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# Theoretical Analysis of the Relationship between Heave and Net Heat Extraction Rates Based on Freezing Experiments

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## ABSTRACT

*In order to improve the current design of roads against frost action, the Swedish Transport Administration (Trafikverket) has initiated a research programme. The main goals of the research are to revise the existing frost design models and the frost susceptibility classification system for subgrade soils.*

*A qualitative theoretical analysis to establish a relationship between frost heave and net heat extraction rates based on experimental data has been done. Experiments were carried on disturbed (hand compacted), saturated samples of same type of soil without any overburden. Several different cold end temperatures were applied to create different boundary conditions to make a more detailed analysis.*

*Results were analysed and compared to those of other researchers while pointing out the similarities and differences. Potential reasons for these differences have been identified. Based on the findings of the experimental work, suggestions for improvements are given for future testing. Some preliminary results providing hints for the relationship between segregational heave and net heat extraction rates were obtained. At the end it was shown that there exists a significant difference between the findings of the experimental work and the current system being used in Sweden in order to quantify heave.*

**Keywords: Frost Action in Soils, Laboratory Freezing Tests, Frost Depth, Frost Heave, Heat Flow in Soils**

## 1 INTRODUCTION & BACKGROUND

The relationship between heave and net heat extraction rates at the frost front has been investigated by several researchers at different points during the history of frost action and cold region soil mechanics studies.

Two kinds of conclusions have usually been reached; while the first group of authors (Beskow, 1935; U.S. Army Corps of

Engineers, 1958; Loch, 1977; Hermansson, 1999) has claimed that there exists no direct relationship between heave and net heat extraction rates, the second group (Kaplar, 1970; Penner, 1972; Horiguchi, 1978; Loch, 1979; Konrad, 1987) has tried to show the opposite.

The subject has been reviewed once again in Sweden for the research programme BVFF (Bana Väg För Framtiden) sponsored by the Swedish Transport Administration

(Trafikverket) in order to improve the existing methods of heave estimation and frost susceptibility classification. This necessity to improve the current design can be better understood with the help of Figure 1.

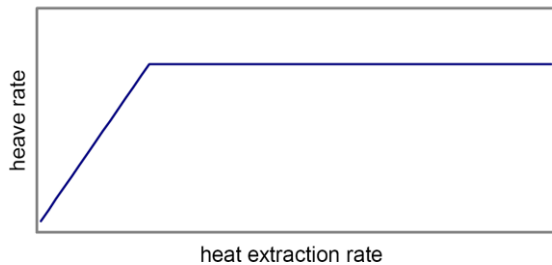


Figure 1 Relationship between heave rate and heat extraction rate at the frost front used in the current Swedish practice (Hermansson, 1999)

Firstly, no attempt is made in the current frost heave model used for pavement design to separate segregational heave from the total heave. In other words, displacements due to heaving are assessed as a whole rather than being treated separately as primary and secondary heave. The importance of secondary heave (also termed as segregational heave) will be explained in the next section. It should be noted that trying to establish a relationship between total heave and net heat extraction rates might result in contradictory results (Konrad, 1987).

Secondly there exists a threshold value of heat extraction rate in Figure 1, which is used in the existing model. Up to this threshold value heave rate is assumed to be proportional with the heat extraction rate. When the threshold is exceeded, heave rate is assumed to be constant and not to be affected by the heat extraction rate. As a result, at relatively higher heat extraction rates the same amount of heave will be estimated which might not necessarily be the case. For such high rates it might also result in overestimation of heave.

To address all these issues a comprehensive experimental study has been undertaken including the development of a freezing test apparatus and laboratory freezing tests.

To this end, laboratory testing of disturbed soil samples under different temperature gradients (i.e. different heat extraction rates) has been conducted. Qualitative assessment of the results has been done. The main focus was on establishing a relationship (by means of plot trends) rather than the numerical accuracy. Details of the experimental work along with the theoretical analyses of results are presented in their respective sections.

## 2 EXPECTED OUTCOME

Beskow (1935) has demonstrated the importance of secondary (segregational) heave by proving that the volume expansion upon freezing is not the main cause of frost heave by experimenting with benzene (benzene is a liquid that shrinks upon freezing). He showed that samples saturated with benzene can still experience significant heave.

Furthermore, ice lens formations in a frozen soil body also provide hints about the significance of segregational heaving.

