samples are prone to collapse during the saturation phase, and selecting an initial void ratio to limit the collapse is key. Further discussion on this is presented in Vermeulen & Archer (2025).

4 RECOMMENDED APPROACH

Based on the authors' experience, a CSL determination approach was developed taking into account the considerations discussed in Section 3. The approach was developed through trial and error and working closely with the laboratory to improve the quality of results.

The basic approach involves targeting the CSL through both contractive and dilative samples, while considering different initial states as shown in Figure 9. Although the basic approach seems broad and open for interpretation, it reiterates the necessary consideration that there is no "one size fits all solution" and that the CSL should be approached using judgement, from multiple directions.

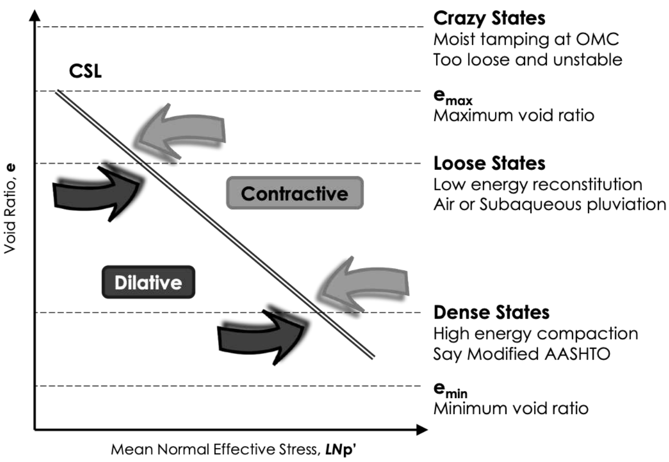

Figure 9. Basic CSL determination approach

The current recommended methodology advocated by the authors can be summarised in the following steps:

1. The first test is a ICUDTx test with a stage consolidation phase as shown in Figure 10. The stage consolidation phase includes loading and unloading stages and typical effective stresses include, 50, 100, 200, 300, 100 (unload stage), 300, 400, and 600 or 800 kPa. The specimen is then sheared at 600 kPa or 800 kPa. Test 1 allows for a contractive specimen, with the stage consolidation providing information on the normal consolidation line and swelling line.
2. The second test is also a ICUDTx test but consolidated to 100 kPa only and then sheared undrained as shown in Figure 11. The aim of this test is to assess the behaviour at high void ratios, i.e. dilative or contractive.
3. The combination of Test 1 and Test 2 provides an initial indication of the CSL with two points on the CSL. As such, the remainder of the tests can be specified as a combination of drained and undrained tests at various initial states as shown in Figure 12.
4. Tests 3 – 8 should be carried out in pairs of two. The results should be analysed for a pair of tests before the instruction is given for the next two tests. This allows for continuous update of the CSL while addressing any shortcomings with the testing regime early on.

In addition to the steps outlined, the following is recommended:

- A minimum of 6 – 8 tests are typically required, while allowance should be made for 10 tests.
- As stated, the initial void ratio is key to the successful tracking of the void ratio by minimising the collapse during the saturation phase. The authors recommend an initial void ratio between 0.7 and 0.9.
- Continuous verification of the results as they become available is crucial, and close cooperation with the laboratory is required.
- Void ratio tracking is key and should be conveyed as such to the laboratory. Refer to Vermeulen and Archer (2025) for more detail.
- Keep the objective of the testing in mind, i.e. only CSL testing or constitutive modelling as well.