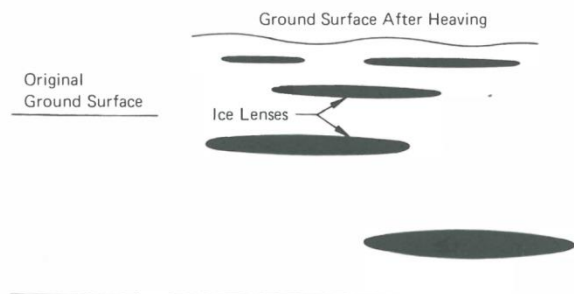


Figure 2 Characteristics of ice lenses and frost heaving (Mitchell, 1976)

Relatively thicker ice lenses observed at the lower depths of a soil body, shown in Figure 2, can be attributed to the segregational heave concept. At the beginning of the winter period where the advancement of the frost line is rapid due to higher heat extraction rates, the thickness of the ice lenses being formed are relatively low. This can be explained due to the lack of time it takes for the surrounding water to reach the frost front due to high frost penetration rates. Towards the end of the winter period, however, frost front almost comes to a halt (i.e. quasi-

stationary) and there usually is enough time for the water to be drawn to the freezing front which causes the formation of thicker ice lenses.

Therefore one might expect that the relationship between segregational heave and heat extraction rates to be of parabolic nature with a peak and approaching to zero for very high and very low heat extraction rates due to the reasons discussed above. This has been verified by the work carried out by Konrad (1987) and can be seen in Figure 3.

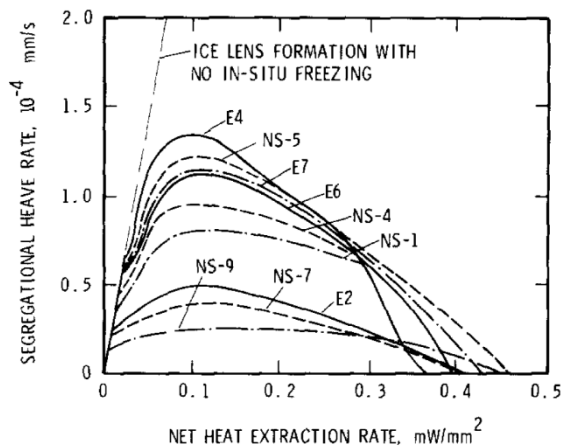


Figure 3 Segregational heave rate versus net heat extraction rate (Konrad, 1987)

Figure 3 shows Konrad's (1987) results in freezing experiments with different temperature boundary conditions conducted on one type of soil (Devon silt). According to this, the samples were found to experience different heave rates for different heat extraction rates with a very clear peak. Interestingly, it has also been found that the same type of soil might undergo different heave rates under a certain value of heat extraction rate. The reason why the same type of soil is experiencing different heave rates for a fixed value of heat extraction is outside the scope of this work but can be attributed to different sample heights and temperature boundary conditions used during experiments.

An in-depth study of segregational heave requires a detailed analysis of water that is being drawn to the frost front during freezing. For this purpose, the movement of water in the surroundings to the frost front

has been investigated by different researchers (Loch, 1979; Ito et al., 1998) and their findings are presented in Figure 4 and Figure 5. In these figures the water is shown to be drawn to the frost front at relatively low rates at the beginning phases of the experiment where the heat extraction rate is higher. As the experiment progresses, water is found to be drawn at higher rates, reaching a peak value and decreasing gradually from then on. Importance of this peak and the time it takes to reach it will be discussed in detail in the coming sections.

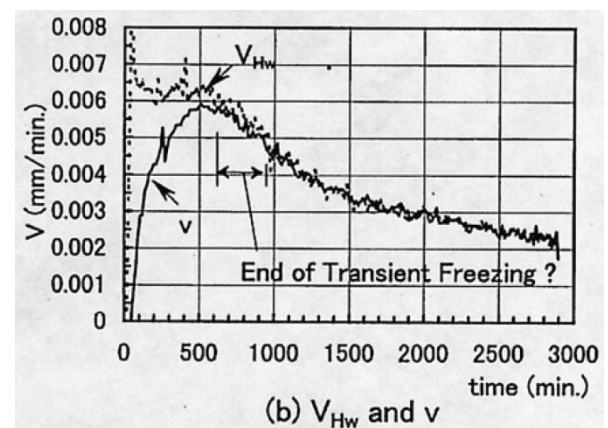


Figure 4 Water intake measurements from a step freezing test (Ito et al., 1998)

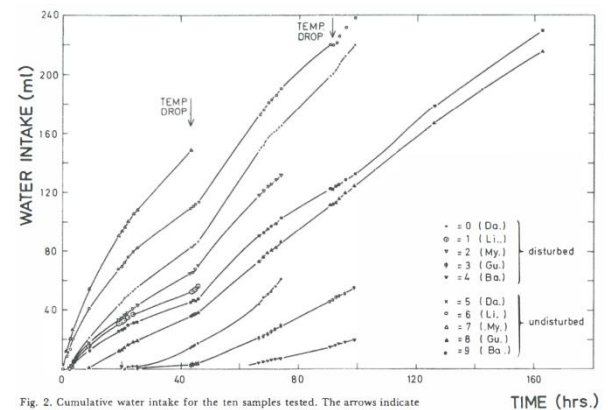


Figure 5 Water intake measurements during freezing tests (Loch, 1979)

### 3 EXPERIMENTAL WORK

#### 3.1 Testing Apparatus

In order to further investigate the relationship between heave and net heat extraction rates, a testing equipment has been constructed. Details of the experimental apparatus are explained in the work by Zeinali et al. (2016).

During frost testing, freezing takes place one dimensionally (from sample top to bottom). The sample is insulated from sides in order to minimize heat losses from other dimensions. There are two cooling units supplying cold and warm temperatures to the top and bottom of the sample, respectively. Temperature sensors are placed (penetrating into the specimen) uniformly within the sample in order to keep track of the temperature profile during the test. There is also a supply of water from the sample bottom, so that the water can be drawn from the surroundings to the frost front. Displacements (due to heaving) are recorded by means of a displacement transducer (LVDT) located at the top of the sample. All measurements (temperature data, water intake and displacements) are recorded by means of a data acquisition system during the test which is connected to a computer for further analyses.

#### 3.2 Soil Properties

The same type of soil (sandy silt) has been used under different temperature boundary conditions during freezing tests. The particle size distribution (psd) of the soil is given in Figure 6.

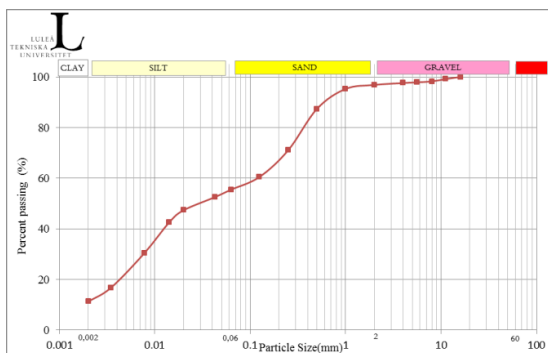


Figure 6 Particle size distribution of the soil tested

The main intention of using the soil which is characterized by the particle size distribution curve in Figure 6 is to conduct freezing tests on a relatively frost susceptible soil. Based on the psd and the silt content it can be concluded that the soil exhibits some degree of frost susceptibility. The specific gravity ( $G_s$ ) of the soil was 2.68.

#### 3.3 Testing Procedure

All tests have been conducted on disturbed samples. Soil samples were prepared by means of hand compaction in the test cell (cylindrical cell with  $\phi=10\text{cm}$ ) by compacting five equal layers of soil to a sample height of about 10cm. As a natural outcome of the compaction method, all the samples had porosity values of about  $n=0.38$ .

In order to avoid problems of redistribution of water during freezing within the specimen and to make analyses relatively easier all tests have been conducted under saturated conditions. Samples were saturated by allowing water movement from the bottom of the sample to the top; under very low hydraulic gradients in order to prevent particle sorting within the sample.

Once the samples were saturated, they were brought to a steady state thermal equilibrium where the temperatures are constant (within the range of 3-5  $^{\circ}\text{C}$ ) along the specimen. After the steady state phase, cold temperatures were applied on the sample top while keeping the temperature at the sample bottom constant at +3-4  $^{\circ}\text{C}$ . The freezing phase usually takes 4 days in order to ensure that there is sufficient time for water intake, development of a thermal equilibrium and formation of ice lenses. During the freezing phase there is free access to water (open system).

In addition, another freezing test with varying temperature boundary conditions has been conducted for comparison purposes. For this test different freezing temperatures have been used. Similar to the case described above, the soil was frozen with a fixed temperature on top and enough time was allotted for the

sample to reach steady state conditions and for the water to be drawn to the frost front. At this point the temperature at the top of the specimen was further reduced to an even lower value to create a new freezing condition. Again, enough time was given for the sample to reach steady state and for water to be drawn to the frost front. The aim with this kind of freezing was to make the tests less time consuming as suggested by Penner (1972).

Temperature boundary conditions for different tests are given in Table 1. Typical plots of the temperature data (steady state + transient freezing phases) and displacements from all tests are given in Figure 7 and Figure 8.