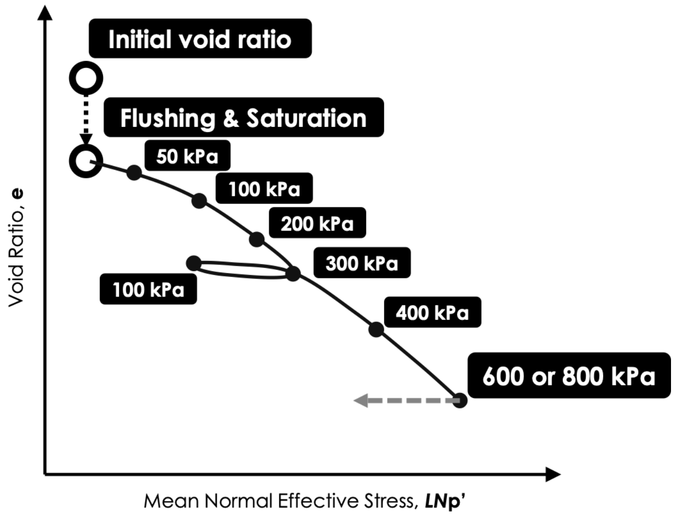

Figure 10. Recommended CSL methodology – Test 1

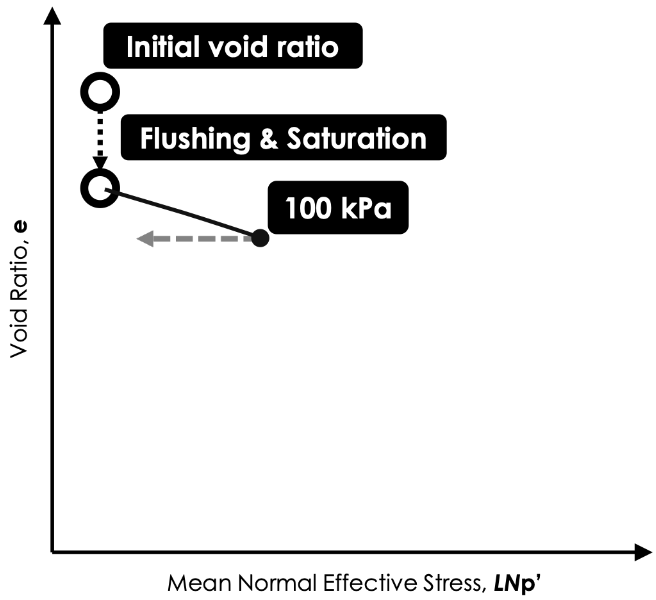

Figure 11. Recommended CSL methodology – Test 2

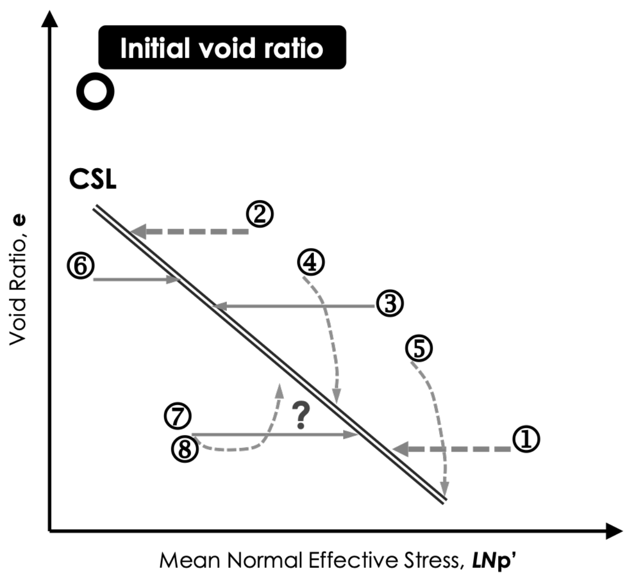

Figure 12. Recommended CSL methodology – Tests 3 - 8

5 CONCLUSION

The authors presented a brief review of CSL determination, including various aspects that should be considered for a successful CSL testing campaign. A CSL determination approach is presented based on the authors experience through numerous CSL testing campaigns.

In addition to the recommended approach outlined in Section 4, the various CSL testing campaigns have highlighted the following general aspects that should be considered:

- Understand what needs to be achieved with the CSL determination. Is the assessment only to have a reference to assess in-situ stress conditions, or will the data be used to calibrate a constitutive model?
- Work closely with the laboratory to ensure high quality testing and accurate tracking of the void ratio. Ensuring the laboratory understands the objectives and required outcome goes a long way to ensure high quality test results.
- Drained dilative tests are as important as undrained contractive tests, and the CSL should be determined with both test types. As such, the CSL should be approached from both contractive and dilative states.
- Always bear in mind that there may only be one chance to obtain samples for testing. As such, there should be limited compromise on the quality of testing and the associated cost.

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