Table 1 Summary of the experimental procedure

Test Type		Cold End Temp. (°C)	Warm End Temp. (°C)	Sample Height (cm)
Single Gradient		-3.2	3.5	10
Multiple Gradients	Step #1	-1.4	2.4	10.2
	Step #2	-2.4	2.4	10.2
	Step #3	-3.4	2.4	10.2
	Step #4	-4.4	2.4	10.2

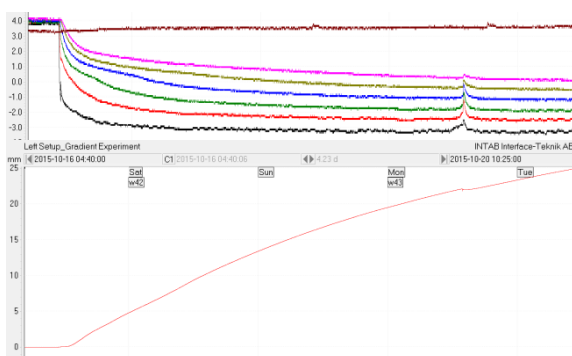


Figure 7 Temperature and displacement data for the single gradient freezing test

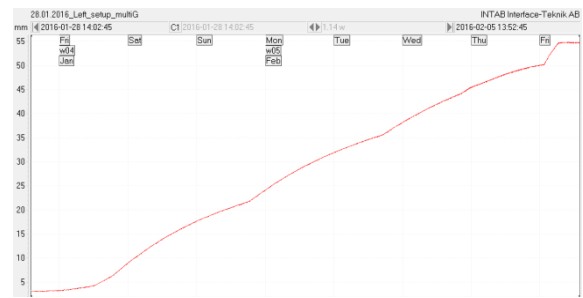
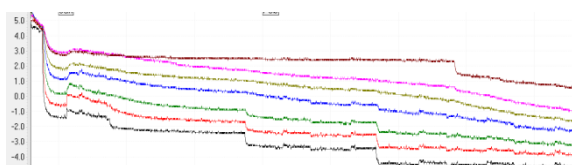


Figure 8 Temperature and displacement data for the multiple gradient freezing tests

#### 4 ANALYSIS RESULTS

In order to establish the relationship between heave and net heat extraction rates for different tests conducted under different temperature gradients, series of theoretical analyses were done. These can be grouped under three categories:

- Calculation of frost depth and frost penetration rate
- Evaluating heave and dividing heave into two parts, namely, “in-situ” heave (due to freezing of pore water) and “segregational” heave (due to water being sucked to the frozen front).
- Calculation of net heat extraction rate at the frost front.

Each of these analyses is treated under its respective section. For the sake of simplicity, the analyses of results for one of the tests have been described in detail. It should be noted that the analyses results given in the forthcoming sections are qualitative, not quantitative. In other words, the accuracy of the numbers is not the primary concern. Therefore, the aim here is to establish a theoretical relationship between heave and net heat extraction rates.

##### 4.1 Calculation of Frost Depth and Frost Penetration Rate

Based on the temperature data given in Figure 7, it is possible to keep track of the frost (0 °C) line. This is done by comparing temperature readings between two adjacent thermocouples and locating the position of the frost line by means of interpolation (see Figure 9). Figure 9 shows that for results

given in Figure 7 frost line penetrates roughly 8.5 cm into the sample.

The main reason for the downward spike (around  $t=75h$ ) in Figure 9 is the minor disturbance recorded by the thermocouples. This is mainly due to an insulation problem around the sample for a very brief moment and has been fixed immediately during the test. It can be seen in Figure 7 that the readings have returned back to their previous values. Thus, the disturbance is believed to have a negligible effect on the results overall.

By fitting a line into the frost depth vs. time plot and taking the derivative, one can obtain rate of frost penetration. This is done in Figure 9 and Figure 10.

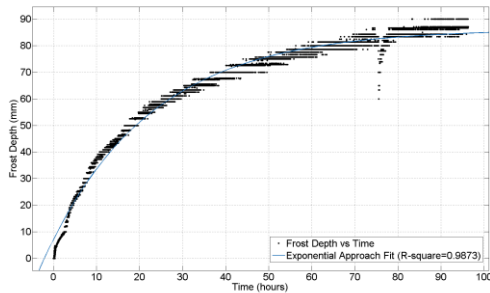


Figure 9 Curve fitting for the frost depth vs. time plot

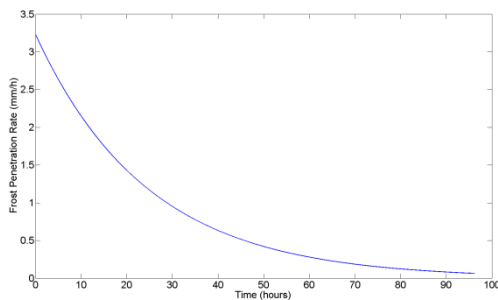


Figure 10 Frost penetration rate vs. time

As expected, the rate of frost penetration is largest at the beginning of the experiment and gradually reduces down to an almost zero value, indicating stationary frost front conditions have been reached towards the end of the experiment.

#### 4.2 Evaluation of Heave

In order to make the analyses as detailed as possible, the total displacement (due to heaving) recorded by LVDT is sub-divided

into two components. This can be done in two ways:

- Calculation of “in-situ” heave based on frost penetration rate; which is then subtracted from total heave to obtain “segregational” heave.
- Calculation of “segregational” heave based on water intake measurements; which is then subtracted from total heave to obtain “in-situ” heave.

Due to unexpected problems with the water intake measurement system the first approach was chosen.

For an open system freezing experiment the total heave rate consists of two components and can be calculated as follows:

$$\frac{dh}{dt} = \underbrace{0.09n \frac{dz}{dt}}_{\text{in-situ heave rate}} + \underbrace{1.09v}_{\text{segregational heave rate}} \quad (1)$$

Where  $n$  is porosity,  $\frac{dz}{dt}$  is the rate of frost penetration (mm/h) and  $v$  (mm/h) is the water intake velocity. Thus, heave rate only due to freezing of pore water can be calculated as:

$$\frac{dh_i}{dt} = 0.09n \frac{dz}{dt} \quad (2)$$

Based on the expression above, the plot of in-situ heave rate vs. time is given in Figure 11.

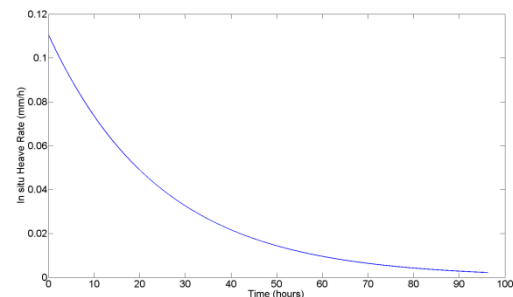


Figure 11 In-situ heave rate vs. time

Integrating the plot given in Figure 11, one can obtain the heave (mm) due to freezing of pore water. If this is then subtracted from the total heave, it is possible to obtain segregational heave, see Figure 12 and Figure 13.

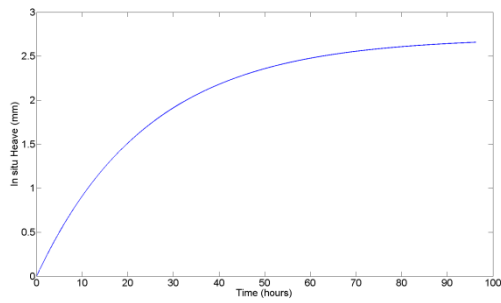


Figure 12 In situ heave vs. time

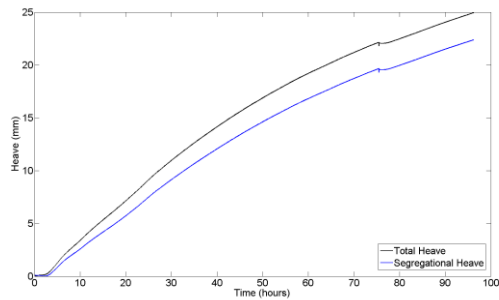


Figure 13 Separation of segregational heave from total heave

Figure 13 is important for two aspects: First, it allows for further treatment of the segregational heave curve to obtain segregational heave rate and water intake velocity. Secondly, it demonstrates the significance of segregational heave. By looking at Figure 13, it is possible to infer that a very large portion of total heave is due to the freezing of water that has been drawn to the freezing front from surroundings.

If a curve is approximated for the segregational heave plot given in Figure 13, it is possible to obtain the segregational heave rate. However, there is one important step that should not be overlooked while doing so. Although it might be tempting to approximate the plot with a single exponential function, doing this will result in a significant flaw for the rest of the analyses. An attempt to clarify this is made in Figure 14 and Figure 15.

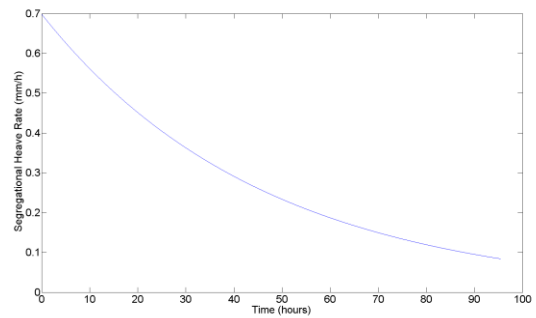


Figure 14 Calculation of segregational heave rate based on a single exponential curve fit

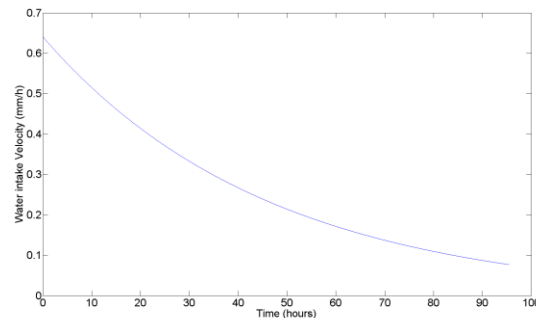


Figure 15 Water intake velocity vs. time plot calculated based on a single exponential curve fit

Figure 14 and Figure 15 would have been obtained if one took the derivative of the exponential function that has been fit to the segregational heave plot in Figure 13. The problem becomes more apparent upon closer inspection of Figure 15. Water intake velocity can be calculated simply dividing segregational heave rate by 1.09 (see Equation 1). However, it does not physically make sense to start with a relatively high value of water intake velocity at the very beginning of the experiment. One would ideally expect the water intake velocity to start from zero at the beginning of an experiment.

The fact that the water intake data should start from zero suggests that the segregation heave curve given in Figure 13 should at least be approximated by two different curves. Thus, in an attempt to estimate the segregational heave rate, segregational heave curve has been approximated by two different fits. The results are given in Figure 16 and Figure 17.



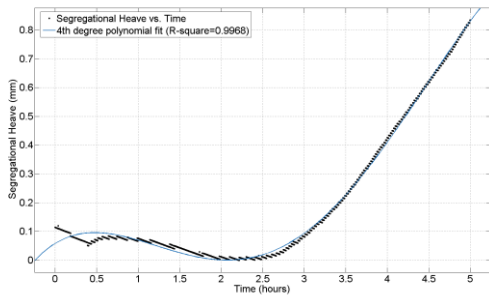


Figure 16 Approximation for the beginning stages of segregational heave curve

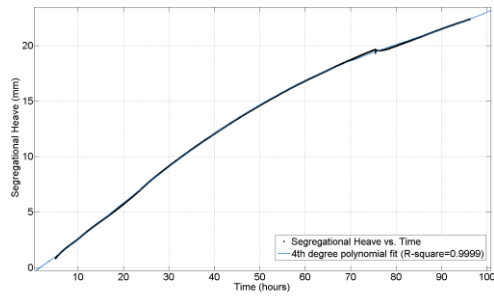


Figure 17 Approximation for the second part of segregational heave curve

Figure 16 demonstrates that the segregational heave rate increases quite rapidly at the very beginning stages of an experiment (during the very first hours of experiment). Later on, the rate of segregational heave reduces gradually. The rate of change in segregational heave can be represented by the plot given in Figure 18.

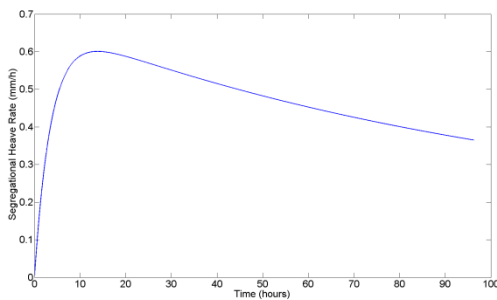


Figure 18 Segregational heave rate vs. time

Figure 18 is generated by taking the derivative of the two curves fit to two parts (Figure 16 and Figure 17) of the segregational heave curve. The derivative functions have then been merged under one plot and approximated by a single curve. There is no doubt that some accuracy has been lost during these approximations, however, the general appearance of the plots (or the trends) still hold true.

Finally, the water intake curve can be obtained dividing segregational heave rate by 1.09. This is given in Figure 19.

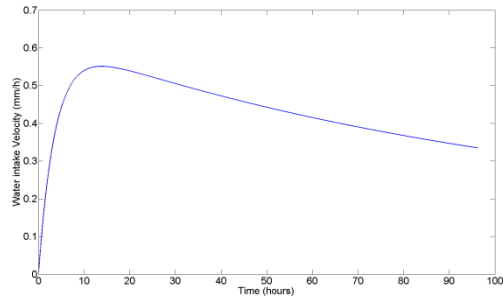


Figure 19 Water intake velocity vs. time

### 4.3 Calculation of Net Heat Extraction Rate

Having calculated the frost penetration rate and the water intake velocity, one can calculate the net heat extraction rate. The expression to calculate the net heat extraction rate is as follows:

$$q_z = Ln \frac{dz}{dt} + Lv \tag{3}$$

Where,  $q_z$  is the net heat extraction rate ( $W/m^2$ ) and  $L$  is the volumetric latent heat of water ( $J/m^3$ ). Based on this, variation of net heat flux during the experiment is given in Figure 20.

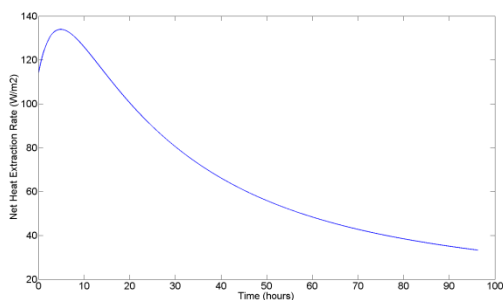


Figure 20 Net heat extraction rate vs. time

With all parameters obtained, the relationship between heave and net heat extraction rates can now be established. This relationship between net heat flux vs. segregational and total heave rates are plotted in Figure 21 and Figure 22, respectively.

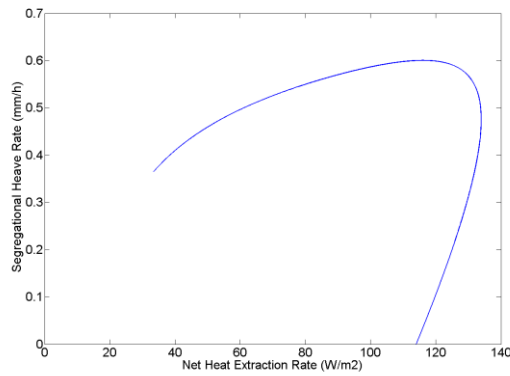


Figure 21 Relationship between segregational heave and net heat extraction rate

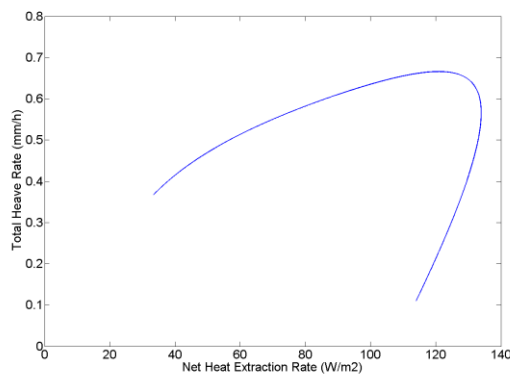


Figure 22 Relationship between total heave and net heat extraction rate

Similarly, the results of the multiple temperature gradient experiments are plotted in Figure 23. Unfortunately, only step numbers 2 and 3 in Table 1 could have been evaluated as the rest of the data were disturbed by outside factors.

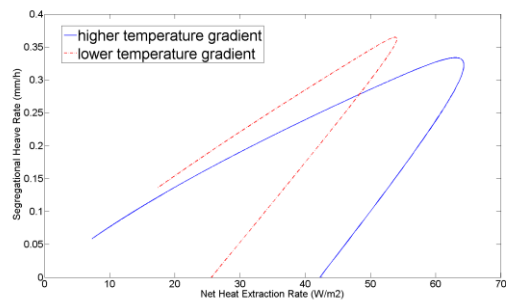


Figure 23 Relationship between segregational heave and net heat extraction rate

## 5 DISCUSSION

Figure 21, Figure 22 and Figure 23 are somewhat unconventional in the sense that the beginning of the experiment is represented by the data point located at the

bottom right of the net heat extraction rate axis. This can be better understood by the help of Figure 20 as the higher rates of heat extraction takes place at the beginning of the experiment and decreases gradually as the test goes on. Keeping this in mind, it can be deduced that higher extraction rates do not always necessarily give the highest heave rates. Furthermore, there seems to be a peak where the highest rate of heave occurs. Increasing the rate of heat removal beyond this point has a negative impact on the heave rate.

The general plot trends obtained in Figures 21, 22 and 23 are also important for discussion. A quick comparison between these figures and Figure 3 (Konrad, 1987) reveals that their shapes are somewhat distorted and gives the impression that the soil might experience different heave rates for a fixed value of heat extraction rate in the early stages of an experiment. It should be pointed out that the shapes of these plots are heavily influenced by their respective water intake vs. time (see Equation 3) curves. For example, in Figure 19 the peak is occurring around the 10 hour mark. If this was happening at a later point (say, 20 hours for instance) the shapes of Figures would have been a lot similar to what one would usually call “normal”. Due to the problems in the water intake measurement system at the time, the authors had no means of accurately determining the location of this peak. Existence of the peak is intuitive, but its location is heavily influenced by the back calculation procedure. As a result, the accuracy of the beginning phases of these plots is affected significantly.

The distinction between the terms *higher* and *lower* thermal gradients that appear in Figure 23 can be better understood with the help of Figure 24. In Figure 24 approximate temperature profiles along the frozen part of the soil body at the end of each testing step is plotted. The initial location of the frost front at the beginning of step #2 experiences a temperature change of  $\Delta T_2$  during that step. Similarly, the initial location of the frost front

at the beginning of step #3 experiences a temperature change of  $\Delta T_3$  during that step. Since the penetration of the frost line is almost identical between these two steps, it can be concluded that the rate of heat extraction is greater for the second step than that of the third.

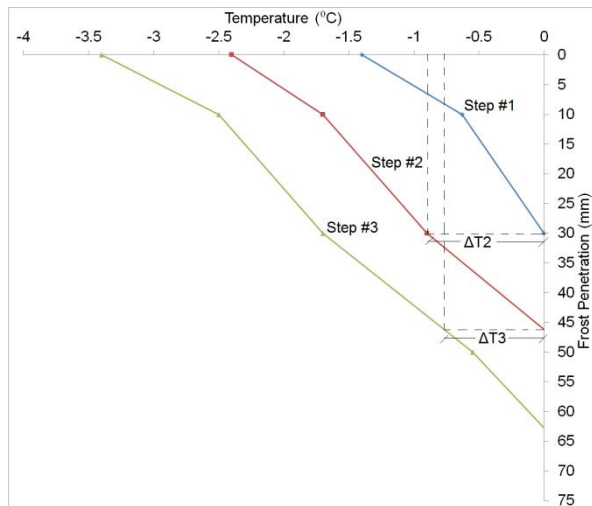


Figure 24 Frost penetration and temperature profiles for multiple gradient tests

Moreover, in Figure 23 there are also some hints that the same type of soil is experiencing similar heave rates for a given rate of heat extraction located at the left side of the peak. The plots get even closer with decreasing heat extraction rates. It is also worth noting that the peaks are located remarkably close to each other. However, more experimental work with a larger variety of thermal gradients needed here to reach more solid conclusions.

Finally, the heat extraction required to freeze a completely unfrozen sample (single gradient experiment) is anticipated to be much higher than the heat extraction required to further freeze down an already partially-frozen sample (multiple gradient experiments) and this is thought to account for the large difference in the range of net heat extraction rate values between Figure 21 and Figure 23.

## 6 CONCLUSIONS & FUTURE WORK

A theoretical analysis of the relationship between heave and net heat extraction rates

has been carried out based on experimental data. It was expected and, to a degree, was shown that segregational heave might come to a halt for relatively high heat extraction rates. For such rates, heave is purely due to freezing of “in-situ” (or pore) water as there is not enough time for the water to be drawn to the freezing front since the frost front is penetrating quite rapidly.

Detailed analyses demonstrated the importance of accurately keeping track of the water intake during experiments. It is essential to be able keep track of the water intake more precisely in order to draw more solid conclusions. Not meeting this criterion was shown to influence the accuracy of the relationship between segregational heave and the net heat extraction rates due to the back-calculation procedure.

Keeping these in mind and considering the similarities between the trends seen in Figure 23, the focus will be on improving the water intake measurement system and continue investigating the relationship between segregational heave and net heat extraction rate as the future work.

## 7 ACKNOWLEDGEMENT

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## 8 REFERENCES

- Beskow, G. (1935). Soil freezing and frost heaving with special application to roads and railroads. Swedish Geol. Soc., Series C, No. 375.
- Cold Room Studies- Third Interim Report U.S. Army, Corps of Engineers. Tech. Rept. 43. 1958.
- Hermansson, Å. (1999). A New Simple Frost Model, Validated and Easy to Use, Proc. Tenth International Conference on Cold Regions engineering, Lincoln New Hampshire USA, August 16 - 19, 1999, (Ed.: Jon E. Zufelt), ASCE, Reston Virginia USA, pp. 199 – 210.

Horiguchi, K. (1978). Effects of the rate of heat removal on the rate of frost heaving. Proceedings of the International Symposium on Ground Freezing, Ruhr-University, Bochum W. Germany, pp. 25 - 30.

Ito, Y., Vinson, T. S., Nixon, J. F. & Stewart, D. (1998). An Improved Step Freezing Test to Determine Segregation Potential. Proceedings of the International Conference on Permafrost. Yellowknife Canada, Collection Nordicana No 55.

Kaplar, C. W. (1970). Phenomenon and Mechanism of Frost Heaving. Highway Research Record 304, pp. 1 - 13.

Konrad, J. M. (1987). The Influence of heat extraction rate in freezing soils. Cold Reg. Science & Technology, 14: 129-137.

Loch, J.P. G. (1977). Frost Heave Mechanism and the Role of the Thermal Regime in Heave Experiments on Norwegian Silty Soils. Norwegian Road Research Laboratory, Meddelelse nr. 50.

Loch, J.P.G. (1979). Influence of the heat extraction rate on the ice segregation rate of soils. Frost i Jord, 20: 19-30.

Mitchell, J. K. (1976). Fundamentals of Soil Behavior. New York: Wiley.

Penner, E. (1972): Influence of Freezing Rate on Frost Heaving. Highway Research Record, No. 393. pp. 56 - 64.

Zeinali, A., Dagli, D. & Edeskär, T. (2016). Freezing-Thawing Laboratory Testing of Frost Susceptible Soils. Proceedings of the 17<sup>th</sup> Nordic Geotechnical Meeting. Reykjavik Iceland.

